Adaptive Relaxation Algorithms for Thermo-Electromagnetic FEM problems

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Abstract — **Coupled thermo-electromagnetic problems are non-linear. Therefore, the solution process involves an iteration algorithm, often in a block iteration scheme. To prevent divergence and accelerate convergence a relaxation scheme has to be applied. An adaptive scheme is proposed and implemented in a coupled solver. The performance of the methods used is demonstrated at an induction motor simulation and an electroheat application.**

Index terms — **Non-linear differential equations, relaxation methods, finite element methods, eddy currents, electrothermal effects**

I. INTRODUCTION

Electric and magnetic field equations contain non-linearly temperature dependent material properties. Electromagnetic heat sources are usually described by a non-linear expression. Therefore, a non-linear coupled solution with the thermal field is required to simulate electromagnetic devices [1] accurately.

Different algorithms exist to obtain the coupled field solution. Due to the non-linearities, the convergence can become troublesome [2]. Precautions in the form of relaxation algorithms have to be taken to prevent divergence, numerical oscillations and to accelerate the convergence.

II. COUPLING ASPECTS

Electrostatic field equations may contain electric characteristics altering with the temperature. Dielectric losses contributes to rises in the thermal field.

The coefficients in the different formulations of the magnetic field equation may alter strongly under varying temperature conditions.

- The reluctivity of certain ferromagnetic materials changes at elevated temperatures; this effect is significant in induction heating applications.
- The characteristics of hard magnetic materials (permanent magnets) shift with temperature. The performance characteristic of permanent magnet machines is influenced by this effect.
- The temperature dependence of the electrical conductivity of current carrying materials is important. In non-static magnetic fields, eddy current effects arise influencing the conductivity locally.

The heat sources associated with the magnetic field are joule losses caused by the driving currents and the induced eddy currents. Iron losses occur in ferromagnetic materials subject to hysteresis phenomena.

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II. SOLVING THE NON-LINEAR COUPLED FEM-EQUATIONS

The following discussion is limited to a coupled magnetic-thermal field problem. The methods can be applied to electric-thermal problems or triple fields such as magnetic/electric/thermal field problems as well.

A. Matrix System Building

After discretisation on a finite-element mesh, the FEM equations of the thermo-magnetic coupled <*A,T*>-problem have the form (1). It is not necessary that the meshes on which the magnetic subproblem and the thermal subproblem are discretised, are identical. For instance, air regions carrying leakage flux are replaced by a convection boundary in the thermal model. The regions on which both fields are present, may be discretised by separate submeshes. In this case, interface equations extending (1) describing the projection have to be added.

$$\begin{bmatrix} K_A^i & C_A^i \\ C_T^i & K_T^i \end{bmatrix} \cdot \begin{bmatrix} A^{i+1} \\ T^{i+1} \end{bmatrix} = \begin{bmatrix} F_A^i \\ F_T^i \end{bmatrix} \quad (1)$$

The stiffness matrix contains two sparse diagonal blocks consisting of the same terms as in the decoupled single field problems. The entries of the terms in the off-diagonal blocks and the right-hand side vector are depending on the non-linear iteration scheme.

- Piccard or successive substitution locally keeps the coupling terms constant and hence they move to the right-hand side, leaving off-diagonal blocks filled with zeros. The matrix equation can be decomposed into two sub-equations solved in successive steps.
- Newton-Raphson: the matrix performs the function of the Jacobian containing the partial derivatives. The off-diagonal blocks are non-zero.

B. Block Iteration Schemes

The obtained system can be solved by different linear block iteration schemes. Hereafter the corrections in the surrounding non-linear iteration loop are determined.

A complete system solution is appropriate to solve the system arising when Newton-Raphson is applied. Due to the different nature of the equations, numerical difficulties may occur.

For the block diagonal systems constructed in a Piccard iteration, two solution schemes are possible. The first one resembles the Jacobi iteration method to solve linear systems. Both equations are solved simultaneously with appropriate linear equation solvers before the updates are determined.

The alternative is a Gauss-Seidel-like iteration scheme in which the non-linear updates for the next block to be solved are determined as soon as the previous block is solved.

If convergence is obtained without relaxation, the Gauss-Seidel scheme needs less iteration steps. The Jacobi method however has the possibility to perform the iterations for both subsets of equations in parallel. If the information vectors can be exchanged fast enough, the solution speed of this scheme is acceptable.

C. Stopping Criteria

A stopping criterion can be set in the L_2- or L_∞-norm of the difference of two consecutive solution vectors. Both subproblem solutions can be used, depending on the objective of the calculation. A weighted combination of the relative norms of both subproblems is appropriate as well.

III. Adaptive Relaxation

A. Relaxation Principle

The steps in the non-linear iteration loop are time-intensive since several large linear systems have to be solved. By applying an appropriate relaxation scheme, some of the steps can be avoided.

During the convergence towards the solution, the values of the non-linear coefficients may strongly oscillate. Possibly these excursions jump to regions with phase transitions and discontinuous changes of the material parameters. This numerical difficulty is avoided by damping the system of equations.

A relaxation scheme with different coefficients for both subproblems is proposed. The intermediate solution vector used to build (1) for the next step $i+1$ is shown in (2). The vectors with the indices $i+½$ represent the last completed solution. The vectors with index i are the previous complete solution vectors.

$$\begin{bmatrix} A^{i+1} \\ T^{i+1} \end{bmatrix} = \begin{bmatrix} A^{i} \\ T^{i} \end{bmatrix} + \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \cdot \left(\begin{bmatrix} A^{i+1/2} \\ T^{i+1/2} \end{bmatrix} - \begin{bmatrix} A^{i} \\ T^{i} \end{bmatrix} \right) \qquad (2)$$

The parameters r_1 and r_2 ought to minimise the residuals (3) [3], calculated in one matrix product and addition:

$$\begin{bmatrix} R_A \\ R_T \end{bmatrix} = \begin{bmatrix} K_A^{i+1} & C_A^{i+1} \\ C_T^{i+1} & K_T^{i+1} \end{bmatrix} \cdot \begin{bmatrix} A^{i+1} \\ T^{i+1} \end{bmatrix} - \begin{bmatrix} F_A^{i+1} \\ F_T^{i+1} \end{bmatrix} \qquad (3)$$

These residuals can be calculated in a period of time which is significantly smaller than the solution of (1). Only one sparse matrix-vector product and a vector addition are required. However, since the non-linearly dependent matrix elements have to be rebuild, the number of residual calculations is limited.

The ideal relaxation parameters minimise the generalised problem residual in (4).

$$M(r_1, r_2) = w_1 \frac{\|R_A\|_{2/\infty}}{S_A} + w_2 \frac{\|R_T\|_{2/\infty}}{S_T} \qquad (4)$$

w_1 and w_2 are weighting factors. S_A and S_T are scaling parameters to allow the comparison of both residuals.

B. Determination of the Relaxation Parameters

The determination of the ideal set of relaxation parameters (r_1, r_2) is a bounded optimisation problem on its own. Several methods exist to solve it [4], but the number of residual calculations has to be restricted. For instance, it is possible to construct a response surface through some points in the (r_1, r_2)-plane. To obtain a reasonable good estimate, at least a 3*3=9 evaluations have to be performed to determine the location of a minimum, plus the evaluation in this point.

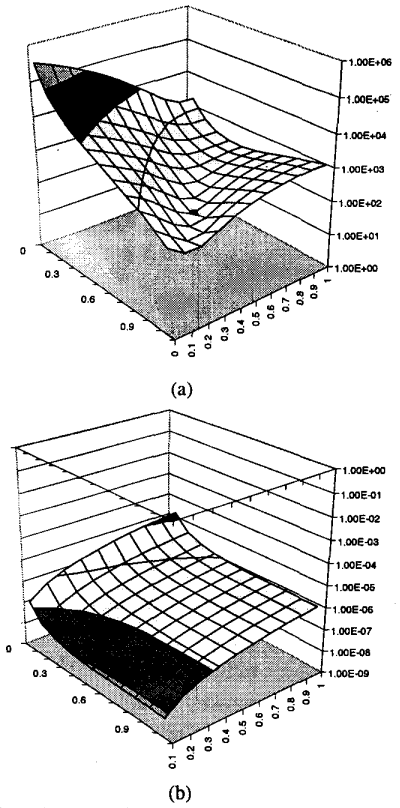
C. A Fast Search Algorithm

Here, a fast search algorithm is proposed in the form of a marching scheme through the (r_1, r_2)-plane. Consecutive sets of parameters are tested to find an estimate for the minimum norm of the residual over the (r_1, r_2)-plane:

- The start set is ($r_{1,0}$, $r_{2,0}$), here fixed at $r_{1,0}$=$r_{2,0}$=1.0. $M(1.0, 1.0)$ is calculated. If overrelaxation is allowed, a higher number can be chosen.
- Next, two other sets are tested: $M(d_1*r_1, r_2)$ and $M(r_1, d_2*r_2)$ with d_1 and d_2 as given pre-set parameters. Every time, the residual norm is calculated and compared to the previously attained minimum, multiplied by a penalty factor that augments at every double pace. The set evolving towards the lowest norm is retained as a start set for the next iteration.
- There is a maximum for the number of tests.

In this way, a local minimum is found. In practice this proves to be almost always the global minimum as well for a proper choice of the parameters.

Fig. 1 shows the residual planes as they are found in the first step of the simulation of an induction heating problem starting from a zero-field initial solution in the magnetic field and at constant temperature. The search algorithm managed



(a)

(b)

Fig. 1. Residuals in the beginning of coupled simulation (a) L2-norm of thermal residual vector (b) L2-norm of the magnetic residual vector.

to locate the minimum according to (3) at (0.3, 0.9) in 10 steps, with d_1 and d_2 both equal to 0.9.

D. Starting solution

The quality of the starting solution is very important in non-linear iterations [5]. If no information is available, A^0 is assumed to be all zeros and T^0 equal to the thermal problem solution without heat sources (usually equal to the ambient temperature).

Better results are obtained when a multi-level approach is employed. The projection of the results of the calculation of coupled problems on a coarser - unrefined - mesh is an interesting starting solution [5].

III. Applications

A. Equations

The proposed relaxation algorithm has been tested on several simulations of thermo-electromagnetic coupled problems. The applications shown here are typical 2D coupled eddy current-temperature problems. The equation used to model the magnetic field is the vector potential in the frequency domain formulation (5).

$$\nabla(\upsilon(A,T)\nabla(A)) - j\omega\sigma(T)A = J_s \tag{5}$$

If the problem is voltage driven, the right-hand side term becomes temperature dependent as well.

$$J_s = \sigma(T)V_s \tag{6}$$

The thermal equation is given by (7), in which the heat source terms of (8) are found.

$$\nabla(k(T)\nabla(T)) = -q_{joule}(A,T) - q_{iron}(A,T) - q_{external} \tag{7}$$

$$q_{joule}(A,T) = \frac{J_{rms}(A)^2}{\sigma(T)} \tag{8a}$$

$$q_{iron}(A,T) = f(\omega)(\nabla \times A(A,T)) \tag{8b}$$

B. Test Models

The calculated models are a voltage driven induction motor extended with circuit equations and theoretical inductive heating problems.

Due to the heating effects in the induction motor, the resistances of the stranded stator and solid rotor windings change. This influences the magnitude and phase of the stator currents and the distribution of the rotor currents and hence the overall operation of the machine. The knowledge of the change of the machine parameters is of importance for control purposes (Fig. 2).

Here, $q_{external}$ is assumed 0.

In an induction heating problem, similar effects are found, but the temperature gradients are higher. The test model consists of two circular solid bars of which one is driven by an external source. The second bar is assumed to be short-circuited. Skin effects occur. Due to the short distance, proximity effects are seen as well.

C. Results

The model has been calculated without relaxation by means of the Jacobi- and the Gauss-Seidel method. The

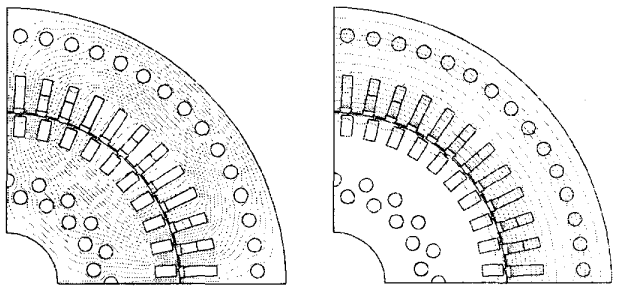
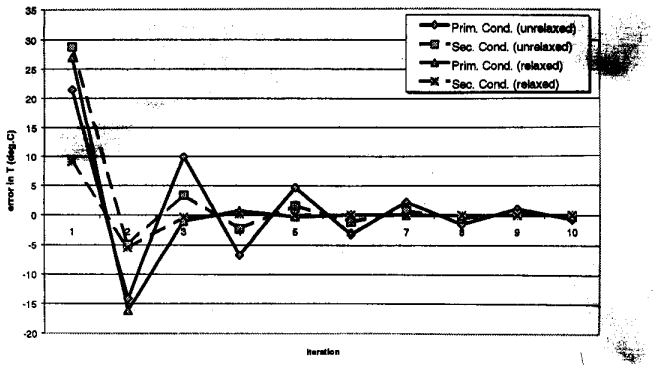
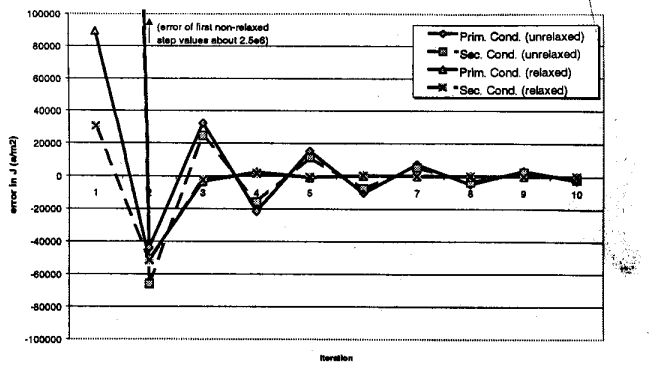


Fig. 2. (a) time-harmonic solution of the magnetic field of an induction motor, (b) isothermal lines.



(a)



(b)

Fig. 3. Evolution of the temperature and the current densities. The graphs show the difference with the value after convergence.

number of non-linear iterations is shown in Table 1. The stopping criterion in all the calculations was set at $\left\|T^{i+1}-T^{i}\right\|_{\infty}<10^{-4}$.

To illustrate the convergence properties, the thermal starting solution is assumed to be equal to the ambient temperature (20°C). In the first iteration step the magnetic calculation results in high current densities since the conductor materials are still 'cold' with a low resistivity. The high density heat sources are causing the temperature rise swiftly. In the following iteration step, the conductors are more resistive and hence carry less current. This leads to moderate heat sources and thus a lower temperature. In the following steps this oscillating behaviour continues. Fig. 3 shows the evolution of the temperature and current densities in an induction heating problem, calculated using a Gauss-Seidel iteration.

If relaxation is applied, less iteration steps are required (Table 1). Less overshoot occurs. The numerical oscillations at the end of the iteration loop vanish. Fig. 4 shows the

TABLE 1
COMPARISON OF # OF ITERATIONS FOR THE DIFFERENT ALGORITHMS

Algorithm	Induction motor	induction heating of a bar
Jacobi	16	85
Jacobi relaxed	11	56
Gauss-Seidel	11	38
Gauss-Seidel relaxed	6	23

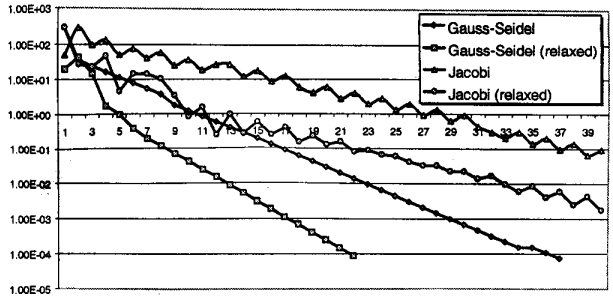


Fig. 4. Evolution of the L_∞-error norm for different iteration algorithms.

evolution of the L_∞-error norm. The damping is the strongest in the first steps, where the error decreases faster than in the unrelaxed version. The error evolution of the first steps of the damped Gauss-Seidel iteration shows that initially a smaller step is taken, after which a larger step directly towards the final solution is chosen.

Initially the thermal residuals have a large impact to the starting solution (Fig.1). In the first iteration steps it can be noticed that the residuals are very sensitive to the corrections of the magnetic solution. This can be explained by the squaring in (8a).

Further on during the convergence, the dependence of the conductivity in the magnetic equation becomes relatively stronger and small damping factors are applied to limit the oscillations.

CONCLUSIONS

A generally applicable adaptive relaxation algorithm to calculate coupled electromagnetic fields by means of the finite element method is presented. It can be applied to non-linear coupled systems that are solved as a complete system or by means of a Jacobi- or Gauss-Seidel-like block iteration algorithms.

The fast searching algorithm used to determine an estimate for the minimum of the generalised problem residual is proposed. The performance of the relaxation is demonstrated by means of coupled field calculations on an induction motor and electrothermal problem.

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