MOTIONAL MAGNETIC FINITE ELEMENT METHOD APPLIED TO HIGH SPEED ROTATING DEVICES

Herbert De Gersem, Hans Vande Sande and Kay Hameyer

Katholieke Universiteit Leuven, Dep. EE (ESAT), Div. ELEN, Kardinaal Mercierlaan 94, B-3001 Leuven, Belgium
E-mail: Herbert.DeGersem@esat.kuleuven.ac.be

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Abstract

Finite element simulations of high speed rotating devices suffer from numerical instabilities due to the presence of a dominant convection term in the magnetomotional partial differential equation. The problem is cured by a combined approach consisting of an artificial diffusion upwind technique and an adaptive mesh refinement. The local refinement of the mesh based on the error estimation of intermediate solutions results in an optimal distribution of the finite elements. The upwind scheme ensures the stability of these intermediate solutions. The braking torque of a high speed rotating magnetic brake is examined. Applying this combined approach, accurate solutions are obtained with minimal computational efforts.

Introduction

Electromagnetic devices involving motional effects are commonly simulated using transient models. Transient finite element simulation techniques have to remesh the geometry, to choose an appropriate time step scheme and to cope with excessive computation times. In the case of moving bodies retaining the original geometry, these problems can be avoided by using a motional formulation. The $\mathbf{v}\times\mathbf{B}$ velocity term is included in the differential equations. Here, some disadvantages of this motional finite element method considering high speed rotating devices are discussed. The oscillations observed for the standard Galerkin method applied to models at higher speeds, are overcome by applying upwind techniques. External electric circuits required to model realistic devices, are considered by additional integral equations.

Motional Finite Element Formulation

The governing partial differential equation for conductive media with a 2D translationary geometry submitted to an applied magnetic field, is

$$-\nabla\cdot\left(\nu\nabla A_z\right)+\sigma\mathbf{v}\cdot\nabla A_z=\frac{\sigma}{\ell}V\,, \tag{1}$$

with A_z the component of the magnetic vector potential in the *z*-direction, V the electric voltages across the conductive parts, $\mathbf{v}=\left(v_x,v_y,0\right)$ the velocities of the moving parts, σ the conductivity and ν the reluctivity which may be non-linear. The first term of (1) represents diffusion whereas the second denotes convection. The standard Galerkin finite element method with nodal finite elements yields a non-linear system of equations with a sparse and non-symmetric system matrix.

It can be shown that the error ε of the approximative solution of an elliptic partial differential equation obtained by linear finite elements is bounded by

$$\|\varepsilon\|<c_1h^2\,. \tag{2}$$

h is the characterisitic mesh size for a uniformly distributed discretisation. The exponent indicates a convergence rate of $O(h^2)$. c_1 is the convergence factor [1]. For 2D problems, the convergence rate in terms of the number of degrees of freedom *(DOF)* is $O(DOF^{-1})$.

It is known that the Galerkin technique shows numerical instabilities when applied to problems with dominant convection [2]. Then, the convergence factor is unacceptable high. The relative importance of convection when compared to diffusion is characterised by the Péclet number

$$Pe = \frac{h\sigma\|\mathbf{v}\|}{2\nu} . \tag{3}$$

If *Pe* exceeds 1, instabilities can be observed. Roughly speaking, the mesh size is to large to model the steep changes of the magnetic vector potential at loci in the model where high velocities and skin effects arise. The oscillations have a numerical nature and do not exist in reality. Equation (3) indicates two techniques to cure the problem: decreasing the mesh size and decreasing the ratio $\sigma\|\mathbf{v}\|/\nu$.

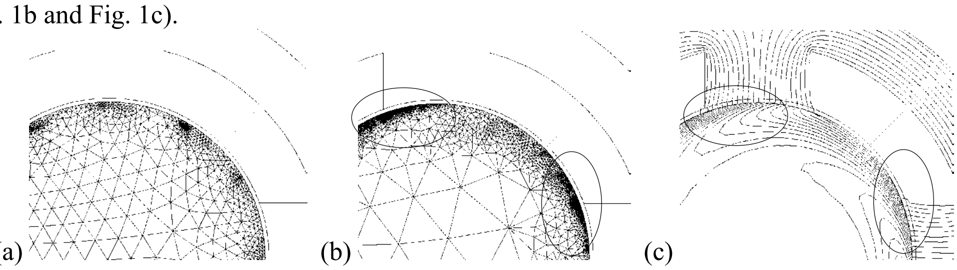
It is possible to choose a discretisation with a grid size small enough to avoid instabilities. However, in the presence of high velocities, the required refinement will lead to unacceptable computation times. It is recommended to apply an adaptive refinement procedure controlled by an appropriate error estimator [3]. The error estimator relies on the approximative solutions on intermediate meshes. As a consequence, this approach requires stable solutions even on rough meshes.

Decreasing the relative importance of convection with respect to diffusion is performed by artificially augmenting the reluctivity. An additional numerical diffusion term

$$-\nabla\cdot\left(\nu_{add}\nabla A_z\right) = -\nabla\cdot\left(\sigma\|\mathbf{v}\|h\nabla A_z\right) \tag{4}$$

is appended to (1). This particular choice yields a Péclet number lower than 1 and avoids numerical oscillations. An error of order *O(h)* is introduced in the differential equation. As a consequence, the second order convergence of the Galerkin approach with linear form functions is lost [2]. This scheme is called consistent (decreasing the mesh size, the approximative solution converges towards the exact solution) but not strongly consistent (the exact solution of (1) is not a solution of the differential equation modified by (4)). This artificial diffusion method is a popular and easy to implement upwind technique to stabilise finite element solutions of convection-diffusion equations. The additional diffusion yields a less steep front in the approximative solution compared to the exact solution.

Because the velocity inside rotational devices depends on the radius, upwinding has to be applied locally. In each element of the mesh, the Péclet criterion is checked and extra diffusion is added if required. The Péclet criterion can serve as an a priori error estimator. The resulting refinement exhibits concentric layers with a decreasing mesh size from the middle to the surface of the rotating body (Fig. 1a). However, error estimation based on intermediate solutions provides the possibility to refine only at loci with large eddy currents and yields smaller models (Fig. 1b and Fig. 1c).

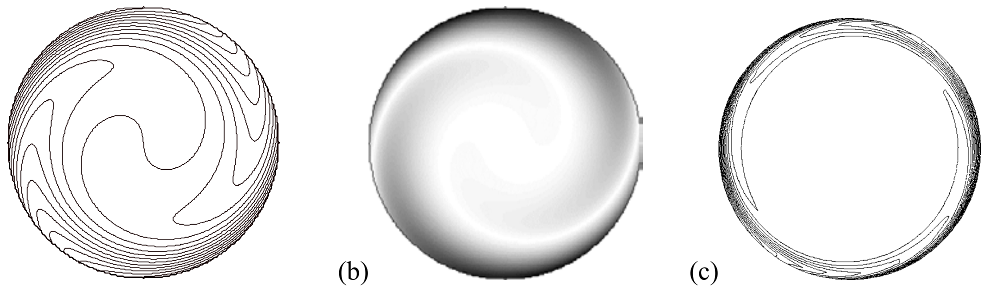


(a) (b) (c)

Fig. 1. Refined mesh applying (a) the a priori Péclet criterion and (b) adaptive refinement using the intermediate solution (c).

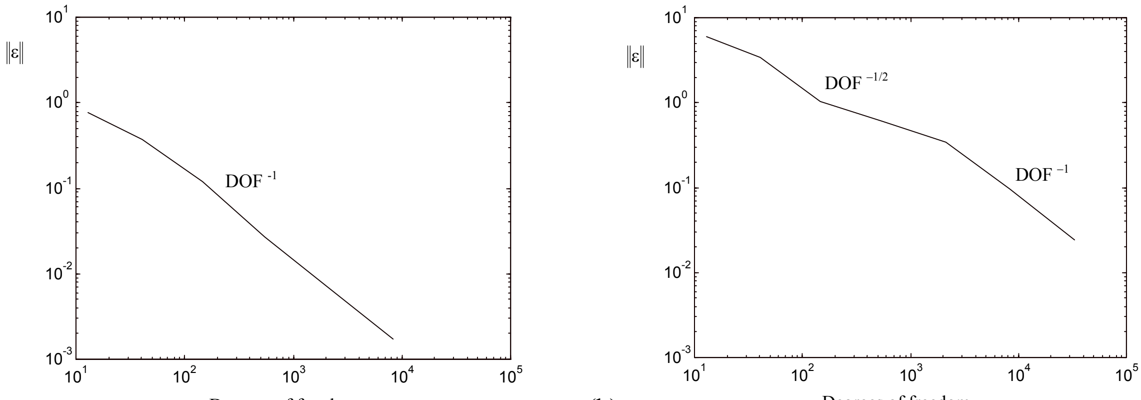
Benchmark Problem

A uniform magnetic field is imposed upon a rotating copper disk with a radius of 1 m [4]. Due to motional eddy current effects, the magnetic field is driven forward into the direction of motion (Fig. 2). For increasing velocities, the flux lines get more twisted and the eddy current effect appears in a smaller skin of the disk (Fig. 2c). For small velocities, the diffusion is dominant. The discretisation error decays quadratically with respect to the decreasing mesh size (Fig. 3a). For large velocities, the standard Galerkin technique fails to determine a stable solution (Fig. 4a). Applying upwinding provides stable solutions for rough meshes but implies linear convergence (Fig. 4b and Fig. 3b). With sufficiently small mesh size, the artificial diffusion vanishes and the quadratic convergence is regained (Fig. 3b).



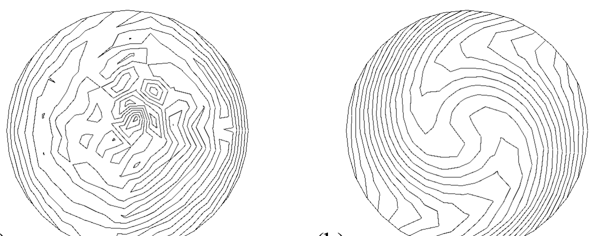
(a) (b) (c)

Fig. 2. (a) Flux lines and (b) magnetic vector potential of the benchmark model rotating at –10 rad/s and at (c) 100 rad/s.



(a) (b)

Fig. 3. Convergence rate for (a) the Galerkin approach applied for a velocity of 1 rad/s and (b) the artificially diffusion technique applied for a velocity of 100 rad/s.



(a) (b)

Fig. 4. Benchmark model: flux lines for a velocity of 100 rad/s of (a) unstable solution obtained by the Galerkin method and (b) stable solution by applying artificial diffusion.

External Circuit Condition

The 2D electromagnetic model is completed by an external circuit condition. The voltage V_{sol} across the rotating cylinder is defined as an additional unknown whereas the total current I_{sol} through its cross-section Ω_{sol} is forced to zero. This condition is imposed by adding the discrete form of the integral equation

$$I_{sol} = \int_{\Omega_{sol}} J \, d\Omega = \int \left(\frac{\sigma}{\ell} V_{sol} - \sigma \mathbf{v} \cdot \nabla A_z \right) d\Omega = 0 \qquad (5)$$

to the system of equations. A general method to couple an arbitrary electric circuit to the motional magnetic model is pointed out in [5].

Application: Electromagnetic Brake

A electromagnetic braking device consists of a steel disk posted between four magnetic poles with windings excited by a DC current (Fig. 5). The steel is conductive and exhibits non-linear magnetic properties. A part of the flux induces an electromotive force in the disk, causing eddy currents and therefore producing torque. Another part closes through the coils and the air gap. Both the electromotive force and the leakage flux become more and more important for increasing velocities. As a result, the dependence of the torque on the rotation speed is difficult to predict.
The motional magnetic finite element simulation coupled to the electric circuit condition is repeated within the adaptive mesh refinement procedure. If the magnetic energy converges up to 0.1%, the torque operating upon the rotating disk is computed using the Maxwell stess tensor. The torque reaches a maximum at 110 rad/s but decays significantly for higher velocities (Fig. 6).

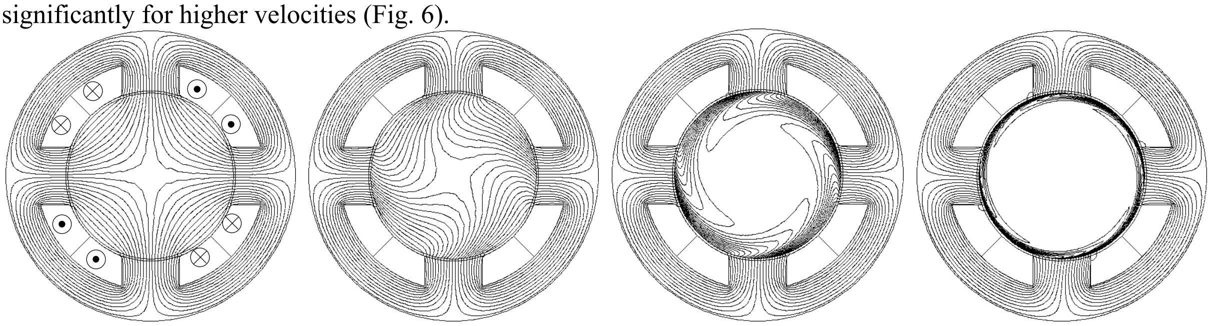


Fig. 5. Flux lines in a magnetic brake rotating at (a) 0 rad/s, (b) 1 rad/s, (c) 10 rad/s and (d) 100 rad/s.

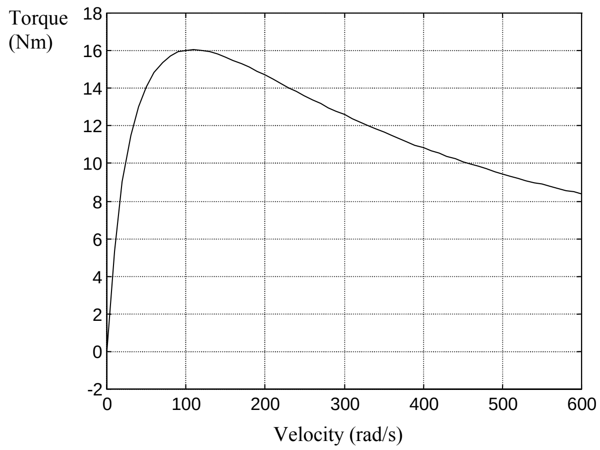


Fig. 6. Braking torque operating on the rotating disk with respect to the velocity.

Conclusion

In the case of fast rotating devices, a local upwind scheme is combined with adaptive mesh refinement. The upwind technique ensures non-oscillatory solutions even for rough meshes. The adaptive refinement procedure yields a sufficiently accurate final solution keeping the mesh as small as possible to prevent large computational efforts. The external electric connections are modelled as a boundary condition. All techniques are applied to the simulation of the braking torque generated by a 4 pole cylindrical magnetic brake. Measurements at a prototype brake will be performed to verify the computed results.

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