Int. J. of Applied Electromagnetics and Mechanics 6 (1995) 367–376

Automated optimal design of a permanent magnet DC motor using global evolution strategy and FEM

M. Kasper[a], K. Hameyer[b] and A. Kost[c]

[a]*Forschungsschwerpunkt Mikroperipherik, Technische Universität Berlin, D-13355 Berlin, Germany*
[b]*Katholieke Universiteit Leuven, E. E. Dept., Div. ESAT/ELEN, B-3000 Leuven, Belgium*
[c]*Institut für Elektrische Maschinen, Technische Universität Berlin, D-10857 Berlin, Germany*

Received 30 March 1995

Abstract

Automated Optimal Design (AOD) of electromagnetic devices turns out to be a task of increasing significance in the field of electrical engineering. Often the direct relation between desired quality of the technical product and the design variables is unknown. Stochastic optimization methods in combination with general numerical field computation techniques like the finite element method (FEM) offer the most universal approach in AOD. The application to a nonlinear magneto-static problem of technical significance is demonstrated by minimizing the overall material cost of a small DC-motor by optimizing the rotor and stator shape.

1. Introduction

The combination of numerical field computation methods with stochastic search algorithms permits computer simulations to be extended to the design stage. Nowadays, numerical methods become more and more common in the design of electromagnetic devices. With increasing computer speed, complex problems can be solved with a high economical efficiency. To accelerate development expenditure, field computation of complicated geometries with various types of material can be performed thus avoiding expensive prototyping. As a consequence, numerical optimization algorithms are combined with field calculation methods.

Depending on the determination of the step length, the search direction and the stopping criterion various numerical optimization algorithms are introduced [4]. There is a distinction between deterministic and heuristic techniques. In this paper a heuristic search algorithm is introduced.

An advantage of stochastic search methods is their insensibility to disturbances of the objective function caused by numerical evaluation. This follows from non-deterministic search and disuse of derivatives. A second important property is the easy treatment of constraints. Hence a complicated transformation into an unconstrained problem formulation is unnecessary.

To solve the non-linear field problem a suitable method of wide application range has to be chosen. Here field calculation is accomplished by the FEM. Error estimation, adaptive mesh

0925-2096/95/$06.00

generation and refinement are used. This method guarantees the greatest possible facility in modeling and allows optimization without severe geometrical restrictions. Thus, compared to the gradient based optimization method reported in [9], no topological restrictions concerning the finite element mesh have to be considered.

2. Optimization method

Optimization in general means to find the best solution for a given problem with the consideration of several restricting conditions. In mathematical terms:

Define a point $\boldsymbol{x}_0 = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ with the independent variables $x_1, x_2, \ldots, x_n$ in such a way, that by the variation of the design variables, inside the admissible space, the value of a quality function $Z(\boldsymbol{x}_0)$ reaches a minimum or maximum. The point $\boldsymbol{x}_0$ is described as the optimum.

Therefore, optimization requires the concentration of all design aims into a single quality function

$$Z(\boldsymbol{x}) = Z(x_1, x_2, \ldots, x_n) \to \min. \tag{1}$$

This function depends on all independent design parameters $\boldsymbol{x} = \{x_i\colon i = 1(1)n\}$ and represents the quality of the specific design. Additional constraints limit the admissible parameter variation.

Stochastic strategies do not use the derivatives and only evaluate the objective itself. If the objective function is complicated and includes local optima or saddle points deterministic methods in general do not converge to the global optimum.

In this contribution a combination of evolution strategy and simulated annealing is used. The evolution strategy is well known to be a stable local optimizer, but has only poor global convergence

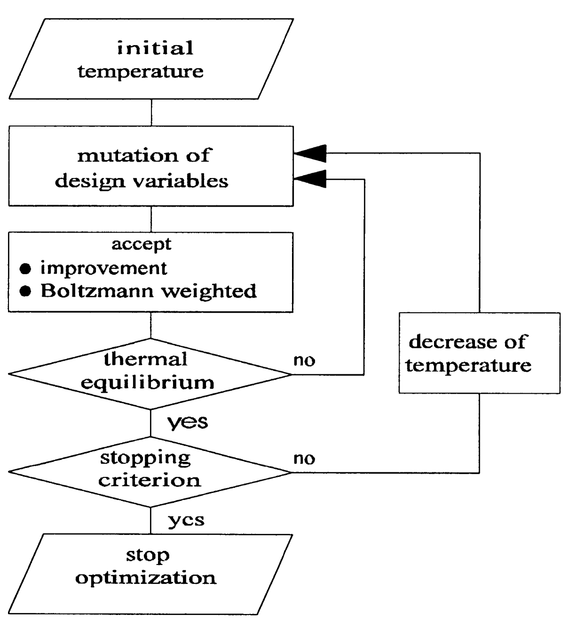

Fig. 1. Block diagram of the combined search algorithm.

properties. Simulated annealing is a global optimizer with slow local convergence speed. In the combined optimization algorithm the random variations of the design variables x_i are done using the principles of evolution strategy and the selection of the new parameter vectors is done according to the rules of simulated annealing (Fig. 1).

The evolution strategy copies the natural principles mutation and selection (*survival of the fittest)* of the biological evolution into the technical optimization problem. The basic concept of the evolution strategy is found in the substitution of Darwin's notion of fitness for the quality of a technical problem. The driving force in the optimization process is the repetition of mutation and selection in successive steps.

Rechenberg [3] transformed the scheme of biological evolution into a simple algorithm. This elementary procedure is termed the $(1+1)$-strategy. One parent (valid solution vector) generates a descendant, which differs by mutation of the design variables from the parent. After evaluation of the objective function the vector design variables of better quality is chosen to form the parent for the following generation etc.

Based on the $(1+1)$-strategy more general and powerful strategies were introduced by Schwefel [4].

The mutation of the design variables of an initial generation of μ admissible design vectors (parents) leads to a number of children. The variables of one child may depend on ρ parent variable vectors. The μ best children of λ are selected to form the next parent generation. This procedure is termed a comma-strategy $(\mu/\rho,\ \lambda)$. Compared to Rechenberg's [3] $(1+1)$-strategy the comma variant is more efficient [4].

In order to transfer this procedure to an efficient optimization method, a self-adaptive step length control is necessary. If the step length is too small, i.e., descendant and parent vectors are very similar, the method will exhibit a poor convergence rate. An improvement of quality will not be noticed. On the other hand, if the mutation step length is too large, the algorithm results in pure random variation. The convergence rate of evolution strategy is maximal in a narrow band of step length (evolution window). A decreasing step length *in average* ensures convergence during the optimization. Thus, the step length serves as a convergence criterion.

Simulated annealing describes the physical process of heating up a solid to a maximum temperature at which all molecules are freely moving and the process of slowly cooling down until a state of minimal inner free energy E_i is reached. This process describes a natural optimization, the minimization of the free energy. Simulated annealing introduces by the Boltzmann distribution the control parameter temperature into the search process [1, 2]. Thus the probability of the change of energy can be expressed by:

$$\mathrm{prob}\{\Delta E_\mathrm{i}\} = \exp\Big(-\frac{\Delta E_\mathrm{i}}{k_\mathrm{B}T}\Big). \tag{2}$$

This acceptance criterion aims to avoid the system getting stuck in a local minimum. Barriers of height $\sim k_\mathrm{B}T$, where k_B is the Boltzmann constant and T the temperature, can be surmounted on the way to a better solution. During the optimization the artificial temperature T is reduced by a simple schedule $T^{(k)} = T^{(0)}\alpha^k$, where k denotes the step of iteration and the reduction factor is $0 < \alpha < 1$.

To illustrate the simplicity and its universal applicability Fig. 1 shows a scheme of the algorithm described more detailed in [5].

3. Field computation

Two-dimensional field calculation is performed with standard finite elements using linear shape functions over triangular elements to approximate the vector potential. The method applies to nonlinear magneto-static or eddy current problems. Since a two-dimensional geometry is assumed, the vector potential consists of only one component [6, 7].

To guarantee global convergence of the nonlinear vector equation for the nodal potential $\boldsymbol{V}$

$$\boldsymbol{F}(\boldsymbol{V}) = 0 \tag{3}$$

a damped variant of the Newton method is used. This method exhibits good global (at least linear) and quadratic local convergence. It is important for the convergence of this method to determine a suitable step length. If $\boldsymbol{d}^{(k+1)}$ is the correction vector obtained from the solution of the linear system (Jacobian matrix), then a damping parameter α is chosen such that

$$\left\|\boldsymbol{F}\left(\boldsymbol{V}^{(k)} - \alpha \boldsymbol{d}^{(k+1)}\right)\right\|_2 \leqslant (1 - \sigma\alpha)\left\|\boldsymbol{F}\left(\boldsymbol{V}^{(k)}\right)\right\|_2 \tag{4}$$

is fulfilled. This condition guarantees that the residual decreases in each iteration.

$$\alpha = \beta^j \quad \text{with} \quad j = 0(1)j_{\max}. \tag{5}$$

Damping is performed in successive steps and the first j fulfilling condition (3) is chosen. Damping parameters are in the range $\beta = (0, 1)$ and $\sigma = (0, 1/2)$. Adequate values are $\beta = 0.5$ and $\sigma = 0.01$. An additional regularization of the search directing is not necessary. Only if the angle between Newton and the steepest decent direction is near to or larger than $\pi/2$ the Levenberg–Marquardt method gives an improvement.

The accuracy needed to solve the linear system by an iterative method can be derived from the accuracy of the last Newton step. An adequate error bound of the residual of the linear system is given by

$$\varepsilon_{\text{lin}} \leqslant \left\|\boldsymbol{F}\left(\boldsymbol{V}^{(k)}\right)\right\|_2. \tag{6}$$

Due to the low accuracy in the first Newton steps, computation effort in solving the linear system is reduced. The sparse linear system is solved by a preconditioned conjugate gradient method. To ensure controlled accuracy an adaptive mesh generation is employed.

An initial mesh is generated from any geometry represented by non-overlapping polygons. This mesh consists of a minimum number of triangles, i.e., no inner points are generated inside the polygons.

To control discretization and numerical errors the local error distribution is evaluated and local mesh refinement is used in successive steps until a given error bound is reached [8].

Figure 2 shows an initial and adaptive mesh for a small brushed DC-motor excited with permanent magnets in the stator. The adaptively refined mesh is coarse in regions where the local change of field quantities is small, e.g., outside the stator and fine inside regions with high saturation respectively. The resulting flux density distribution is plotted in Fig. 3.

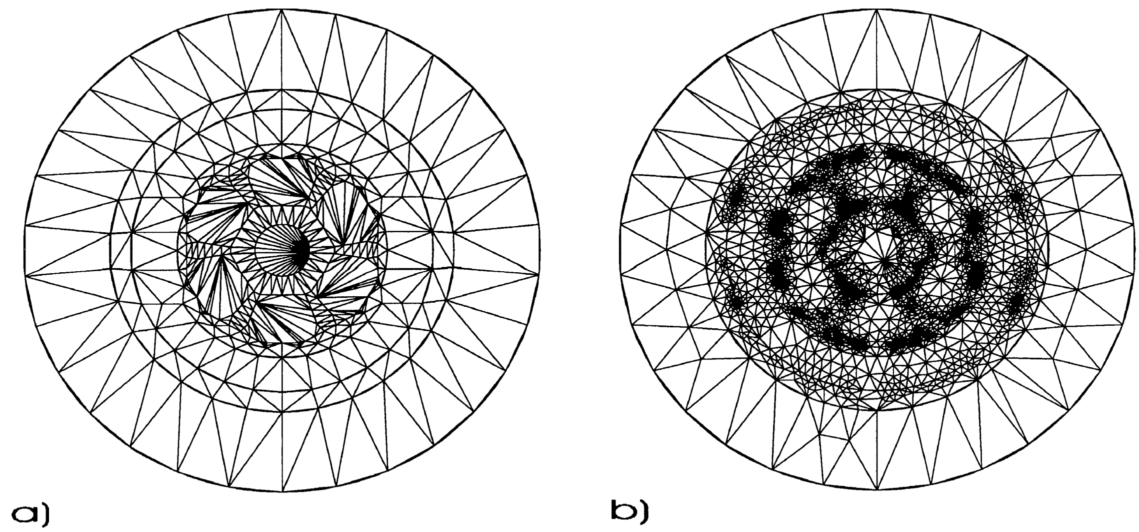

Fig. 2. a) Initial and b) adaptively refined finite element mesh.

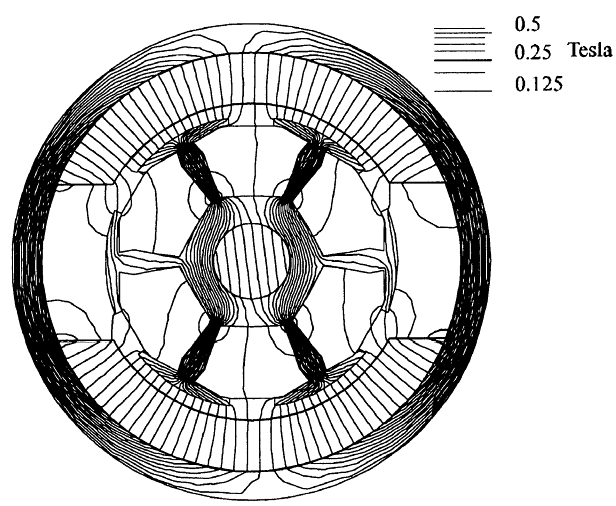

Fig. 3. Magnetic field distribution (iteration step 120).

4. Problem description

The application of the method is demonstrated by the shape optimization of a fractional horse power DC-motor. The objective is to minimize the overall material costs $cost(\boldsymbol{x})$ subject to a given torque T_{min}. Permanent magnet material, winding copper and iron lamination are taken into consideration. The quality function is defined to be a function of the total material costs.

$$Z(\boldsymbol{x}) = 10^{\frac{cost(\boldsymbol{x}) - cost_{\text{max}}}{cost_{\text{max}}}} + C(\boldsymbol{x}). \tag{7}$$

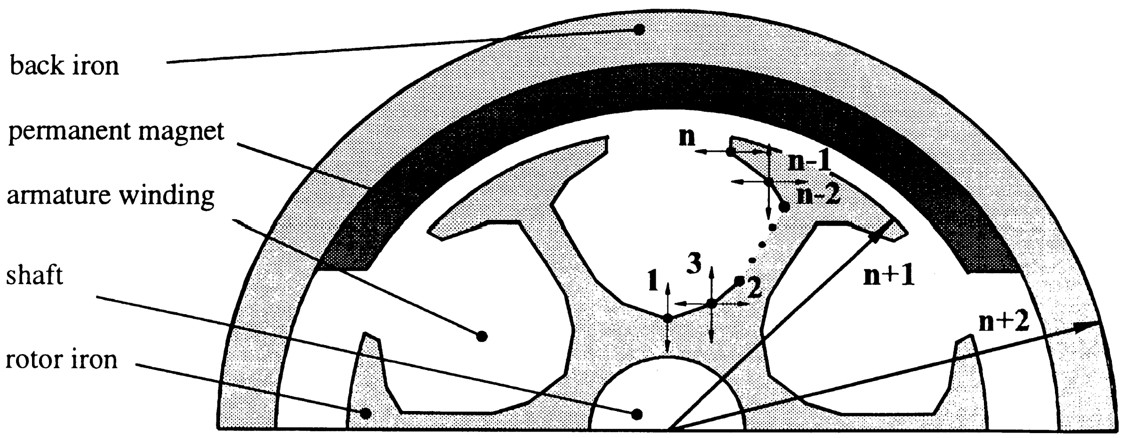

Fig. 4. Motor geometry and definition of design variables.

The use of a penalty term $C(\boldsymbol{x})$ in the form

$$C(\boldsymbol{x}) = \begin{cases} 10^{\frac{T_{\min} - T(\boldsymbol{x})}{T(\boldsymbol{x})}}, & T < T_{\min}, \\ 1, & T \geqslant T_{\min}, \end{cases} \tag{8}$$

allows to evaluate the objective function even if the torque constraint is violated. The sum of material costs is computed with the specific costs for winding copper: 5 US \$/Kg, lamination: 1 US \$/Kg and permanent magnet: 46 US \$/Kg. The terms (7), (8) were scaled to be equal to one for $cost\,(\boldsymbol{x}) = cost_{\max}$ and $T(\boldsymbol{x}) = T_{\min}$. The constraints were set to a torque of $T_{\min} = 0.22$ Nm/m and $cost_{\max} = 20$ US \$/m.

The torque $T(\boldsymbol{x})$ is computed by integrating the Maxwell stress tensor in the air gap region of the machine. Flux density depending rotor iron losses were taken into consideration at a rated speed of 200 RPM and subtracted from the air gap torque to form $T(\boldsymbol{x})$. The losses were computed by integrating the loss density across the rotor iron.

The geometry is modeled with 14 design variables. The free parameters are the $n/2$ edges of the polygon describing the rotor slot contour and the outer dimensions of rotor and stator as defined in Fig. 4.

Additional constraints result from fabrication conditions:

- magnet height fixed to 2.0 mm;
- constant air gap, set to 0.5 mm;
- minimal tooth width, set to 0.5 mm;
- minimal slot opening, set to 1.7 mm;
- pole arc of the permanent magnet, set to 121°;
- diameter of the shaft, set to 3.0 mm.

Some additional geometrical restrictions have to be introduced in order to guarantee the function of the motor expressed by its topology, e.g., the rotor has to be smaller than the stator.

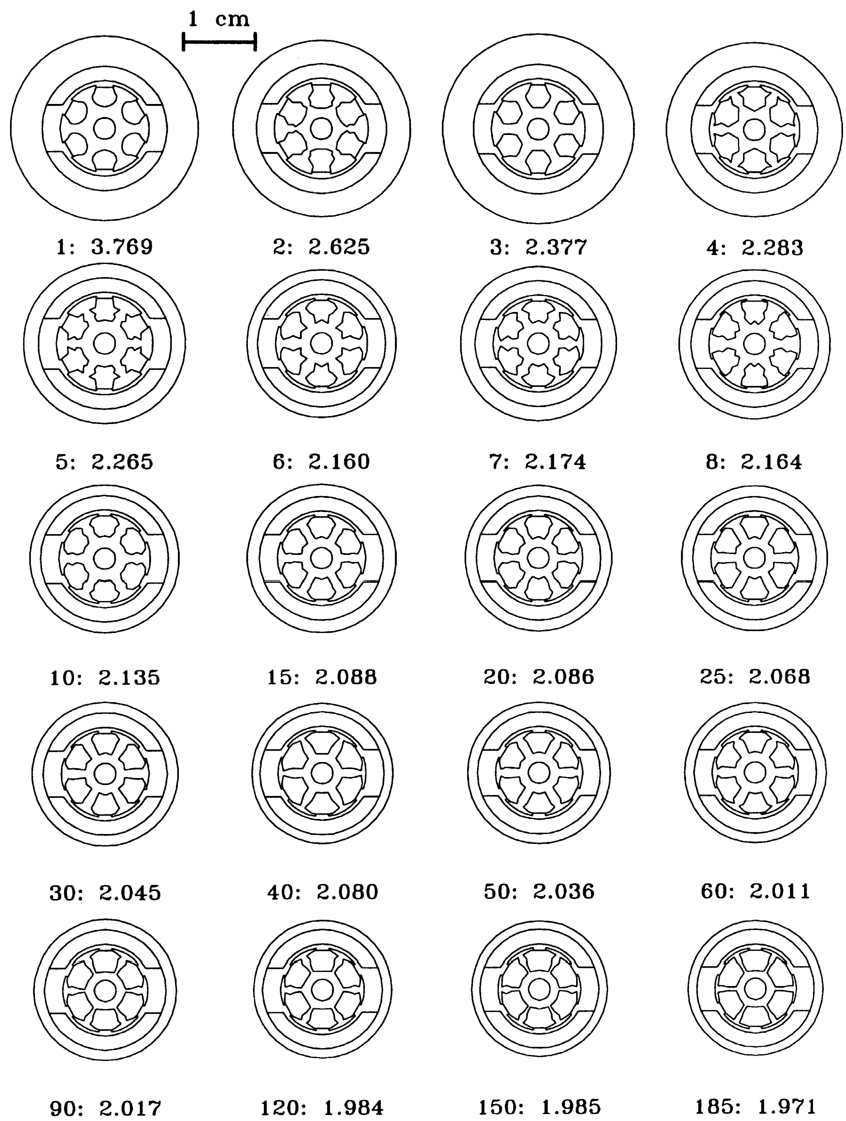

Fig. 5. Motor geometry variation during optimization of the best child in each generation. The given data indicate the iteration step with its value of quality.

5. Results

The evolution strategy used was the (12/12, 60)-strategy. Due to the complex and highly non-linear problem formulation, a reliable strategy with a rather great number of parents and children is recommended. The use of a *comma*-variant for the evolution strategy is advantageous with regard to the numerical inaccuracies of field and torque computation.

Starting from an initial value of 1 K the temperature was successively reduced every 30 iteration steps. The value of the starting temperature is problem dependent. The starting temperature has

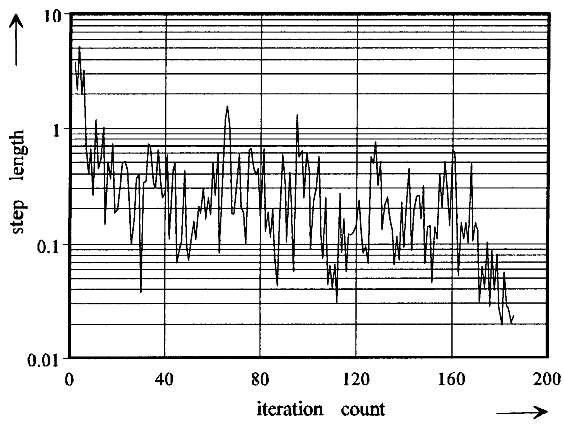

Fig. 6. Step length versus iteration count of the best child in each generation.

to be adjusted according the relative variation of the objective function. Within the picture of simulated annealing the starting temperature corresponds to a temperature near the melting point. Different cooling schedules have been investigated. It was found that a geometrically decreasing temperature schedule is sufficient.

During optimization an accuracy of at least 5% (energy norm) was required for the FEM field calculation. The number of elements constituting of the mesh was restricted to 20,000 elements.

Using this maximum number of elements and the mentioned accuracy, the computational costs for the evaluation of the quality function for each variant on a PC 486/66 lies in the range of eight minutes. The optimization process was stopped after 185 iterations. Thus, using a (12/12, 60)-strategy means 60 function evaluations per generation all along the 185 iteration steps.

The change of shape from a suboptimal initial geometry via temporary and the final shape of the motor is shown in Fig. 5. It can be noticed, that the iron parts of the initial geometry are over dimensioned. The actual torque of this configuration was approximately 25% lower than the desired value $T_{\min}$. The optimization algorithm very quickly decreases the overall dimensions of the starting configuration. After approximately 10 iterations the final outer dimensions are nearly reached. The fine-tuning of the remaining design variables controlling the shape of the slots is done in the following iterations. Thus, a fast convergence of the optimization strategy used can be stated.

The optimized motor holds the torque recommended, which mainly is achieved by enlarging the winding copper volume of about 20%. The most significant change from the start to the final geometry can be seen in the halving of the iron volume. Consequently the iron parts are highly saturated, especially the teeth regions. In comparison to this, a test optimization with neglected rotor iron losses resulted in a 10% smaller rotor diameter.

The total material costs are governed by the permanent magnet. The quality expressed by the material expenditure decreased approximately by 10%.

After optimization, the overall volume of the motor was reduced by 38%. In Fig. 5 this rapid decline of the outer dimensions in the first iteration steps is shown. Due to the large step length the geometry varies strongly and shows the examination of several unusual shapes. The improvement of quality is shown in Fig. 7. Comparing the parameters of the final geometry with the above

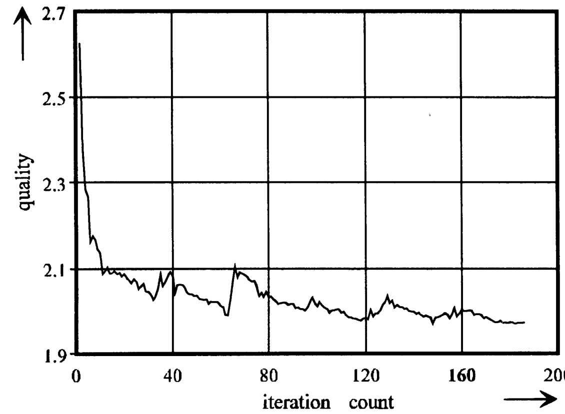

Fig. 7. Quality versus iteration count of the best child in each generation.

mentioned constraints resulting from fabrication conditions, it can be noticed that all minimal size constraints are active. On average, a continuing descent of step length in between successive temperature steps can be seen out of Fig. 6. Due to the use of evolution strategy as a local minimizer only a limited number of temperature steps are needed. At the temperature steps the optimization process is destabilized and step length strongly rises. This avoids the algorithm to get stuck in a local minimum and to ensure global convergence as Figs 6, 7 illustrate.

6. Conclusion

A method for global minimization of functions of continuous variables based on a combination of evolution strategy and simulated annealing was presented.

The highly non-linear electromagnetic field of a small DC-motor excited with permanent magnets has been optimized. Among different approaches, stochastic search strategies offer a simple and reliable alternative, since these methods do not require derivatives of the objective function. The main advantage of stochastic procedures is their stability with respect to discretization or numerical errors.

The objective function evaluation was performed with the standard finite element method using linear shape functions, error estimation and adaptive mesh generation. The methods introduced guarantee the greatest possible facility in modeling and allow the optimization without severe geometrical restrictions. A variant of damped Newton iteration has been introduced to guarantee convergence of the field computation during the optimization. The application of the method was demonstrated by shape optimization of a brushed fractional horsepower DC-motor.

References

[1] S. Kirkpatrick, C.D. Gelatt and M.P. Vecchi, Science 220 (1988) 671; also in: Neurocomputing, Foundations of Research, eds S. Kirkpatrick, C.D. Gellatt and M.P. Vecchi (1988) p. 554.

[2] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller and E. Teller, J. Chem. Phys. 21 (1953) 1087.

[3] I. Rechenberg, Evolutionsstrategie – Optimierung technischer Systeme nach Prinzipien der biologischen Evolution (Frommann-Holzboog, Stuttgart, 1973).
[4] H.-P. Schwefel, Numerical Optimization of Computer Models (Wiley, Chichester, 1981).
[5] K. Hameyer and M. Kasper, in: Optimization of Structural Systems and Applications (Elsevier, London, 1993) p. 15.
[6] C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method (University Press, Cambridge, 1987).
[7] P.P. Silvester, H.S. Cabayan and B.T. Browne, IEEE Trans. Power Apparatus Systems 92 (1973) 1274.
[8] R.E. Bank and A. Weiser, Math. Comp. 44 (1985) 283.
[9] S. Subramaniam, A.A. Arkadan and S.R. Hoole, IEEE Trans. Magn. 30(5) (1994) 3455.