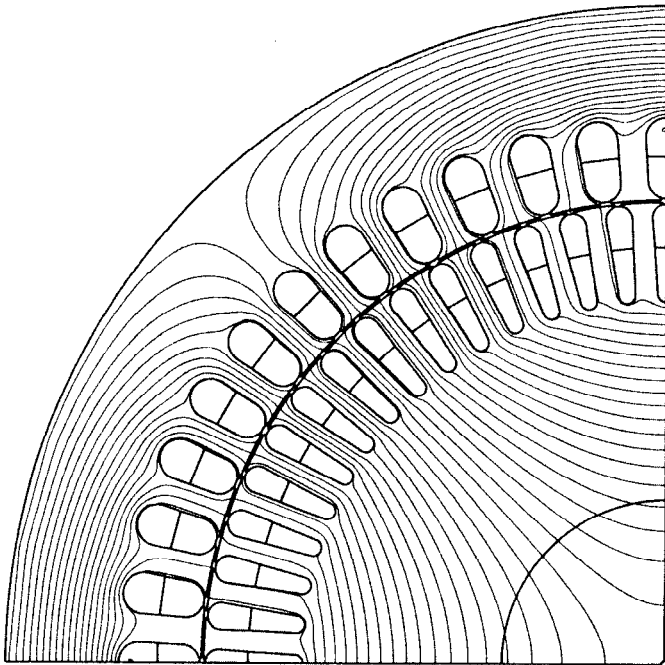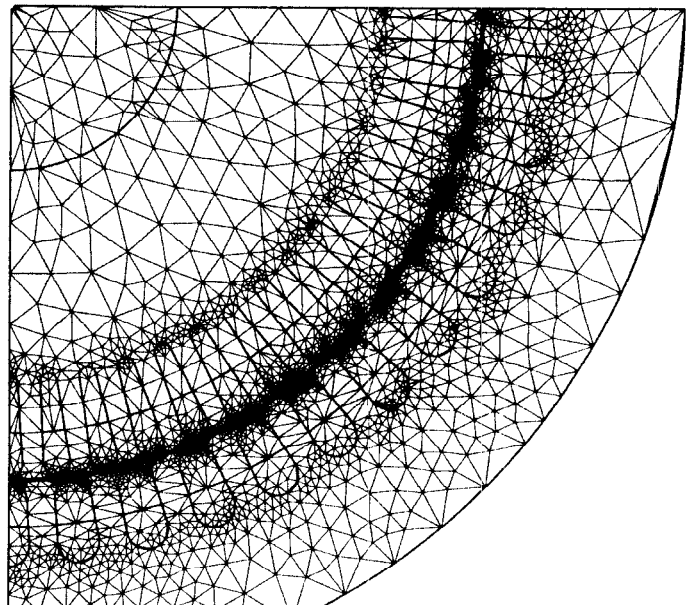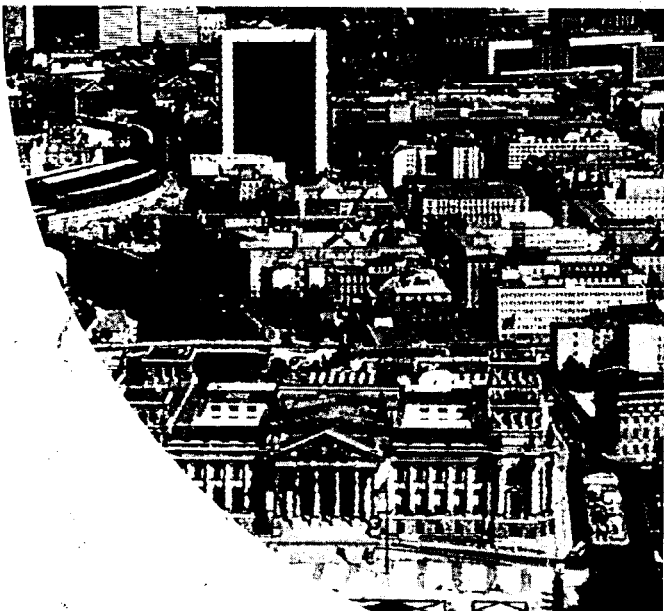# COMPUMAG

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Parameter computation of squirrel-cage induction motor models using finite elements

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Abstract- Induction motor analysis is mostly based on the use of either the equivalent circuit (for steady-state analysis) or the two-axis model (for dynamic behaviour). An extended motor model where each rotor and stator phase is considered to be linked with a winding of all other phases, offers a third possibility. The link between the different models is discussed in the paper. Depending on the model to be calculated, different approaches of the finite element method are considered.

INDUCTION MOTOR MODELS

In general, an induction motor can be represented by a set of differential equations describing the motor as a set of coupled windings:

$$[u] = [R][i] + \frac{d[\psi]}{dt} \tag{1}$$

$$[\psi] = [L][i]$$

where $[u]$ describes the voltages across the windings, $[R]$ a diagonal matrix containing the resistance values of the phases, $[L]$ the inductance matrix describing the coupling between phases, $[\psi]$ the flux vector and $[i]$ the current vector. When all motor phases are considered, the model is referred to as the coupled winding model. For a squirrel-cage induction motor a number of possibilities exists for defining the rotor phases. A single rotor bar is considered to be a rotor phase.

When the number of phases in the stator and the rotor is reduced to two and the rotor quantities are referred to the stator, the model is called the two-axis or d-q model.

$$
\begin{bmatrix} u_{sd} \\ u_{sq} \\ u_{rd} \\ u_{rq} \end{bmatrix} =
\begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}
\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} +
$$

$$
\frac{d}{dt}\left(
\begin{bmatrix}
L_M + L_{s\sigma} & 0 & L_M \cos(p0) & L_M \sin(p0) \\
0 & L_M + L_{s\sigma} & L_M \sin(p0) & L_M \cos(p0) \\
L_M \cos(p0) & L_M \sin(p0) & L_M + L_{r\sigma} & 0 \\
L_M \sin(p0) & L_M \cos(p0) & 0 & L_M + L_{r\sigma}
\end{bmatrix}
\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}
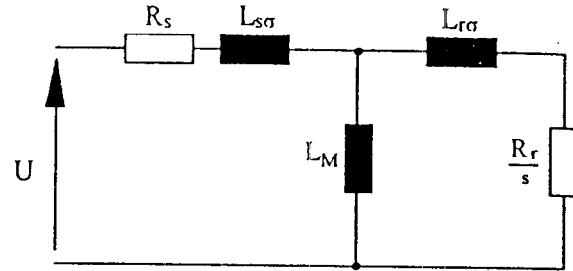\right)
$$

$$\tag{2}$$

Fig. 1: Induction motor per-phase equivalent circuit

For steady state analysis, the d-q model can be transformed to the per-phase equivalent circuit (Fig. 1). Both, d-q and the equivalent circuit model contain the same parameters. Therefore, a transformation between both models is trivial.

PARAMETER COMPUTATION

In the classical approach, the model parameters are calculated from analytical formulae [1]. Leakage inductances are split in different components and computed on a per phase basis for the stator or on a per bar basis for the rotor. The analytical approach has the disadvantage that not all rotor bar geometry's can be considered. To compute the magnetising inductance L_M the CARTER-factor is required. For special slot shapes, this factor is very difficult to assess. Also motors having closed rotor slots are difficult to consider analytically. During the last decennia, finite element techniques are used to overcome some of these drawbacks and adjust the analytical formulae. When the model parameters are calculated using only finite element analysis, different options exist.

Simulation of tests The parameters of the equivalent circuit or d-q model can be calculated by simulating a no-load and a locked rotor test [2]. Inductance values are calculated from the magnetically stored energy W_m in the model.

$$L = \frac{2W_m}{i^2} \tag{3}$$

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The advantage of this approach is a direct comparison with the measurements. The disadvantage is, that from the locked rotor simulation or measurement, rotor- and stator leakage can not be split. A more substantial disadvantage is, that the model contains parameters calculated at different operating points. Using the model for other points assumes that the parameters remain constant, which is not true in reality.

Calculation of the flux-linkages In this approach, a specific operating point for the motor is assumed. In a first step, the saturation level in each element is computed. This can be done using the method of the effective reluctivities [3] or an iteration process with static and time-harmonic solutions [4]. The calculated reluctivities are then frozen, the inductances can be calculated using the proper excitations. Self- and mutual inductances can directly be calculated from the vector potential A and the current density J in the model [5]

The calculation of the coupled winding model is straightforward. Each of the different phases is excited separately with a unit current. The flux linkages with the different phases immediately provides one row or column in the inductance matrix [L] in (1). The rotor quantities in this matrix are not referred to the stator. Due to slotting and saturation, the self inductances of the different stator or rotor phases are not equal. To transform the coupled winding model to the d-q model, the leakage component must be separated. If the rotor or stator consists of n phases, an average value for the self inductance $l + l_\sigma$ of one phase can be calculated as

$$l + l_\sigma = \frac{\sum\limits_{i=1}^{n} L_{ii}}{n} \tag{4}$$

where L_{ii} is the ith diagonal element of the calculated matrix [L]. For $n>2$, the leakage component l_σ can be calculated as

$$l_\sigma = \frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} L_{ij}}{n} \tag{5}$$

When a rotor phase is considered to consist of a rotor bar, (5) provides the bar leakage. By referring it to the stator, the rotor leakage $L_{r\sigma}$ is found. The magnetising inductance L_M needed in both, equivalent circuit and d-q model, can be calculated from the inductance l as

$$L_M = \frac{n}{2} l \tag{6}$$

The magnetising inductance may also be calculated in one step by exciting the stator with a unit current I_{unit} in the d-axis. The inductance is found by considering the voltage induced in the q-axis or by computing the flux-linkage according to the d-axis. In the latter case, the leakage inductance $L_{r\sigma}$ has to be substracted from the calculated value. For a three-phase winding with the U-phase aligned with the d-axis, the input currents for U, V and W-phase are

$$I_U = 2/3\ I_{unit},\ I_V = -1/3\ I_{unit},\ I_W = -1/3\ I_{unit}$$

The advantage of this approach is, that rotor and stator quantities are splitted, resulting in more accurate model parameters enabling a better motor control. Since the model is calculated at a certain operating point, the calculations have to be repeated to obtain the motor model in the overall operating range. This increase in computational cost is rewarded with an increase in accuracy.

CONCLUSIONS

The calculation of different induction motor models using finite elements is discussed. The method of the flux-linkages is shown to provide a more detailed motor model. Transformation from the extended coupled winding model to the equivalent circuit or d-q model is presented.

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