Magnetostriction in a Finite Element Model

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***Abstract*— The paper has two objectives. The first one is to demonstrate how magnetostriction formulae found in literature can be implemented in a finite element (FE) formulation. Such an implementation is not obvious because the deformation of the material needs to be adequately taken into consideration in the magnetic finite element equations. The formulae needed to do so are presented. They allow to draw clearly the distinction between the form effect and the magnetostriction itself. The second objective of the paper is, thanks to the finite element model, to compare various local models of magnetostriction with measurements.**

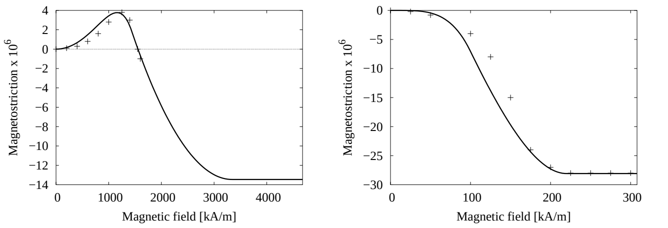

Fig. 1. Comparison of the model proposed by Likachev [2] and the measurements reported by [3] for pure Ni (left) and pure Fe (right).

FE WITH MAGNETOSTRICTION

Let Ω be a piece of a magnetostrictive material. The natural variable to discuss magnetic phenomena in magnetic materials is the magnetic field $\mathbf{h}$. One has usually

$$\mathbf{h} = h_1\,\mathbf{e}_1 + h_2\,\mathbf{e}_2 + h_3\,\mathbf{e}_3 \qquad (1)$$

where the h_i's are the coefficients of $\mathbf{h}$ in the basis vectors $\{\mathbf{e}_i\}$, $i = 1, 2, 3$. Constitutive laws and energy are then algebraic expressions of the h_i's. This representation of the magnetic field works fine as long as Ω remains undeformed.

If now Ω deforms, an important question arises that is worth being stated explicitly. One needs indeed to decide whether the reference frame $\{\mathbf{e}_i\}$ will or not be involved in the deformation, i.e. whether $\mathbf{h}$ can be represented in an external global non-moving frame (attached to the laboratory for instance) or in a local deforming material frame. This is actually not a matter of choice. As the material laws are related with the local properties and the state of the matter, the field must necessarily be defined on the material domain, and its components must be expressed in the (comoving) material frame.

In such conditions, the coefficients h_i's do not any longer hold a complete information about the variation of $\mathbf{h}$. A more involved mathematical framework is required, which allows to handle adequately fields defined over deforming domains. This formalism, called *differential geometry*, tells that the magnetic field must be represented by a 1–form and provides the rules to compute its material derivative $\mathcal{L}_{\mathbf{v}}$ (i.e. its variation as Ω deforms).

All calculations done, the formulae governing the magnetomechanical coupling under the assumption of small displacements, can all be translated back in terms of the components of $\mathbf{h}$ in a non-moving frame attached to the undeformed state of Ω. Given an expression $\rho^\Phi(\mathbf{h}, \varepsilon)$ of the coenergy density as a function of the magnetic field and the strain tensor, these formulae (which will be presented in the full paper) allow to derive systematically the correct and energy-consistent FE equations of a magnetostrictive material. One finds for the variation of ρ^Φ

$$\delta \int_\Omega \rho^\Phi = \int_\Omega \left\{ \partial_{\mathbf{h}}\rho^\Phi \cdot \mathcal{L}_{\mathbf{v}}\mathbf{h} - \sigma_M \, : \varepsilon \right\} \qquad (2)$$

with the coupling stress tensor $\sigma_M = \partial_{\mathbf{h}}\rho^\Phi \mathbf{h} - \rho^\Phi \mathbb{I} - \partial_\varepsilon \rho^\Phi$. Complete definitions will be given in the full paper. One can already note that the stress due to reluctivity forces and responsible for the so-called form effect (first 2 terms), and the stress due to magnetostriction strictly speaking (last term) are clearly distinguished in σ_M. These physically indissociable effects can thus easily be analysed separately with the FE model.

APPLICATION

The FE model model will be used to compare various local models of magnetostriction found in the literature with measurements, Fig. 1. In particular, a magnetic ring core is considered. A coil wound around the core creates a magnetic field at a frequency of 0.1 Hz. The induction field and the deformation, in radial and tangential directions are measured. In such a closed-loop magnetic circuit, magnetostrictive effects cease to be overriden by reluctivity forces. This application is therefore very useful to discriminate between different models of magnetostriction found in literature.

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