Numerical techniques for electrical machines: A personal view

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In the last thirty years, fast developments of the numerical techniques for the particular questions concerning electrical machinery can be noticed. This development can be seen in parallel with the efforts and success of the soft- and hardware computer industry in producing powerful tools to treat realistic numerical models of technical relevance.

A very rough figure can be drawn in saying that this development of numerical methods for electrical machines started with the finite difference method which quickly was followed and further overtaken from the finite element method.

With the first numerical models of electrical machines, in the 70s, electromagnetic fields considering imposed current sources could be simulated by applying first order triangular elements [1]. Further developments of the finite element method lead via the definition of external circuits to problem formulations with imposed voltage sources, which where more accurate models with respect to the realistic machine that is operated by a voltage. Today's developments are directed to all aspects of coupled fields. There are the thermal/magnetic problems or structure dynamic field problems coupled to the electromagnetic field; e.g. acoustic noise in transformers and rotating machines excited by electromagnetic forces.

Parallel with the developments of the finite element method first attempts to numerically optimise the finite element models where made. Optimisations of realistic machine models where first performed in the late 80s and early 90s. Various deterministic and heuristic methods have been applied and can be found in the conference proceedings e.g. COMPUMAG and CEFC of that time.

This paper is intended to give a more detailed overview of some particular methods and their applications for the simulation of electrical machinery than given in the rough view in this abstract. It must be seen as an incomplete and personal view of the author because the importance of the various techniques and methods depends always on a particular point of view. My electrical engineering life started with the finite difference method, which I used during my time as development engineer at the Robert Bosch GmbH in Schwieberdingen, in Germany and beside other duties there where permanent magnet excited synchronous machines to develop. At that time Dr.-Ing. G. Henneberger was the head of the development departments and I worked in one of his divisions. Therefore, all for me important methods where at that time related to electrical machines and actuators.

1. INTRODUCTION

At the time, when not too many computers where available yet, the design of electrical machines was performed in the classical way, by using one-dimensional models. Particular electromagnetic parts of the electrical machine are considered to form a homogenous element in a magnetic circuit approach. In this approach the knowledge of particular "design" factors is assumed. Such models enabled the calculation of specific stationary working points of the machine. Laplace transformations applied to such models makes it possible to analyse the machine for the dynamic behaviour. The refinements of the 1D models yield the technique of magnetic equivalent circuit models.

More and more powerful computer hardware enables the simulation of more and more complicated field problems in two- and three-dimensions. Figure 1a shows the exponential increase in circuit integration respectively in clock frequency (Fig. 1b), which indicates the enormous growth in the computer hardware development. The figure displays Moore's law, which says that the computer power will double each 18 months.

The development of numerical algorithms and methods can be seen in parallel to this. This opportunity with respect to hard-, software and numerical techniques delivered the possibility to develop novel technical products. By using the numerical simulation of technical devices, a shorter "time to market" is realised. The cost for prototyping can be minimised as well. Due to

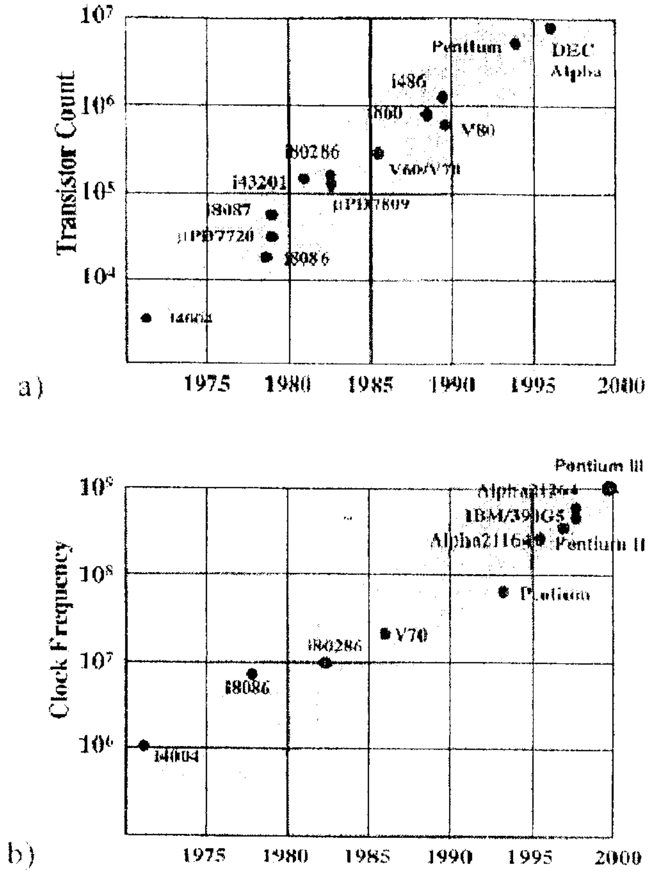


Fig. 1. Moore's law – Exponential increase in (a) circuit integration and (b) clock frequency [2].

simulations, it is now possible to develop miniaturised devices, such as high dynamic motor drives for computer hard-, floppy and CD-ROM applications in a very much faster way. An increase in the ratio power/weight of the electromagnetic energy transducers, the actuators, is a result of these opportunities. If we plotted this ratio for e.g. electrical generators, we could see a comparable growth for the development of such electrical machines.

To have a brief look to the future of the developments in numerical techniques, in Table 1 the predicted hardware developments are collected. It seems that the Moor's law will still be applicable.

With the ongoing developments in computer hard- and software and numerical research, difficulties concerning computational costs and numerical problems are continuously moving to the background. Today, efficient numerical solutions can be obtained for a wide range of problems beyond the scope of analytical methods. In particular the limitations imposed by the analytical methods, their restrictions to homogeneous, linear and steady state problems can be overcome using numerical methods.

Table 1. 2000 and beyond [3].

	1999	2002	2014
Technology (nm)	180	130	35
Wafer diameter (mm)	300	300	450
Memory size (bits)	1 G	4 G	1T
Transistors/cm^2	6.2 M	18 M	390 M
Wiring levels	6-7	7	10
Clock speed (MHz)	1250	2100	10000
Chip size (mm^2)	340	430	901
Power supply	1.5–1.8	1.2–1.5	0.37–0.42
Maximum power (W)	90	130	183
Number of pins	700	957	3350

Nowadays, various state of the art two- and three-dimensional numerical program packages are commercially available to helping to design electrical machines. Complicated geometries, non-linear material properties can be studied by using such software. The question, addressed by the machine producing and material manufacturing industry, asks for the appropriate material for the particular technical application. Such software products can partially give the answer to this issue.

Research groups at universities have code available that goes further in modelling. For example, it can consider external electrical circuits with power electronic components operating the machine model. Such models are consuming a lot of computational time. However, research directed to accelerate the numerical solvers is going on as well.

But after all developments of numerical techniques around the electrical machine analysis, it must be noticed that the classical approach, the enormous expertise and knowledge to model the various electromagnetic effects accurately, can not be replaced by the numerical models. The modelling/solving/post-processing of numerical motor models still requires some engineering time and cannot replace the quick and correct answer of an exercised motor design engineer. Both experts, probably in one person, numerical modelling and electromagnetic motor design engineer, are required for successful novel developments in this engineering discipline.

2. COMPUTER AIDED DESIGN IN MAGNETICS

For designing and constructing electromagnetic devices an accurate knowledge of the field quantities inside the magnetic circuit is necessary. In many cases the air gap is of particular importance (e.g. motors, switches, relays, contactors, actuators). Here the conversion from electrical to mechanical energy and vice versa takes place. In the air gap the field quantities such as flux density and field strength have to be calculated very accurately in order to be able correctly to asses the operational behaviour of the device.

Although Maxwell equations have been known for more than a century, in the past the task in calculating a magnetic circuit was to find as many assumptions and simplifications as possible. Then, results could be obtained with rather low numerical efforts. Using this approach, only devices or problems with a strongly simplified geometry could be studied. It was a design following simple rules, found empirically. Physical effects were considered by correction factors applied to the existing rules. In the following period of time this design through rules has changed into another design philosophy: design analysis. Here, computer models were used to solve the field problem. Analysis means the treatment of the field problem by numerical simulation.

In general, the procedure for analysing an electromagnetic device can be divided into three steps of pre-processing, processing and the post-processing.

In the first step, the field problem is defined and prepared to be solved. The second step delivers the numerical solution of the physical problem. During the post-processing, the obtained solution is prepared to calculate the required field quantities or to evaluate forces and other macroscopic quantities. This threefold approach of defining, solving and evaluating is typical for every analysis procedure, numerical or analytical. The different techniques, data structures or algorithms used in the individual steps, influence and/or limit the overall procedure during the analysis of a field problem (Fig.2).

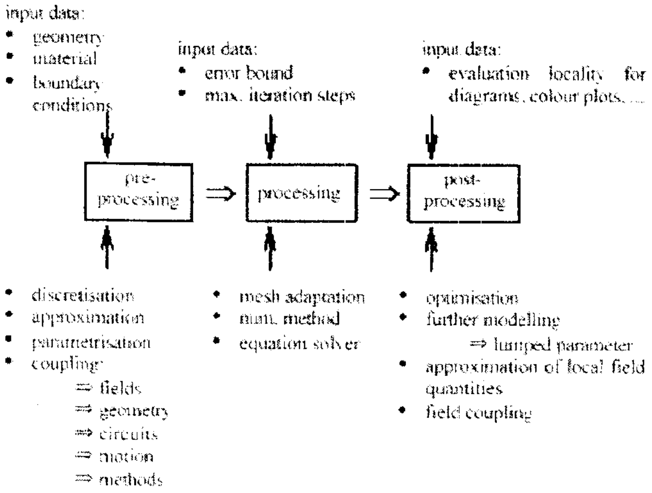


Fig. 2. Field analysis steps.

To define a field problem, the input data describing the geometry of the domain of interest, the material representation and the boundary conditions are always required. Even with enhanced CAD drawing techniques, most of the analysis time will have to be spent on the pre-processing. Given error bounds will support a desired accuracy of the solution. Often, the user cannot influence this step. During the post-processing, the solution must be prepared to study the local field effects. The post-processing represents an open-ended process, because the user of the analysis can evaluate the calculated solution in various ways for different aspects.

The methods and algorithms used in the single steps of the overall procedure can form an efficient analysis or design tool and determine the quality of the results of the analysis. For example a use of particular internal data structure can enable very quick search routines to obtain an efficient, fast and automated discretisation with parameterised geometries and materials. The various possible coupling mechanisms of different fields, circuit equations, methods such as FEM/BEM combinations, motion term or geometries yield into an accurate approximation of a realistic physical problem. The properties of the coefficient matrix decide which equation solver or algorithms must be used to solve the problem.

2.1. Design strategies using numerical methods

The development and design of electromagnetic devices reflects a complex process. Originating from an initial idea, the construction runs through different phases. This procedure is terminated when a final concept is selected and considered to be designed, subject to various targets and constraints. As a whole, the task of the design engineer is to find solutions for technical problems. On the way to the latter physical and technical product, certain aspects have to be considered. Technological and material-dependent questions as well as cost effectiveness and ecological constraints have to be taken into consideration. A cut-set of the mentioned boundary conditions controls the feasibility of the final design. With emphasis on electromagnetic devices, Fig. 3. shows a simplified scheme of interdependencies of targets and constraints. This simple pattern clarifies that the design process is strongly dependent on the experience of the engineer and reflects an optimisation procedure with often contradictory aims. Therefore, the necessity of a systematic and strategic design with engineering tools is obvious. Here, solution strategies using modern numerical methods to accelerate and ensure a high-standard technical product in an overall design process are discussed.

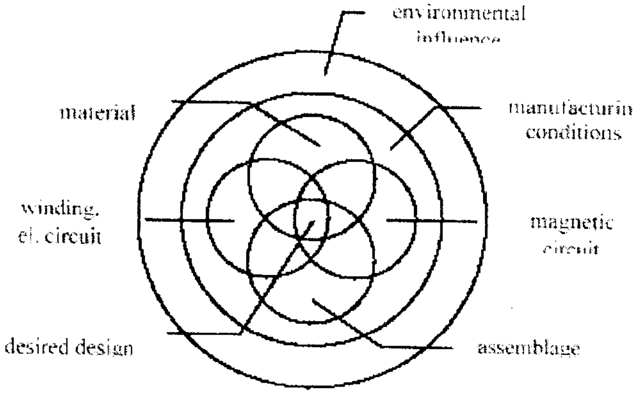


Fig. 3. Interdependencies in the design of electromagnetic devices.

Designing electromagnetic devices includes the calculation and analysis of the electromagnetic field distribution. From the local field quantities forces, torques and losses can be derived to make predictions concerning global quantities such as converted power and efficiency. For complicated geometries analytical field solutions are non-existent or very hard to obtain. Using numerical field computation techniques of a general application range, the microscopic field solution leads via a lumped parameter approach to the desired time-dependent behaviour of the device The microscopic field solution itself delivers important knowledge regarding the material utilisation. Such results offer the opportunity to reduce material, weight and the costs of the latter product. To accelerate development, extensive field computations with various types of material can be performed avoiding expensive prototyping. It is even possible to predict system behaviour before new materials are actually available on the market. With this knowledge, the design engineer can order special material to be developed at the material manufacturer or, vice versa, if the material supplier uses such numerical tools he can suggest and offer the right choice of material for a particular device.

2.2. Knowledge based design

The main aspect of the structured development of novel technical products is analysis followed by a detailed synthesis. Analysing means obtaining information on partial functions of the desired overall function, by investigating single elements and their mutual interactions. In this way overall links between various principles of the partial functions are found.

In Fig. 4, a structured and knowledge-based development process is illustrated in a simplified scheme. In this example, the final technical product has to be designed, able to fulfil three partial functions. Those individual functions to be connected to the overall task of the product are a linear motion, a continuous rotation and some reverse operation. After the analysis phase, in the synthesis step different physical working principles are selected and evaluated. The selection process is governed by simple qualitative rules. In this way the partial functions are evaluated with regard to their feasibility with respect to the given

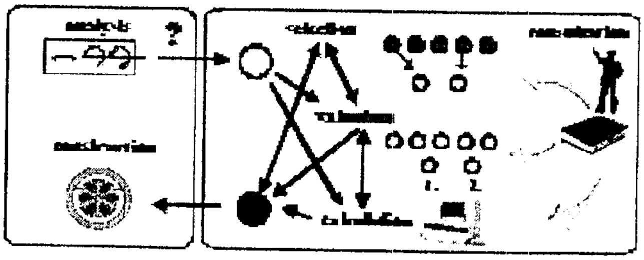


Fig. 4. Knowledge-based and structured design.

constraints and limitations. In this step, the feasible principles are ranked qualitatively by weighted constraints and limitations.

The process of synthesis leads from qualitative decisions to quantitative statements in a following design step. The whole process is accompanied by the consultation of experts and expert knowledge. A detailed investigation and ranking, i.e. the precise calculation of the operating conditions, leads by a comparison to the final technical product. In this loop of iterations, between validation and the performance of detailed predictions of qualified concepts, a numerical optimisation combined with field computation methods is found as an important and powerful engineering tool for the design of electromagnetic devices.

The quantification and ranking of the working principles is governed by the choice of materials or other components such as electronic circuits. Their interdependency on the studied principle can be distinguished into an object and a rule world. The various, for instance ferromagnetic, permanent magnet, conductive or dielectric materials, and respectively components such as the electronic hardware have, considered inside an object world, particular properties and characteristics. To employ such object properties in order to obtain a physical working principle fulfilling a desired function, appropriate rules determining the function of the object have to be considered. In both object and rule world, constraints are found to govern a decision to consider the principle further in the ranking or to reject it. Numerical techniques can help to employ the rules accurately to the studied object.

3. MAXWELL'S EQUATIONS AND POTENTIALS

It all starts with Maxwell's equations (Table II). Every electromagnetic phenomenon can be attributed to the seven basic equations, the four Maxwell equations of the electrodynamic and those equations of the materials. The latter can be isotropic or an-isotropic, linear or non-linear, homogenous or non-homogenous.

Fig. 5. James Clerk Maxwell (1831-1879).

Table II. Maxwell's equations.

	differential form	Integral form
(i)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{r} = -\frac{d\phi}{dt}$
(ii)	$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_C \mathbf{H} \cdot d\mathbf{r} = I$
(iii)	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$
(iv)	$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \rho$
(v)	$\mathbf{B} = \mu \mathbf{H}$	
(vi)	$\mathbf{D} = \varepsilon \mathbf{E}$	
(vii)	$\mathbf{J} = \mathbf{J}_0 + \mathbf{J}_c = \mathbf{J}_0 + \sigma \mathbf{E}$	

The Maxwell equations are linked by interface conditions. Together with the material equations they form the complete set of equations describing the fields completely. Φ is the magnetic flux, *I* the conducted current, *Q* the charge, *C* indicates the contour integral and *S* the surface integral.

The seven equations describe the behaviour of the electromagnetic field in every point of a field domain. All electric and magnetic field vectors **E**, **D**, **B**, **H**, and **J** and the distance **r** are in general functions of time and space. The conducting current density can be distinguished by a material/field dependent part $\mathbf{J}_c$ and by an impressed and given value $\mathbf{J}_0$. It is assumed that the physical properties of the material's permitivity ε, permeability μ and conductivity σ are independent of the time. Furthermore it is assumed that those quantities are piecewise homogenous.

The Maxwell equations represent the physical properties of the fields. To solve them, mainly the differential form of the equations and mathematical functions, the potentials, satisfying the Maxwell equations, are used. The proper choice of a potential depends on the type of field problem.

The electric vector potential for the displacement current density will not be introduced here, because it is only important for the calculation of fields in charge-free and current-free regions such as hollow wave-guides or in surrounding fields of antennas.

Various potential formulations are possible for the different field types. Their appropriate definition ensures the accurate transition of the field problem between continuous and discrete space. Using these artificial field quantities reduces the number of differential equations. Considering a problem described by *n* differential equations, a potential is chosen in such a way that one of the differential equations be fulfilled. This potential is substituted in all other differential equations, the resulting system of differential equations reduces to *n-1* equations.

It is distinguished between magnetic and electric vector respectively scalar potentials (Table III). The potentials are the basis of the finite element approach. Applying such potentials to Maxwell's equation and using the Galerkin method yields the discrete equation [4]. The choice of type of finite elements, e.g. 2D-triangular, 2D-rectangle or 3D-tetrahedron determines the final system of equations to be solved.

Table III. Potentials.

	scalar	vector
electric	$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$	$\mathbf{J} = \nabla \times \mathbf{T}$
magnetic	$\mathbf{H} = \mathbf{T} - \nabla \phi$ $\mathbf{H} = -\nabla \phi$	$\mathbf{B} = \nabla \times \mathbf{A}$

4. FINITE ELEMENT MODELLING

In this section, the modelling of particular electrical machine problems with the finite element method will be discussed. Particular attention is paid to the modelling of the ferromagnetic material.

4.1. Eddy currents in laminated ferro-magnetic steel

High permeable steel laminations are usually used for the design of electrical machines. This is because unfortunately the material has very good magnetic properties, but is also a very good electrical conductor. As a result of this, eddy current losses have to be considered. To lower eddy current losses in AC fields several iron laminates with coating material on both surfaces are applied. The coating material is less conductive and less permeable than the iron, preventing excessive eddy current losses but at the expense of a higher reluctance of the global magnetic flux path [5] (Fig. 6).

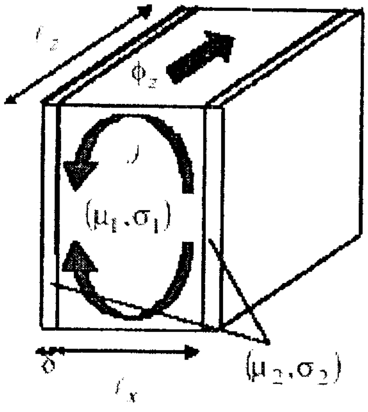


Fig. 6. Ferromagnetic lamination.

In analysing electrical machines, usually a 2D-FEM electrodynamic model is built, which lies in the plane of the flux. For this case, no eddy currents, as shown in Fig.6 for the laminated steel, in the x-plane can be considered. To be able to simulate the eddy currents in laminations, the model has to be built in the xy-plane and the flux is perpendicular to it. To obtain a solution for this case, an electrodynamic in-plane formulation using the electric vector potential **T** is chosen:

$$\nabla \times \mathbf{T} = \mathbf{J}\,, \tag{1}$$

this yields the time-harmonic formulation

$$-\frac{\partial}{\partial x}\left(\rho \frac{\partial T_z}{\partial x}\right) + j\omega\mu T_z = -j\omega\mu \frac{V_m}{l_z}\,. \tag{2}$$

ρ is the resistivity, μ the permeability, $j\omega$ is the angular frequency, V_m the magnetomotive force and l_z the length of the problem in z-direction.

With this model it is possible to determine the losses for single sheets and for multiple layer arrangements of ferromagnetic lamination (Fig.7). For the simulation of multiple layers an FEM approach with external magnetic circuits for the flux in z-direction has to be chosen [9].

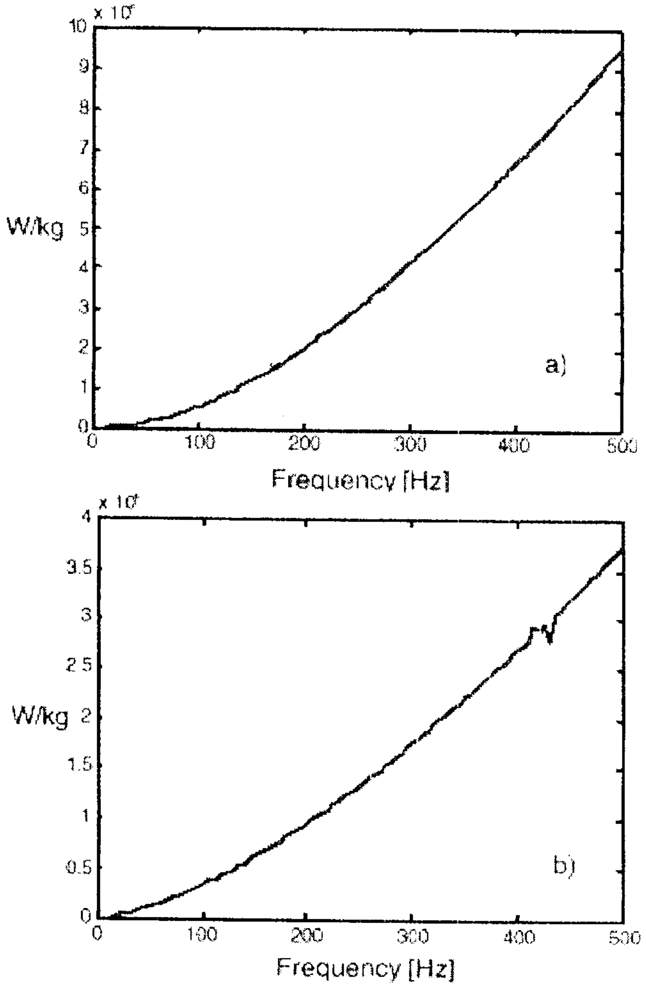


Fig. 7. Losses in a ferromagnetic steel (**a**) single sheet and (**b**) in a multiple layer arrangement.

The model consisting of multiple iron laminates with coating material on both surfaces is drawn in Fig.8. Semi-analytical simulations consider the losses in a single laminate and neglect the conductivity and permeability of the coating material. The model presented here deals with coating material with a finite resistivity. As a consequence, the closing path of the current may cross the coating layers. The eddy current losses are completely different from the simplified analytical model. As an external condition, the total magnetic flux through the model has to equal the applied flux. The magnetic equivalent circuit represents the parallel connection of all domains in the electrodynamic model, excited by a flux source (Fig.8).

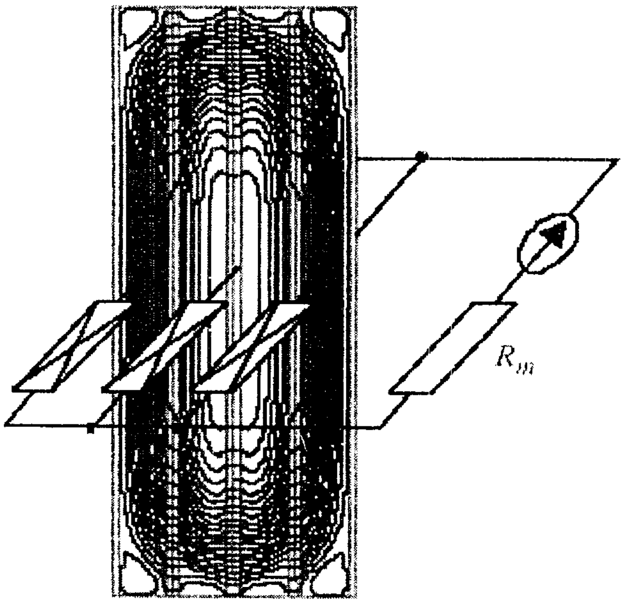


Fig. 8. 2D electrodynamic model of a laminated material of various layers combined with a magnetic equivalent circuit [9].

By using this hybrid modelling technique, the coupled FEM and magnetic equivalent circuit model, dependencies of the eddy current losses can be studied with respect to the frequency and the number of iron layers (Fig. 9a, b, c). Such results can further be employed to simulate the materials behaviour in electrical machines at various frequencies. To consider the complicated shape of e.g. an induction machine these simulated results have to be transformed in such a way that they can be used in the 2D model of a realistic electrical machine.

The second, something more technical example, in which this approach is illustrated, is a dielectric heating device (Fig.10). A cylindrical dielectric is placed between two circular electrodes, both constructed from ferromagnetic material. Both, dielectric and conductive heating effects are considered [6]. The geometry and the excitation are modelled by an axisymmetric model in this case. As a consequence, the magnetic equivalent consists of the short-circuit connection of all magnetic paths. In contrast to the discussed lamination model, the excitation is of electric nature and is therefore applied as a difference in electric vector potential. The combination of

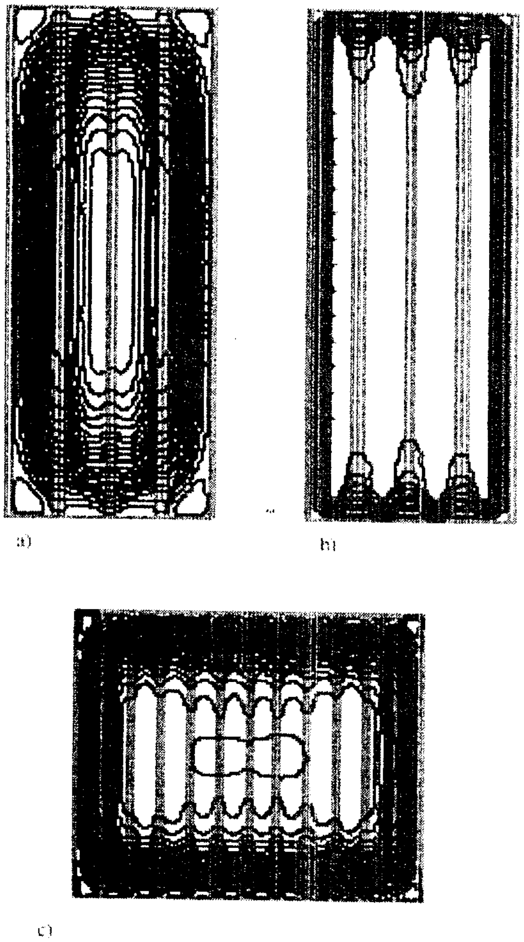


Fig. 9. Eddy current density distribution for a field at (**a**) 50 Hz, (**b**) at 500 Hz and (**c**) for comparison, multiple laminations at a frequency of 50 Hz [9].

conductive and dielectric effects involves a complex valued resistivity in the potential formulation of this problem type.

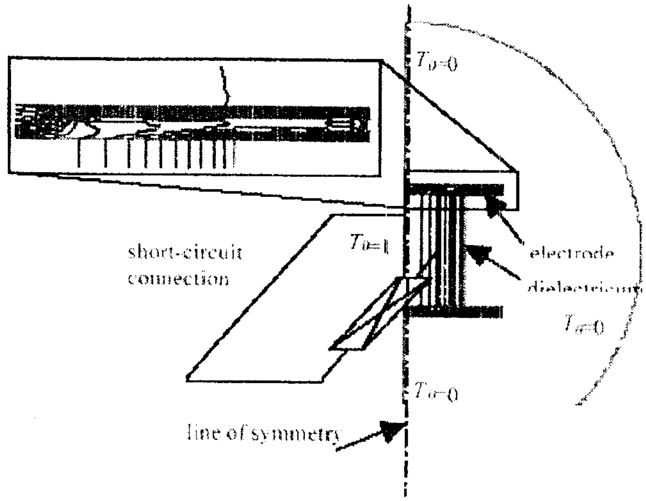


Fig. 10. Heating device as axis-symmetrical hybrid model [9].

4.2. Eddy current losses and harmonics

The figures 9 and 10 show already simulated current distributions in the lamination used in electrical machines. These simulations where performed assuming a single frequency for the FEM electrodynamic models. In this time harmonic approach, only the fundamental magnetic field frequency component is considered [7]. Losses can be considered by an elliptic approximation of the magnetic *BH*-characteristic [8]. However, in reality various harmonics have to be considered. The multi-harmonic and the harmonic-balance approach are possible alternatives to solve this type of problem. For the multi-harmonic first approach, it is assumed that the magnetic field is dominated by the fundamental frequency component, it can further be assumed that the *DC*-saturation level is dominated by this component. A typical example of this situation is a magnetic device connected to a strong sinusoidal power grid operating a non-linear load, which is generating the current harmonics. The total magnetic field can then be computed by means of an uncoupled set of equations. First the harmonic problem at fundamental frequency is solved and in further steps the problem is solved at the harmonic frequencies. The level of the ferromagnetic saturation is determined by the element-wise constant permeability obtained from the solution of the problem at fundamental frequency.

In many cases, however, the field harmonics are important, e.g. when the device's supplying voltage is non-sinusoidal and contains higher harmonics and the device is not highly saturated. Therefore, in the multi-harmonics second approach, all of the harmonics have to be considered to obtain the *DC*-saturation. It is assumed that the field components do not mutually interact directly. Therefore, all the equations are non-linearly coupled. In the multi-harmonics second approach, the different equations are solved for a particular operating point, in which the saturation level is temporarily kept constant.

The harmonic-balance approach considers the mutual interactions between the various harmonics, which are caused by the saturation. The different equations become intensely coupled [10]. Therefore, an entire harmonic-balance is required. Both, harmonics in the field as well as in the material series are considered. FFT algorithms can be used to calculate this non-linear reluctivity updates.

In general the multi-harmonic second approach seems to overestimate the saturation effects when compared to multi-harmonic first approach. Multi-harmonic first approach works accurate in the case of a dominant fundamental frequency. It is applicable for devices operated by pulse-width-modulated power electronic circuits. The harmonic-balance can account for the time-local occurrence of saturation up to a pre-set maximum frequency and delivers the best approximation.

However, it results in longer computation times. Figure 11 shows the field plots of some simulation results obtained by the harmonic balance approach.

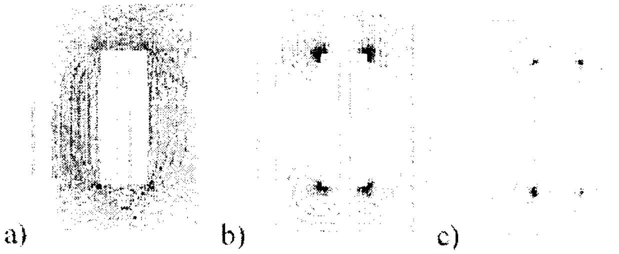


Fig. 11. Field for a single-phase transformer model (**a**) fundamental, (**b**) 3 rd and (**c**) 5 th harmonic frequency [11].

5. NUMERICAL OPTIMISATION OF FINITE ELEMENT MODELS

The design process of electromagnetic devices reflects an optimisation procedure. The construction and step-by-step optimisation of technical systems in practice is a trial and error-process. This design procedure may lead to sub-optimal solutions because its success and effort strongly depends on the experience of the design engineer. To avoid such individual parameters and thus to achieve faster design cycles, it is desirable to simulate the physical behaviour of the system by numerical methods such as the finite element method. In order to get an automated optimal design, numerical optimisation is recommended to achieve a well-defined optimum. Optimisation of electro-magnetic devices turns out to be a task of increasing significance in the field of electrical engineering. The term of Automated Optimal Design (AOD) describes a self-controlled numerical process in the design of technical products. Recent developments in numerical algorithms and more powerful computers offer the opportunity to attack realistic problems of technical importance.

In general, the development of numerical optimisation algorithms can be distinguished in their evolution into three generations:

- First generation: Direct evaluation of a quality function; end 80s and early 90s.
- Second generation: Evaluation of a polynomial approximation of the quality function; around 96s.
- Upcoming third generation: The unknown quality function is constructed by using artificial intelligence, such as Neurofuzzy models.

The two first generations of numerical optimisation algorithms with known objective function can be distinguished in deterministic (gradient methods, linear programming) and stochastic approaches (genetic algorithms, evolution strategy, simulated annealing). A prerequisite of the deterministic approach is the availability of derivatives, which is for the optimisation of FEM models mainly not the case. Their advantages can be seen in its efficient iterative search. However, such algorithms can easily be trapped in local minima.

Stochastic search methods do not use derivatives, are robust in identifying the global optimum but have the disadvantage of very large computational costs. This can be partly overcome by searching the optimisation space in parallel.

A hybrid optimisation can be obtained by identifying a global optimum by stochastic methods that are further evaluated by a deterministic search to extract the global minimum. These combined algorithms have the advantage of a higher accuracy and are more efficient regarding computational expenses.

The distinctive feature of an electromagnetic optimisation problem is its complexity, which results from a high number of design parameters, a complicated dependence of the quality on design parameters and various constraints. Often the direct relation of the desired quality of the technical product on the objective variables is unknown. Stochastic optimisation methods in combination with general numerical field computation techniques such as the finite element method (FEM) offer the most universal approach in AOD.

To be able to select the appropriate optimisation algorithms to form an overall design tool together with the numerical field computation, the properties of typical electromagnetic optimisation problems has to be considered. Electromagnetic design and optimisation problems reflect mainly the following categories:

- constrained
- problem type:
  ☒ parameter- or static optimisation, $f(\mathbf{x}) \rightarrow \min$.
  ☐ trajectory, or dynamic problem, $f(\dot{\mathbf{x}}, \mathbf{x}) \rightarrow \min$.
- non-linear objective function
- design variables:
  ☒ real
  ☐ mixed real/integer
- multi-objective function
- interdependencies of the quality function and the design variables are unknown; no derivative information available
- the quality function is disturbed by stochastic errors caused by the truncation errors of the numerical field computation method.

In reality electromagnetic optimisation problems are constrained due to the various reasons mentioned (Fig. 3). Nowadays optimisations are performed mainly as static problems. Numerical optimisations require huge amounts of computation time. Therefore, the optimisation as aimed at here, combined with the FEM, of the dynamic system with time-derivative $\dot{\mathbf{x}}$ is not yet commonly performed. For transient problems an evaluation of the quality function by numerical methods (FEM) is still too time consuming. Considering mixed real/integer design variables results in long computation times as well. The tick boxes in the list, which are not marked, represent future developments. In general, optimisation means to find the best solution for a problem under the consideration of given constraints and it does not mean to select the best out of a number of given solutions.

The application of an optimisation using evolution strategy and the FEM is demonstrated by the optimisation of a small DC motor [4].

The objective is to minimise the overall material expenditure, determined by permanent magnet-, copper- and iron volume subject to a given torque of the example motor.

$$Z(x) = 10^{\left(\frac{\cos t(x) - \cos t_{\max}}{\cos t_{\max}}\right)} + penalty \rightarrow \min. \quad (3)$$

The use of penalty term in the form:

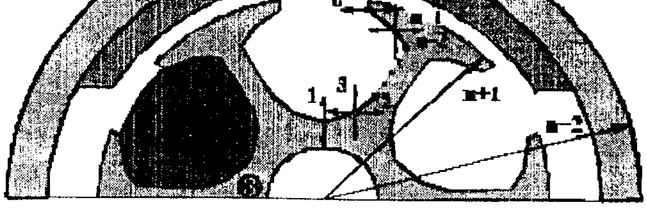
$$penalty = \begin{cases} T < T_{\min} & : 10^{\left(\frac{T_{\min} - T(x)}{T(x)}\right)} \\ T \geq T_{\min} & : 1 \end{cases} \quad (4)$$

allows the evaluation of the objective function even if the torque constraint is violated.

The torque is computed by integrating the Maxwell stress tensor in the air gap region of the motor. Flux density dependent rotor iron losses were taken into account at a rated speed of 200 rpm and subtracted from the air gap torque to form the resulting output torque.

The overall dimensions and the slot geometry of the DC motor are described by 15 free design parameters. The free parameters are the n/2 edges of the polygon describing the rotor slot contour and the outer dimensions of rotor and stator as indicated in Fig. 12. The motor consists of a stator back iron with a 2-pole Ferrite permanent magnet system and a rotor with six slots.

The two-dimensional field computation to determine the quality function (eq.'s 3 and 4), to



1. stator back iron
2. permanent magnet
3. rotor iron
4. winding slot

Fig. 12. Geometry definition for the shape optimisation of a DC motor [4].

compute the torque of the machine, is performed by standard two-dimensional finite element analysis. To ensure controlled accuracy, adaptive mesh generation is applied until a given error bound is fulfilled.

The change of shape from a sub-optimal initial geometry to the final shape of the motor can be taken from Fig. 14. It can be noticed that the iron parts of the initial geometry are over dimensioned. The actual torque of this configuration was approximately 25% lower than the desired value min *T*. The optimised motor holds the torque recommended, which is achieved mainly by enlarging the winding copper volume by about 20%. The most significant change from start to final geometry can be seen in the halving of the iron volume. Consequently the iron parts are highly saturated, especially the teeth regions. In comparison to this, a test optimisation with neglected rotor iron loss results in a 10% smaller rotor diameter. Unfortunately, the permanent magnet material is brittle, which limits the minimum magnet height. The magnet volume decreases slightly. Along optimisation the overall volume off the motor was reduced by 38%. The rate of convergence is plotted in Fig. 13. It can be noticed that the used evolution strategy is relatively fast converging towards the desired minimum. However, from generation 60 to the final solution no large changes can be observed. Therefore, technically interesting results e.g. with regard to fabrication tolerances can already be obtained in an early stadium of the optimisation.

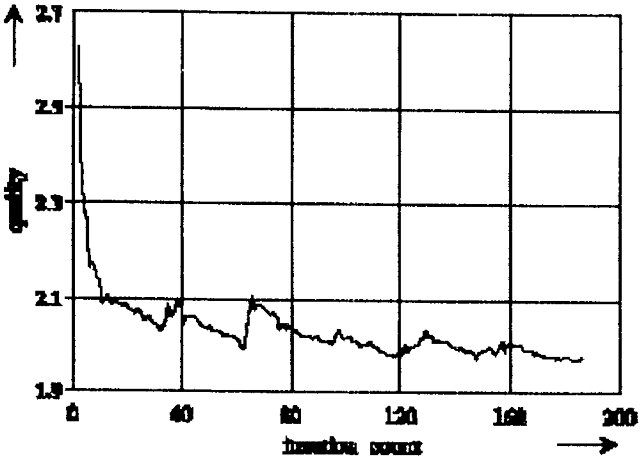


Fig. 13. Course of the optimisation.

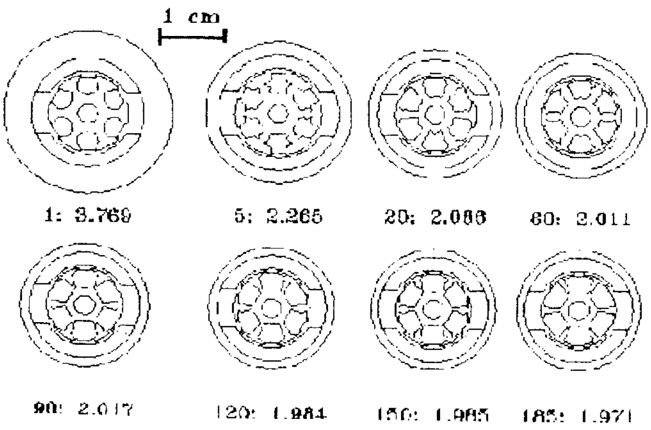


Fig. 14. Design variants during optimisation (# generation: quality).

6. CONCLUSIONS

This paper is intended not to be a theoretical "firework". It is an incomplete and a personal view on the topic of numerical techniques used for electrical machines. It shows the possible potential for further development of the methods that can be used for the efficient design of electrical machines. The potential and the possibilities of the numerical models are closely linked to the capabilities and availability of computer hardware and can therefore be derived from these developments. Many difficult field problems are still unsolved.

This paper shows some techniques to model electrical machines, such as electric motors and transformers. To restrict eddy current losses, these machines are usually constructed with ferromagnetic laminations. Here, a closer insight is presented to the modelling of ferromagnetic materials using the finite element method. Single frequency time-harmonic models are presented as well as multi-harmonic methods.

With increasing computation power and by assuming further developments in the numerical algorithms, it is expected that numerical optimisation algorithms will be employed more commonly in the future. An optimisation example is given to illustrate the operating of a numerical optimisation.

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