

Local Magnetostriction Forces for Finite Element Analysis

Koen Delaere, Ward Heylen*, Kay Hameyer, Ronnie Belmans

Abstract—The magnetic material's deformation caused by magnetostriction is represented by an equivalent set of mechanical forces, giving the same deformation to the material as magnetostriction does. This is done in a way similar to how thermal stresses are usually incorporated. The resulting magnetostriction force distribution is summed to other force distributions (external mechanical forces, magnetic forces) before starting the mechanical deformation or vibration analysis. This procedure is incorporated into a weakly coupled cascade solving of the magnetomechanical problem.

Index Terms—Coupled Problems, Finite Element Methods, Magnetomechanics, Magnetostriction.

I. INTRODUCTION

An important source of vibrations and noise in electric devices, rotating as well as non-rotating, are the deformations caused by magnetostriction (MS). These MS deformations can be of the same order of magnitude as the deformations caused by reluctance forces on the iron-air interface [1].

The incorporation of the MS effect in the (numerical) design process is usually impaired since detailed data on the magnetic material behaviour are hard to obtain. Versatile experimental methods to obtain all needed technical data on MS effects, permeability, losses, etc. are reviewed in [2]. Once the MS behaviour of the material is known, it has to be incorporated in the magnetic and mechanical analysis. First, the coupled magnetomechanical finite element model [3] is briefly reviewed and then it is illustrated how to use this model to take the MS material characteristic into account by a set of MS forces. The MS material characteristic, e.g. in $\lambda(B)$ form, MS strain λ as a function of magnetic flux density B , is assumed to be known.

II. THE MAGNETOMECHANICAL SYSTEM

Both magnetostatic and elasticity finite element methods are based upon the minimisation of an energy function. The total energy E of the linear electromechanical system consists of the *elastic energy* U stored in a body with deformation a [4] and the *magnetic energy* W stored in a magnetic system with vector potential A [5]:

$$E = U + W = \frac{1}{2} a^T K a + \frac{1}{2} A^T M A, \quad (1)$$

Manuscript received February 10, 2000. This work was supported by the "Fonds voor Wetenschappelijk Onderzoek Vlaanderen (FWOV)"; K. Delaere has a FWOV scholarship. The authors thank the Belgian Ministry of Scientific Research for granting the IUAP No.P4/20 on Coupled Problems in Electromagnetic Systems. The research Council of the K.U.Leuven supports numerical research.

K. Delaere, K. Hameyer, R. Belmans are with the Katholieke Universiteit Leuven, ESAT-ELEN, Kardinaal Mercierlaan 94, B3001 Leuven, Belgium, kay.hameyer@esat.kuleuven.ac.be.

W. Heylen is with the Katholieke Universiteit Leuven, Mechanics Dept.-PMA, Celestijnenlaan 200C, B3001 Leuven, Belgium, ward.heylen@mech.kuleuven.ac.be.

where K is the mechanical stiffness matrix and M is the magnetic 'stiffness' matrix. Considering the similar form of the energy terms, the following system of equations represents the numerically coupled magnetomechanical system:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \quad (2)$$

where T is the magnetic source term vector and R represents external forces (not of electromagnetic origin). The partial derivatives of the total energy E with respect to the unknowns $[A \ a]^T$ identify with the combined system (2):

$$\frac{\partial E}{\partial A} = M A + \frac{1}{2} a^T \frac{\partial K}{\partial A} a = T, \quad (3)$$

$$\frac{\partial E}{\partial a} = \frac{1}{2} A^T \frac{\partial M}{\partial a} A + K a = R, \quad (4)$$

where $T \delta A$ and $R \delta a$ represent energy input into the system. Using (2), (3) and (4), the coupling terms D and C are recognised as

$$D = \frac{1}{2} a^T \frac{\partial K}{\partial A}, \quad (5)$$

$$C = \frac{1}{2} A^T \frac{\partial M}{\partial a}. \quad (6)$$

The coupling term D is related to the elastic energy U by

$$D a = \frac{1}{2} a^T \frac{\partial K}{\partial A} a = \frac{\partial U}{\partial A} \quad (7)$$

and represents the increase in elastic energy U due to an increase in the magnetic field A , with deformation a held constant, e.g. caused by magnetostriction effects. The term D is used to represent MS in a *strong coupling* scheme [6], but will not be considered further, since here MS is dealt with by a *weak coupling* procedure.

The coupling term C (6) represents the dependency of magnetic parameters in M on the mechanical displacement a . Right-multiplying C by vector potential A immediately renders all magnetic forces F_{mag} (Lorentz forces as well as reluctance forces):

$$-C A = -\frac{1}{2} A^T \frac{\partial M}{\partial a} A = -\frac{\partial W}{\partial a} = F_{mag}. \quad (8)$$

This approach is extended to the non-linear magnetic case in [3].

III. MAGNETOSTRICTION FORCES

A. Concept

When the coupling term D is not used ($D=0$) and the magnetic forces $F_{mag} = -CA$ are shifted to the right hand side of (2), the system becomes uncoupled:

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} \end{bmatrix}. \quad (9)$$

Now MS effects are built into the analysis using a force distribution F_{ms} that can be added to R and F_{mag} . By *magnetostriction forces* F_{ms} we indicate the set of forces that induces the same strain in the material as the magnetostriction effect does. This approach is similar to the use of thermal stresses due to heating [7]. In calculating thermal stresses, the thermal expansion of the free body (no boundary conditions) is calculated based on the temperature distribution, and the thermal stresses are found by deforming the expanded body back into its original shape (or back inside the original boundary conditions). In calculating MS forces, the expansion of the free body (no boundary conditions) due to magnetostriction is calculated based on the magnetic flux distribution, and the MS forces are found as the forces needed to deform the expanded body back into its original shape (or back inside the original boundary conditions).

For FE models, this can be done on an element by element basis. The midpoint of the element is considered to be fixed. The MS deformation of the element, i.e. the displacement of the nodes with respect to the midpoint, is found using the element's flux density B^e and the $\lambda(B)$ characteristic of the material. If a set of $\lambda(B, \sigma)$ characteristics are given, one has to be chosen for the appropriate value of tensile stress.

B. Strain for Isotropic Materials

Fig.1 shows a typical MS characteristic for isotropic 3% SiFe (solid lines) as a function of tensile stress. For isotropic materials, the local xy -axis of the element are rotated so that the flux density vector \mathbf{B} coincides with the local x -axis [8]. The strains λ_x and λ_y in the local frame are then given by

$$\begin{aligned} \lambda_x &= \lambda \\ \lambda_y &= \lambda_t = -\lambda/2 \\ \lambda_z &= \lambda_t = -\lambda/2 \end{aligned} \quad (10)$$

where $\lambda = \lambda(B)$ is the MS strain in the direction of \mathbf{B} (x -direction) and λ_t is the MS strain in the transverse direction (y and z -directions). Usually, magnetostriction will leave the total volume and density unchanged [9], so that $\lambda_y = \lambda_z = -\lambda_x/2$. This volume invariance is equivalent to a magnetostrictive 'Poisson modulus' of 0.5, which is bigger than the mechanical Poisson modulus of about 0.3. Therefore, when the MS deformation is represented by a set of mechanical forces in the direction of the vector \mathbf{B} , there is always a set of forces perpendicular to \mathbf{B} to correct this difference in Poisson modulus (Fig.2). In a 2D plane strain analysis, the thickness (z -direction) of the material has to remain constant and an additional tensile z -stress needs to be applied in order to obtain $\lambda_z = 0$. This adjusts the values of λ in (10) to

$$\begin{aligned} \lambda_x &= \lambda + \nu\lambda_t \\ \lambda_y &= \lambda_t + \nu\lambda_t \\ \lambda_z &= \lambda_t - \lambda_t = 0 \end{aligned} \quad (11)$$

where ν is the mechanical Poisson modulus of the material and $\lambda_t = -\lambda/2$.

C. Strain for Anisotropic Materials

Fig.1 shows a typical MS characteristic for anisotropic M330-50A steel (dashed lines). As an approximation of the anisotropic behavior, the flux density vector is decomposed into a B_x and a B_y component in the element's local xy -axis, arranged so that the x -axis coincides with the rolling direction, and the y -axis with the perpendicular direction. The rolling direction curve $\lambda_{RD}(B)$ is then used with B_x as input, and the perpendicular direction curve $\lambda_{PD}(B)$ with B_y as input, giving

$$\begin{aligned} \lambda_x &= \lambda_{RD}(B_x) - \lambda_{PD}(B_y)/2 \\ \lambda_y &= \lambda_{PD}(B_y) - \lambda_{RD}(B_x)/2 \\ \lambda_z &= -\lambda_{RD}(B_x)/2 - \lambda_{PD}(B_y)/2 \end{aligned} \quad (12)$$

Depending on the actual anisotropic behavior of the material, a more accurate strain description can be used, e.g. taking MS shear λ_{xy} into account [8]. A similar correction as above can be made for the plane strain case.

D. Displacement

Still working in the local xy -axis, the strains λ_x, λ_y are converted into a displacement $a_{ms} = (a_{x,i}, a_{y,i})$ considering the midpoint of the element (x_m, y_m) as fixed:

$$\begin{bmatrix} a_{x,i} \\ a_{y,i} \end{bmatrix} = \begin{bmatrix} x_i - x_m \\ y_i - y_m \end{bmatrix} \begin{bmatrix} \lambda_x \\ \lambda_y \end{bmatrix}, \quad i=1,2,3 \quad (13)$$

where i indicates the three element nodes with co-ordinates (x_i, y_i) .

E. Magnetostriction Forces

The mechanical stiffness matrix K^e for an element gives, after multiplication with the MS displacement a_{ms} of the nodes, the nodal *magnetostriction forces*

$$F_{ms}^e = K^e a_{ms}. \quad (14)$$

Equation (14) has to be performed immediately and element by element (using K^e) and not for the whole mesh at once (using the global matrix K), because the N different displacements $a_{ms,j}, j = 1 \dots N$, due to MS in the N elements surrounding a node, should not be summed. It is however allowed to sum the forces F_{ms}^e on one node.

The MS forces are now introduced in (9) giving

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} + F_{ms} \end{bmatrix}. \quad (15)$$

First, the magnetic equation of system (15) is solved to give A , from which the magnetic forces F_{mag} are calculated using (8) and the MS forces F_{ms} using (14). Then the mechanical equation of (15) is solved to give deformation a due to all forces. This weakly coupled cascade approach suffices for

magnetostriction strains $\lambda < 250 \mu\text{m/m}$. For larger strains (in linear motors or actuators based on magnetostriction), the system (15) has to be iterated a few times, or a strong coupling with $D \neq 0$ has to be used [6].

The force distribution F_{ms} can also be used for any other kind of post-processing based on force distributions, e.g. calculating mode participation factors with stator mode shapes [3].

IV. EXAMPLE

Fig.3 shows the magnetic field in one pole of a six-pole synchronous machine. B_{max} in the teeth is 1.26 T corresponding to $\lambda = 2.3 \mu\text{m/m}$ for 3% SiFe with 1 MPa tensile stress. Fig.4 shows the magnetostriction forces on the stator for the magnetic field of Fig.3: Fig.4a for a stator of isotropic non-oriented 3% SiFe and Fig.4b for a stator of anisotropic M330-50A (both materials were modelled with Young modulus $E = 2.2 \cdot 10^{11}$ Pa and $\nu = 0.3$). In the areas of high flux density in the stator, there are MS forces parallel to the flux lines and also a set of MS forces perpendicular to the flux lines, both seeking to increase the circumference of the stator. In the anisotropic case, the general MS force pattern remains but the forces appear slanted. Fig.5 repeats Fig.4a but also shows the reluctance forces F_{mag} for the magnetic field in Fig.3. It can be seen that F_{ms} and F_{mag} are of the same order of magnitude (the size of the nodal force vectors on the teeth tips is 25 N).

V. REFERENCES

- [1] L. Läftman, *The contribution to noise from magnetostriction and PWM inverter in an induction machine*, Ph.D. thesis, Department of Industrial Electrical Engineering and Automation, Lund Institute of Technology, KF Sigma, Sweden 1995.
- [2] H. Pfützner, "Rotational magnetization - an effective source of numerical data on multidirectional permeability, losses and magnetostriction of soft magnetic materials," *Xth Int. Symposium on Theoretical Electrical Engineering ISTEET'99*, Magdeburg, Germany, September 1999, pp.377-383.
- [3] K. Delaere, R. Belmans, K. Hameyer, W. Heylen, P. Sas, "Coupling of magnetic analysis and vibrational modal analysis using local forces," *Xth Int. Symposium on Theoretical Electrical Engineering ISTEET'99*, Magdeburg, Germany, 6-9 September 1999, pp.417-422.
- [4] O.C. Zienkiewicz, R.L. Taylor, *The Finite Element Method*, McGraw-Hill 1989.
- [5] P.P. Silvester, R.L. Ferrari, *Finite Elements for Electrical Engineers, Third Edition*, Cambridge University Press, 1996.
- [6] K. Delaere, K. Hameyer, R. Belmans, "Introducing magnetostriction into magnetomechanical analysis," *International Conference on Electric Machines ICEM 2000*, Espoo, Finland, 28-30 August 2000, submitted for publication.
- [7] B.D. Cullity, *Introduction to Magnetic Materials*, Addison-Wesley (Series in Metallurgy and Materials) Philippines 1972.
- [8] H. Pfützner, A. Hasenzagl, "Fundamental aspects of rotational magnetostriction", in *Nonlinear Electromagnetic Systems*, eds. A.J. Moses, A. Basak, pp.374-379, IOS Press, Cardiff 1996.
- [9] D. Jiles, *Introduction to Magnetism and Magnetic Materials*, Chapman & Hall 1991.

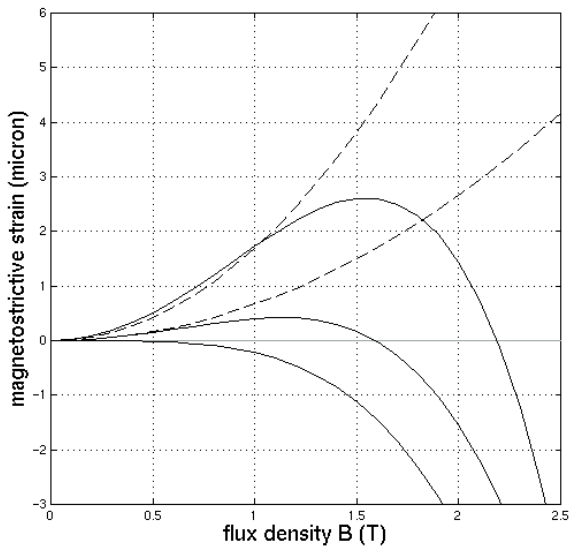


Fig. 1. Magnetostriction characteristics of non-oriented 3% SiFe (solid lines, as a function of tensile stress) and M330-50A (dashed lines, for rolling and transverse direction).

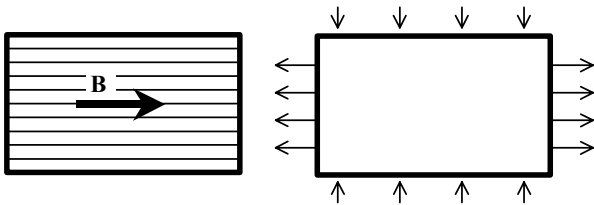


Fig. 2. The set of forces (right) representing the strain caused by magnetostriction due to the magnetic field B (left), consists of a set forces parallel to B and a set forces perpendicular to B .

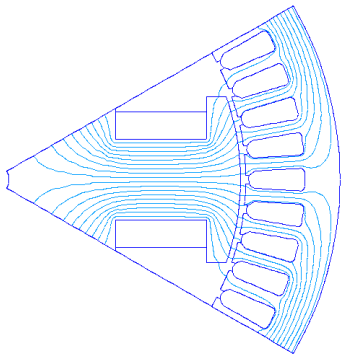


Fig. 3. Magnetic field for one pole of a six-pole synchronous machine.

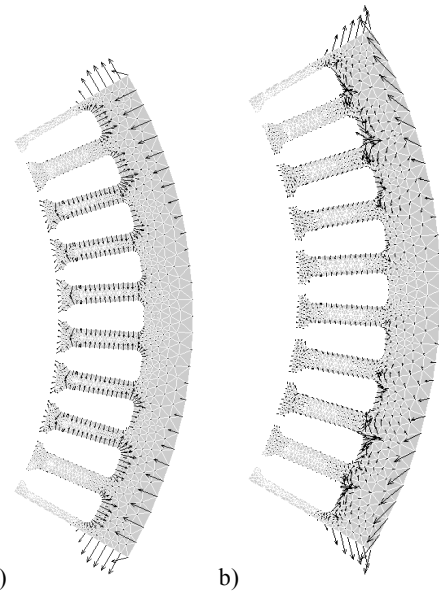


Fig. 4. Magnetostriction forces on stator for a) isotropic non-oriented 3% SiFe, b) anisotropic M330-50A.

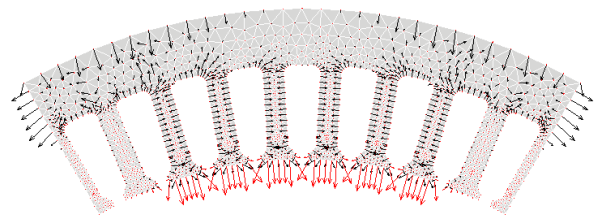


Fig. 5. Combined view of reluctance forces on stator teeth and magnetostriction forces on stator yoke and teeth sides.