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Design and Optimization of Electrotechnical Devices

KAY HAMEYER & RONNIE BELMANS

SUMMARY *The development and design of electromagnetic devices reflects a complex process. Originating from an initial idea, the construction runs through different phases. This procedure is terminated when a final concept is selected and considered to be optimized, subject to various targets and constraints. As a whole, the task of the design engineer is to find solutions for technical problems. On the way to the final physical and technical product, certain aspects have to be considered. Technological and material questions, cost effectiveness and ecological constraints have to be taken into consideration. A subset of the boundary conditions controls the feasibility of the final design. With emphasis on electromagnetic devices, Fig. 1 shows a simplified scheme of the interdependencies of targets and constraints. This simple pattern shows that the design process is strongly dependent on the experience of the engineer and reflects an optimization procedure with often contradictory aims. Therefore, the necessity of a systematic design using engineering tools is obvious [1]. In this paper, solution strategies using modern numerical methods to accelerate and ensure a high-standard technical product in an overall design process are discussed. A numerical optimization tool is used to automate the design process to obtain an automated optimal design according to given constraints and limitations. Practical design examples conclude the paper and demonstrate the behaviour of the strategies and methods used. The combination of numerical field computation and optimization methods produces an automated and structural development procedure and uses the analysis tools in a powerful design methodology with a general application range.*

1. Introduction

Designing electromagnetic devices includes the calculation and analysis of the electromagnetic field distribution. From the local field quantities, forces, torques and losses can be derived to make predictions concerning converted power and efficiency. For complicated geometries, analytical field solutions are non-existent or very hard to obtain. Using numerical field computation techniques of general application range, the microscopic field solution leads via a lumped parameter approach to the desired time-dependent behaviour of the device (Fig. 2). The microscopic field solution itself delivers important knowledge about material utilization. Such results offer the opportunity to reduce the material, weight and costs of the final product. To accelerate development, extensive field computations with various types of material can be performed avoiding expensive prototyping. It is even possible to predict the system behaviour before new materials are actually available. As a consequence, numerical optimization algorithms are combined with field calculation methods. This paper

K. Hameyer and R. Belmans, Katholieke Universiteit Leuven, EE Department, Division ESAT/ELEN, Kardinaal Mercierlaan 94, B-3001 Leuven, Belgium. E-mail: kay.hameyer@esat.kuleuven.ac.be.

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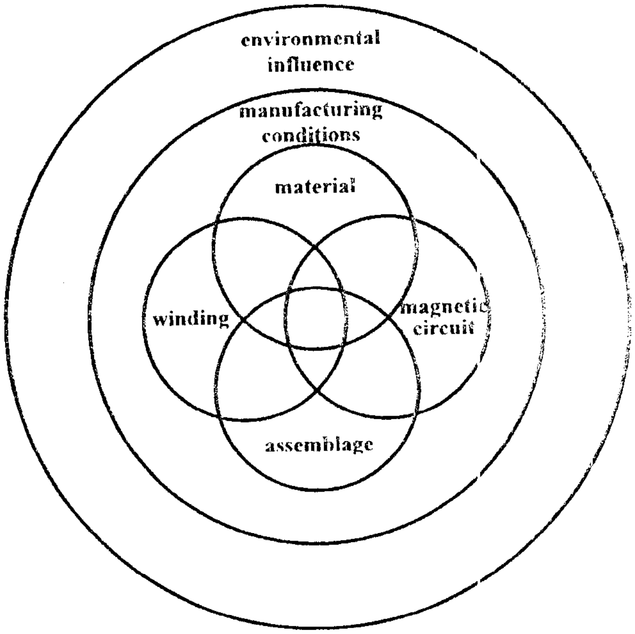


FIG. 1. Interdependencies in the design of electromagnetic devices.

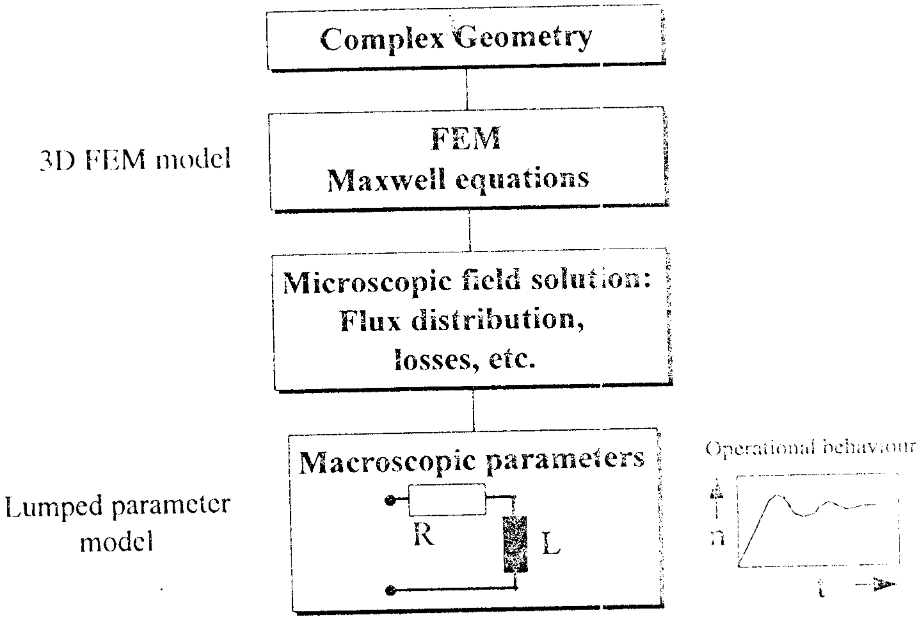


FIG. 2. Analysis scheme using the finite element method.

introduces a heuristic optimization technique as a combination of evolution strategy and simulated annealing [2]. The application of the whole design tool consisting of field analysis and the numerical optimization method is demonstrated, with discussion of an automated optimal design (AOD) of a small consumer product, a brushed dc motor.

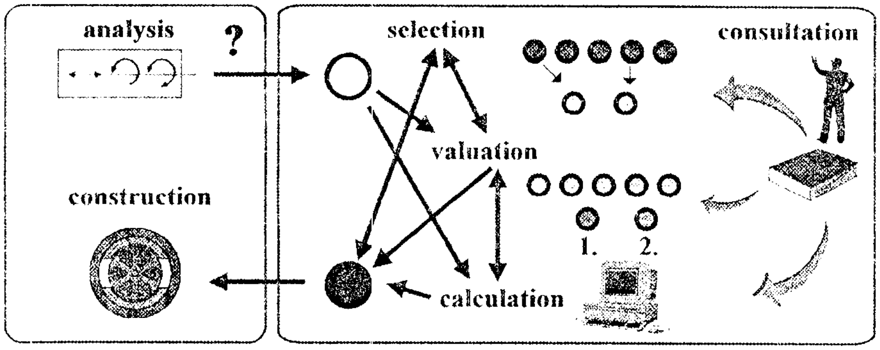


FIG. 3. Knowledge-based and structured design.

To illustrate the generality of the methods used, a micromotor design is discussed and as a third example the optimization of the pole structure of a high-voltage transmission line is shown.

2. Development and Design Strategies

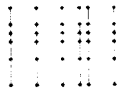
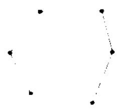
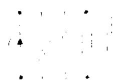
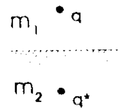
To put the numerical optimization procedure in its proper place, the development process is briefly commented on. The main part in the structured development of novel technical products is analysis followed by detailed synthesis. Analysing means obtaining information about partial functions of the desired overall function, investigating single elements and their mutual iterations. In this way, links between various principles of the partial functions are found. In Fig. 3, a structured and knowledge-based development process is illustrated. In this example, a final technical product has to be designed, which is able to fulfil three partial functions. Those individual functions to be connected to the overall task of the product are a linear motion, a continuous rotation and a reverse operation. After the analysis phase, in the synthesis step different physical principles are selected to be evaluated. The selection process is governed by simple qualitative rules. In this way the partial functions are evaluated with regard to their feasibility with respect to given constraints and limitations.

The process of synthesis leads from qualitative decisions to quantitative statements [3]. The whole process is accompanied by the consultation of experts and expert knowledge (Fig. 3). A detailed investigation and ranking, i.e. the precise calculation of the operating conditions, leads by a comparison to the final technical product. In this loop of interactions between validation and performing detailed predictions of qualified concepts, numerical optimization combined with field computation methods is found to be an important and powerful engineering tool for the design of electromagnetic devices. This paper focuses on the practical use of numerical design tools applied to electromagnetic problems of technical importance. For the right choice of field computation method, a brief review of modern numerical field simulation techniques is provided in the next section.

3. Electromagnetic Field Computation

Depending on the class of field problem, different computation methods are feasible. To select the most efficient one, again a decision process is necessary. The various

TABLE I. Field computation methods

Method	Principle of the discretization	Geometry approximation	Non-linearities	Computational costs
FEM		Extremely flexible	Possible	High
FDM		Inflexible	Possible	High
BEM		Extremely flexible	Troublesome	High
MEC		Specific geometries	Possible	Very low
PMM		Simple geometries	To consider only by constant factors	Low

possibilities range from simplified analytical approaches over semi-numerical to numerical techniques. The drawbacks and strengths of the single techniques are given in Table I. Only the most common computational techniques for electromagnetic field problems are listed. The main difference between the methods is their flexibility in correctly representing the geometry of the electromagnetic circuit, as the accuracy of the solution depends on the discretization of the geometry of the device. Furthermore, the treatment of non-linear material is difficult.

Looking at the point matching method (PMM), it is obvious that the use of this method is limited to geometrically simple shapes. Non-linear ferromagnetic parts of the magnetic circuit can only be considered by constant factors. The magnetic equivalent circuit (MEC) method represents flux paths by magnetic reluctances. The field distribution must be known in broad lines before the magnetic circuit can be modelled [4]. The main advantage of this lumped parameter approach is its computational speed. The boundary element method (BEM) approximates complex geometries very precisely. Only the boundary of the area of interest has to be discretized. The treatment of non-linearities is troublesome. With the finite difference method (FDM), local mesh refinements are not possible and this technique has shortcomings in complex geometries. The field computation method with the most general application range is the finite element method (FEM). Due to the discretization of the field region by non-overlapping triangular elements for two-dimensional and tetrahedron elements for three-dimensional problems, complex geometries can be approximated. Non-linearities and local mesh refinements are possible. The high computational costs of this method may be a drawback. The advantages of using the FEM are predominant. Therefore, this paper focuses on this method.

In principle, field computation is performed in three major steps: pre-processing,

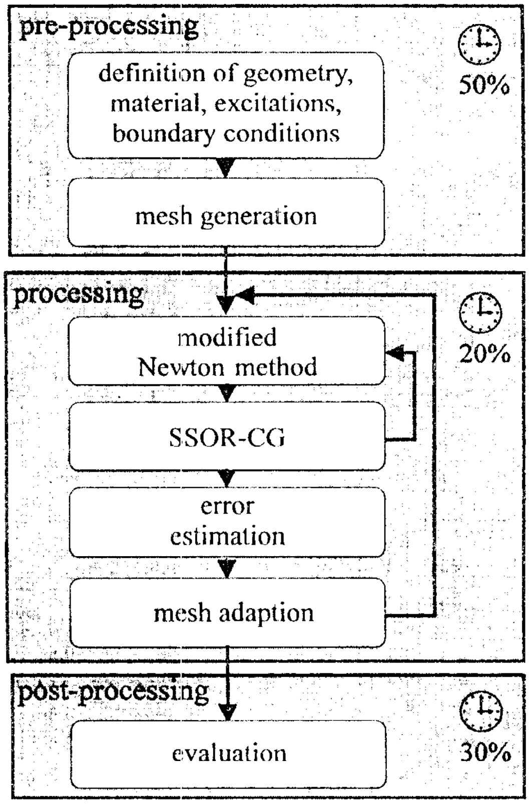


FIG. 4. FEM procedure.

processing and post-processing. Figure 4 shows a typical pattern for the FEM approach. The first step consists of the definition of the geometry of the electromagnetic device. Material properties, electrical excitations, applied voltages or currents and boundary conditions are defined. All the activities have to be carried out by the design engineer. Therefore, the pre-processing is time consuming. An estimated time expenditure for all three steps is given in Fig. 4. When defining complex three-dimensional finite element problems, the time estimates have to be increased for the pre-process and the post-process.

The processing, i.e. the solution of a very large system of equations, is done automatically in the second step. Only parameters to control the solution process have to be defined by the design engineer. In the last part of the FEM procedure, the field quantities of interest are computed from the solution produced by the processing. If the geometrical data can be parametrized, the pre- and post-processing may be automated. This fact and the strict structure of the FEM procedure represent important prerequisites for the possibility of the combination of field computation and numerical optimization.

4. Numerical Optimization

In general, optimization means finding the best solution for a given problem formulation under the consideration of prescribed constraints. In mathematical terms:

> Define a point $\mathbf{x}_0 = (x_1, x_2, \ldots, x_n)^T$ with the independent variables $x_1, x_2, \ldots, x_n$ in such a way that by the variation of the objective variables, inside the admissible space, the value of a quality function $Z(\mathbf{x}_0)$ reaches a minimum or maximum. The point $\mathbf{x}_0$ is described as the optimum.

For the optimization, the exact definition of the space of variables, the restricting constraints and a quality or objective function is necessary. The correct and functional expression of the objective function is of particular importance. The final product of the optimization is a reflection of this function. The quality function must guarantee the existence of an extremum. This can be reached by the right choice of restrictions. The objective variables should be normalized to get a well-conditioned problem formulation, i.e. small variations of the variables result in a small change of quality. When multiple aims have to be followed, a weighted linear combination of the single targets is useful:

$$Z = \sum_{i=1}^{n} a_i Z_i \tag{1}$$

Here, a_i are the valence factors and Z is the overall quality. A prerequisite for a successful optimization is a clear definition of all partial targets and the variability of the objective variables, i.e. the single targets must not contradict too strongly [5]. In general, a mathematical optimization problem can be formulated as follows:

$$Z(\mathbf{x}) = (x_1, x_2, \ldots, x_n) \rightarrow \min \tag{2}$$

with the independent variables $\mathbf{x} = \{x_i; i = 1(1)n\}$ in the space $\mathbf{x} \in \mathbb{R}^n$ and the $j = 1(1)m$ constraints

$$C_j(\mathbf{x}) = C_j(x_1, x_2, \ldots, x_n) \left\{ \begin{matrix} = \\ < \end{matrix} \right\} 0 \tag{3}$$

Optimization algorithms are generally constructed so that the desired optimum is reached step by step. This happens by using determined rules. The generation of the step width that varies the objective variables, the search direction on the way to the optimum and the stopping criterion distinguish between the various numerical optimization algorithms. Detailed investigations and descriptions of different techniques can be found in [5]. Due to complicated dependencies of the free variables, the direct computation of derivatives of the quality function is troublesome or sometimes not possible. Stochastic methods work without the use of derivatives. They are easy to implement and the treatment of constraints is simple. These are the reasons why stochastic methods are a reliable tool in the field of numerical optimization of electromagnetic fields. In the following, two stochastic search techniques and their methodology are described.

4.1 Evolution Strategy

The evolution strategy copies the principles of biological evolution into the mathematical process of optimization. The term 'mutation' describes the variability of the objective variables and the term 'selection' is the equivalent of Charles Darwin's (1809–1882) postulate 'survival of the fittest' [6]. The driving force of this strategy is the repetition of the mutation of variables and selection in successive steps. In Fig. 5, a simple pattern of the evolution strategy is shown. Some important features and possibilities of the algorithm can be observed from this drawing. If each fruit represents

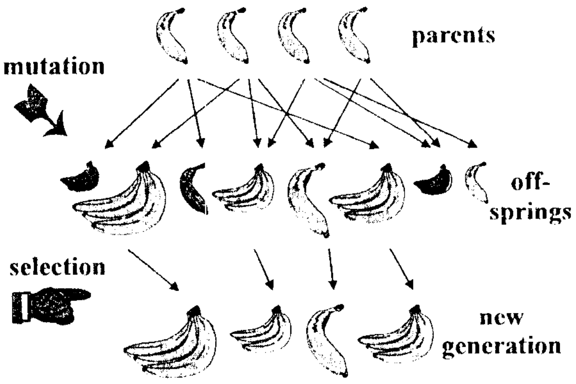


FIG. 5. Schematic of the evolution strategy.

a set of variables, a vector describing the technical product containing all the design parameters, the procedure starts with a given number of μ parent vectors. To generate the λ offspring vectors, mutations of the elements of the design parameter vector are performed by random additions to the parent variables. The arrows indicate that ρ parent variables contribute to the generation of a single child. With the hereditary factor ρ, on average $1/\rho$ of the properties of the parents are transferred to an offspring.

The mutation of the design parameters is done by random additions to the parent variable vector with $\delta_{O,i}^{(k)}$ as the step length and $p_i^{(k)}$ the search direction:

$$x_{O,i}^{(k)} = x_{P,i}^{(k)} + \delta_{O,i}^{(k)} p_i^{(k)}, \qquad \text{with } |\mathbf{p}^{(k)}| = 1 \tag{4}$$

If the step width is too small, the procedure is slowed down and the algorithm gets trapped in a local optimum. If the step width is chosen too large, this results in pure fluctuation of the process. To get an efficient algorithm, the step width must be self-adaptive. Therefore, an intermediate value of step length of the parents $\delta_{P,i}^{(k)}$ is transferred to the offspring $\delta_{O,i}^{(k)}$:

$$\delta_{O,i}^{(k)} = \frac{1}{\rho} \sum_{j=1}^{\rho} \delta_{P,j}^{(k)}(z_j^{(k)}(\mu)), \qquad \text{with} \begin{cases} i = 1\,(1)\,n \\ j = 1\,(1)\,\rho \end{cases} \tag{5}$$

The final step length is generated by applying a step length factor γ to ensure searching in long and short distances in the variable space:

$$\delta_{O,i}^{(k)} = \begin{cases} \delta_{O,i}^{(k)} \cdot \gamma, & \text{for } i = 1\,(1)\,\lambda/2 \\ \delta_{O,i}^{(k)}/\gamma, & \text{for } i = \lambda/2 + 1\,(1)\,\lambda \end{cases} \tag{6}$$

The search direction is generated in such a way that the probability of every direction in the solution space is uniformly distributed. With the previously introduced control parameters λ, μ and ρ, several evolution strategies can be distinguished, differing in the type of selection only. The most powerful strategy is the listed 'comma' variant.

- $(\mu + \lambda)$-plus-strategy: The population of the next generation is selected from μ parents and λ children.
- (μ, λ)-comma-strategy: The population of the next generation is selected from the μ best children. Parents survive only one single generation.
- $((\mu/\rho) + \lambda)$-strategy: ρ parents contribute to the creation of a child. $1/\rho$ of the properties of one parent are transmitted to a child.
- $(\mu/\rho, \lambda)$-strategy: Comma variant of the last listed strategy.

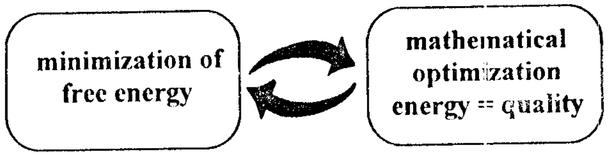


FIG. 6. Basic idea of simulated annealing.

4.2 *Simulated Annealing*

Annealing describes the physical process of heating up a solid to a maximum temperature at which all molecules are freely moving and the process of slowly cooling down to a state of minimum free energy. The decrease of temperature must be slow enough so that at every temperature the solid is in a thermal equilibrium. This process describes a natural optimization with the object of minimizing the free energy. The probability of the energy E_i in a state i at temperature T is described by the Bolzmann distribution. Therefore, the probability of the change of energy can be expressed by

$$\xi < prob\{\Delta E_i\} = \exp\left(-\frac{\Delta E_i}{k_B T}\right) \qquad (7)$$

Substituting the term energy with the term quality, as indicated in Fig. 6, the physical process and equation (7) can be used in the mathematical optimization as an acceptance criterion. If the variation of objective variables results in a better quality, the configuration is accepted. On the other hand, if the change in quality is larger than zero, the configuration is treated as follows. A uniformly distributed random number ξ from the interval [0, 1] is generated. If the Metropolis criterion (7) is true [7], the new configuration is accepted. Otherwise, a new mutation on the free variables is performed. To complete the algorithm, a temperature schedule is necessary, the simplest one being linear [8].

4.3 *Combined Search Algorithm*

Test calculations discovered a weakness of the evolution strategy. Using this method, the global optimum is not always reached. In contrast, simulated annealing is well known for its reliability in finding a global extremum. Therefore, the evolution strategy is combined with simulated annealing. The final algorithm as a combination of both methods is shown in Fig. 7. The design parameters are varied according to the rules of the evolution strategy. Then the new sets of design parameter vectors are judged. Improvements in quality are accepted and a decrease in quality accepts or rejects the set of parameters Boltzmann weighted according to the Metropolis criterion. In this step, the geometrical constraints are controlled as well. If constraints are violated the parameter set is rejected and a new design parameter vector is generated to be checked again, and if accepted with respect to the constraints, it will be evaluated by the quality function in the post-process (Fig. 8).

The overall process control is illustrated in Fig. 8. This process introduces an AOD procedure. An initial set of design parameters, obtained from a basic design concept following the structured design strategy described earlier in this paper, accompanied by preset constraints is given to the process as input data. Using the FEM to evaluate the quality function of the electromagnetic device, the geometrical constraints are controlled in the pre-process. Question marks in Fig. 8 indicate in which step the

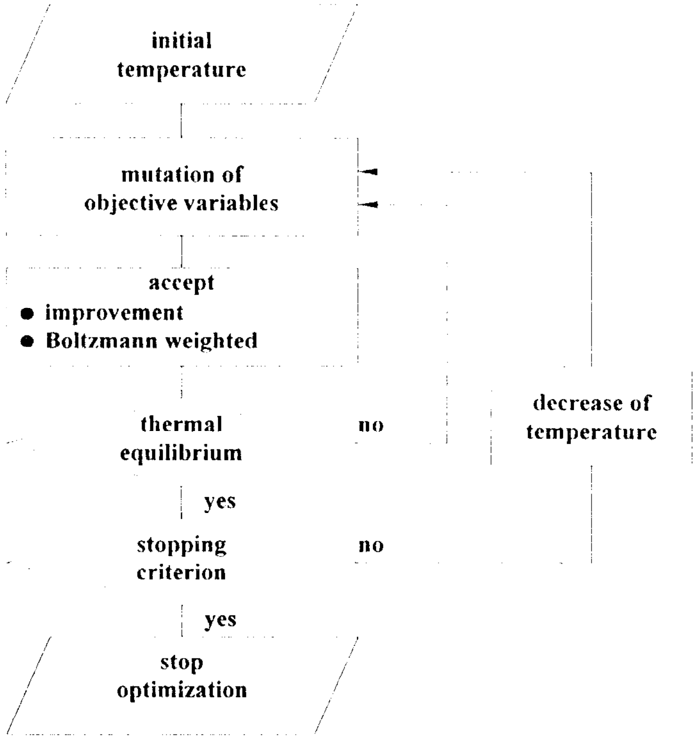


FIG. 7. Combined search algorithm.

constraints are checked. If they are not violated, the field simulation is performed and in the post-process of the FEM the physically and technically founded constraints, in general generated forces and/or minimal/maximal obtained field quantities, are checked. The resulting qualities for the single data set are transferred to the optimization algorithm to be judged according to the strategy used. If a stopping criterion is fulfilled, the optimization is finished. Otherwise, the design parameter sets are varied by the optimization algorithm and the design loop is closed by giving the new parameters back to the field computation.

This type of process offers the possibility of easily changing the numerical strategies used. The two branches, field simulation to evaluate the quality function and optimization method, are independent. Therefore, different techniques may be introduced. Because of the stochastic optimization method, the generated values of the design parameters are equally distributed. Indicated by dots in Fig. 8, this offers the opportunity to use parallel computers in the process to reduce the computational costs.

5. Design Examples

To point out the general application range of the methods introduced, examples from different application fields of electrotechnical importance are chosen. To demonstrate the performance of the algorithms in designing an electromagnetic device the optimization of a small dc motor is shown. The design of an electrostatic micromotor [9] and the optimization of the electrostatic field of an ac high-voltage line are also discussed. In optimizing the ac high-voltage transmission line the point matching

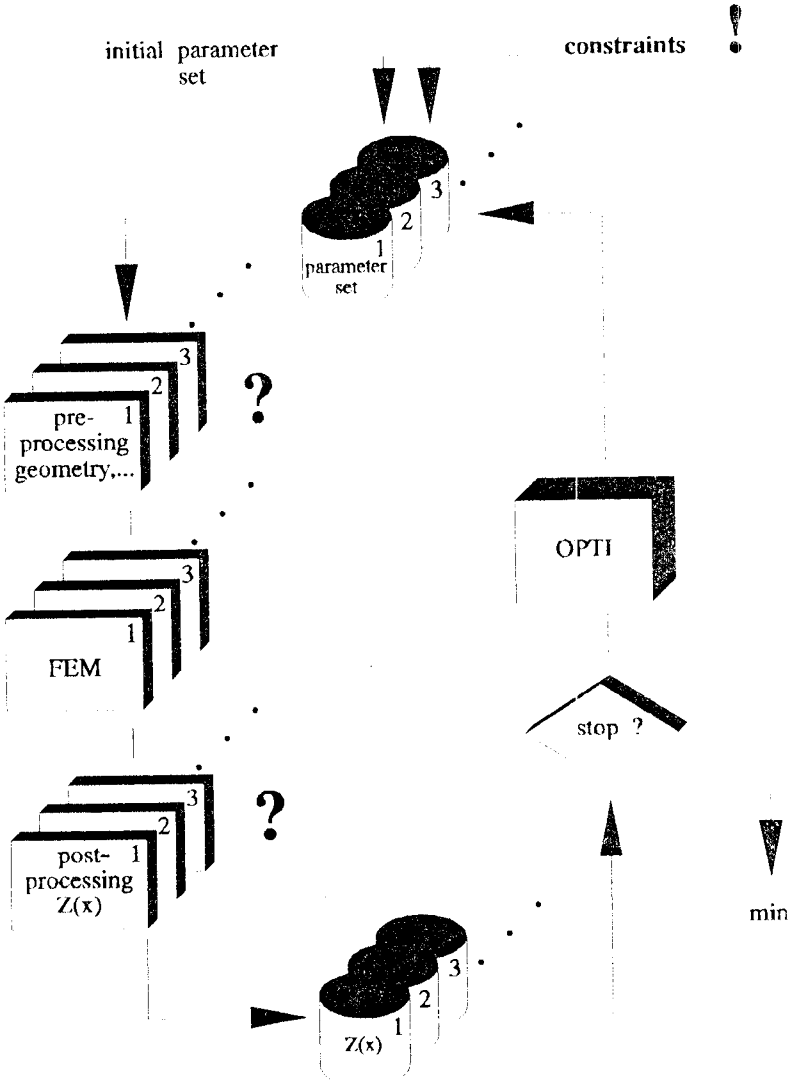


FIG. 8. AOD process control.

method (Table I) is used to compute the electrostatic field strength below the span field.

According to the overall procedure (Fig. 8), a set of initial design parameters (the parent vectors), describing the geometry of the device, are given to the field computation procedure. Geometrical constraints are checked in the pre-process and possible limits of the field quantities in the post-process.

5.1 *Shape Optimization of a Small DC Motor*

With the combined optimization technique, the shape of a small dc motor is optimized, aiming to minimize the overall material cost subject to a given minimum torque. Quality function evaluation is done with the FEM. Figure 9 illustrates the problem formulation. The free design parameters are the coordinates of the data sample of a

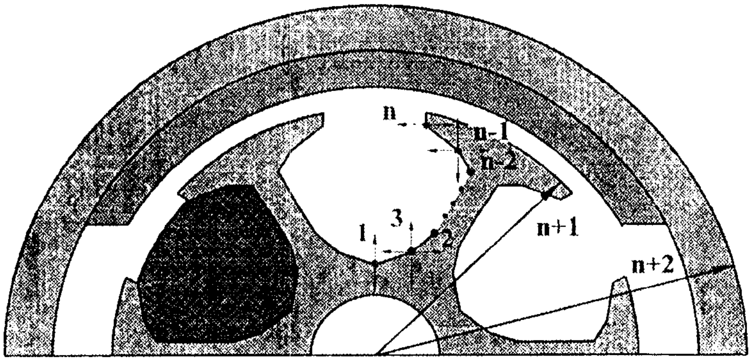


FIG. 9. Motor geometry with the design parameter definition.

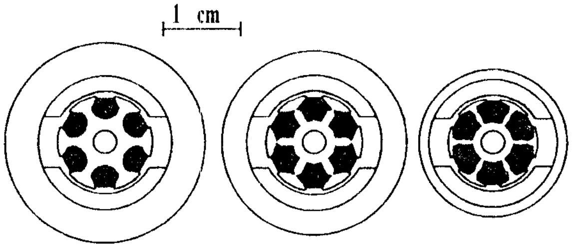


FIG. 10. Initial, temporary and optimized shape.

polygon describing the rotor slot shape, the rotor radius and the outer radius of the motor. In this example, 14 design parameters are tuned. The quality function is formulated in such a way that a minimum quantity of torque $T_{\min}$ is obtained due to a penalty term evaluated in the FEM post-process and the overall costs are computed during the pre-process:

$$Z(\mathbf{x}) = 10^{\left(\frac{\mathrm{cost}(x) - \mathrm{costmax}}{\mathrm{costmax}}\right)} + \mathrm{penalty} \tag{8}$$

$$\mathrm{penalty} = \begin{cases} T < T_{\min}, & 10^{\left(\frac{T_{\min} - T(x)}{T(x)}\right)} \\ T \geq T_{\min}, & 1 \end{cases} \tag{9}$$

The overall costs are estimated by the sum of the copper, lamination and permanent magnet expenditures. Constraints result from manufacturing conditions. The air gap is set to a fixed value, and minimal tooth width and slot opening are given. The initial shape of the motor generates 75% of the desired torque. The change of geometry during optimization is shown in Fig. 10. The quality expressed by the material costs decreased by 10%. In this example, a (12/12, 60)-strategy has been used.

5.2 *Design of an Electrostatic Micromotor*

The same approach as described for the dc motor is used for designing an electrostatic micromotor. Here, the torque production is based on the variation of the capacitance between the rotor and stator electrodes as they move relative to each other. Designing such motors may require the use of the very expensive LIGA technology as reported in

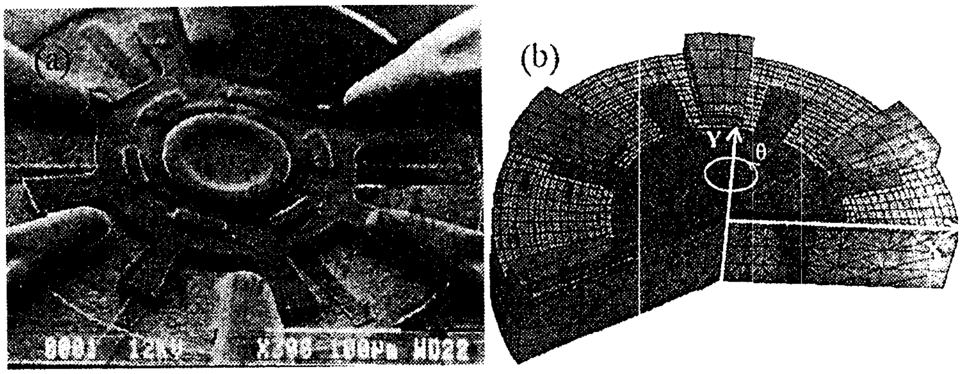


FIG. 11. (a) Axial field type electrostatic micromotor with double stator system and (b) the corresponding three-dimensional FEM. *Source*: ESIEE, Paris.

Mohr *et al.* [10]. Therefore, prototyping to optimize the motor properties is out of the question. Because of the shaft length/diameter ratio, the field computation is performed by the FEM using standard tetrahedron element types to obtain a three-dimensional field solution. Because of the very complicated geometry that may be generated using standard etching techniques, as shown in Fig. 11(a), the three-dimensional field approach is highly recommended (Fig. 11(b)).

The main problem with such electrostatic micromotors is to find a design generating a sufficient torque to fulfil given requirements and motor properties. The average torque must overcome friction losses to guarantee the functioning of the micro device. With a rotor diameter of 320 μm, a rotor thickness of 4 μm and an axial air gap of 3 μm, the generated average torque referred to the applied voltage is in the range of 0.06 pNm V^{-2}. The movement of the rotor is obtained by cyclic switching of the voltage, exciting the stator electrodes. Because of the construction of the motor, the torque is inherently superposed by a torque ripple.

In Fig. 11(b), the construction of the FEM is illustrated. To generate the model, the two-dimensional radial cross-section of the motor is rotated around the y-axis to build the complete extruded three-dimensional model of the motor. The stator is designed as a double system to allow rotor motion between the upper and lower part of the stator electrodes. To consider the torque ripple and to minimize it while maximizing the average torque of the micromotor, several rotor positions, applying different stator excitation sequences, are simulated and evaluated by the optimization algorithm. The variation of the capacitances as a function of the rotor position depends on the geometry of the electrodes. Hence, the pole arc of the stator electrodes and the pole arc of the rotor electrodes are the design parameters. To tune the objective variables, the optimization is performed by a (5/5, 14)-strategy.

Computed results for the average torque obtained by the above-mentioned methods are shown in Fig. 12. Exciting an axial field 6/4 pole configuration with 50 V, the maximum average torque reaches a value of 150 pNm. The optimizer tuned the design parameters for the stator pole arc to 44.4° and to 37.8° for the rotor pole arc to obtain the maximum average torque.

5.3 Minimization of the Electric Field of an AC High-voltage Line

In this example the objective function evaluation is performed by the PMM (Table I), in order to minimize the average value of the electrostatic field generated by

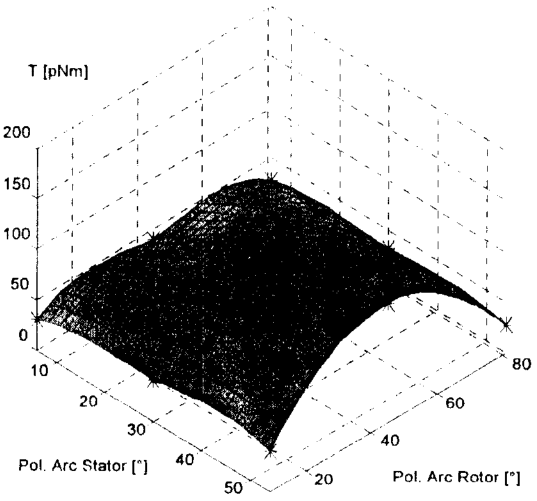


FIG. 12. Average torque as function of stator and rotor pole arc.

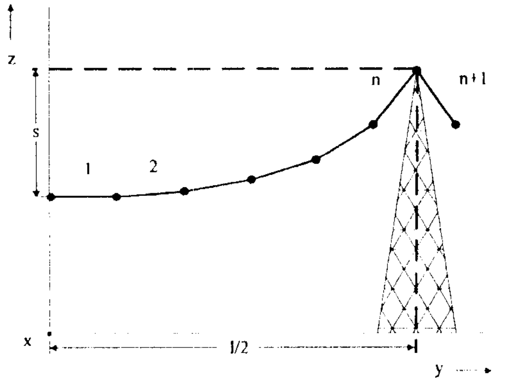


FIG. 13. Modelling of a conductor by a polygon.

high-voltage lines. Because of their influence on the field distribution, the design parameters are the coordinates of the ground lines. With respect to the slack of the phase conductors, a three-dimensional field computation is required. A single transmission line is approximated by a polygon (Fig. 13). Each element of the polygon represents a constant line charge. It is assumed that the ground potential below the high-voltage line is zero. To evaluate the field quantities with respect to this boundary condition, the potential of the line charge has to be mirrored at the ground plane $x-y$. Superposition of the original and the arbitrary mirror charge gives the resulting potential [11].

In Fig. 14, the initial and optimized pole configurations are shown. From Fig. 15, the computed field distribution, one metre above the ground, can be taken. During optimization the average value of electric field-strength decreased by about 8.6% in total. For the computation, a (4/4, 12)-strategy is used.

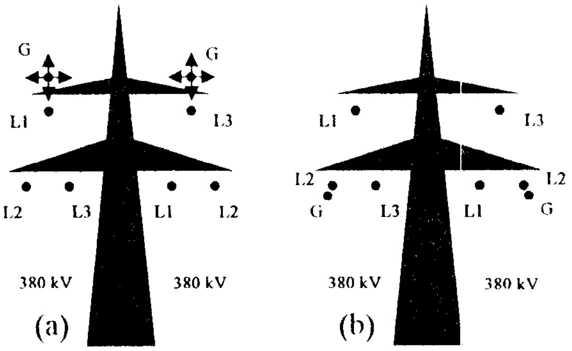


FIG. 14. (a) Initial and (b) final pole configuration.

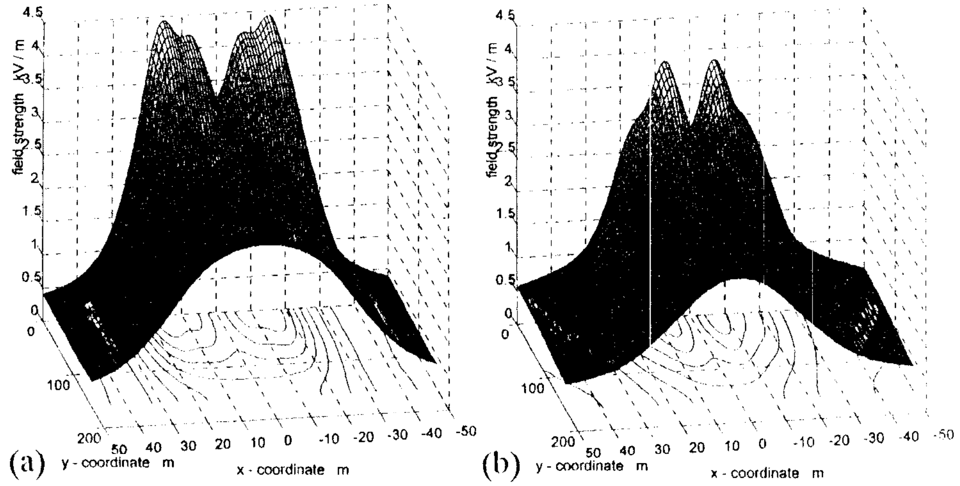


FIG. 15. (a) Initial and (b) optimized field distribution.

6. Conclusions

A brief introduction to a structured and knowledge-based design process is given. Starting from these ideas, the paper explains where the numerical optimization fits in to a design process. A combined stochastic search method is introduced. The basic ideas of the evolution strategy and simulated annealing lead, in combination, to a reliable and robust optimization algorithm. This technique together with field computations yields a powerful tool for the design of electrotechnical devices. Three examples from the area of devices operated by electromagnetic and electric fields demonstrate the general application range and suitability of the methods.

Stochastic search methods inherently need a lot of objective function evaluations. Therefore, optimization using such methods consumes a lot of time. By optimizing electromagnetic or electric fields the right choice of field simulation method can decrease the computational effort. The evolution strategy without the combination with simulated annealing works very well and is rather fast convergent on semi-hard problems (high-voltage transmission line). Hard problems with various design parameters (dc motor) need the combination of both techniques and are very time consuming.

Further investigations concerning the contradictory aims of speeding up the optimization algorithm by simultaneously desired global convergence need to be carried out. Modified simulated annealing algorithms seem to be promising using the FEM to evaluate the quality function and are under investigation at the K. U. Leuven to tackle more complex problems of engineering importance.

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