Staged modelling: a methodology for developing real-life electrical systems Application to the transient behaviour of a PM servo motor

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Abstract – **This paper presents a methodology to achieve a global dynamic model of an electrical system that consists of a battery, an inverter, a permanent magnet servo motor and a turbine. The stress is placed upon the fact that a classical finite element model would not be able to provide a satisfactory representation of the transient behaviour of the whole system. A staged modelling is proposed instead, which succeeds in providing a complete picture of the system and which relies on numerous finite element computations.**

Keywords — Electrical machines, modelling, design.

1. Introduction

One has to analyse the transient behaviour of the system consisting of :

- ◇ a battery,
- ◇ a three-phase inverter bridge,
- ◇ a permanent magnet (PM) servo motor and
- ◇ a turbine (mechanical load).

As this system is foreseen for mass production, all elements are kept as simple and cheap as possible, e.g. very simple inverter with minimum control, standard magnetic iron (high iron losses), low DC supply voltage. Consequently, the system under consideration is characterised by a strong interrelation between :

- the different components (e.g. the waveshapes of the inverted currents depend on the back-emf level, and consequently on the speed, whereas in turn, the torque (and consequently the speed again) depends critically on those waveshapes) ;
- the different physical fields involved (e.g. temperature dependency and magnetic saturation of the cores) ;
- the different time scale (on the one hand the small time constant that characterises the electromagnetic phenomena in the motor and the commutations in the inverter bridge ; on the other hand the large time scale that characterises all mechanical and thermal phenomena).

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The goal of the analysis is to dimension the motor (geometry, magnets and coils) in order to fulfill a set of technical specifications concerning the *dynamics of the system* (ramp-up time, acceleration, . . .) and its *thermal robustness*. It can be noted that the specifications concern non-magnetic properties of the system, whereas the design parameters are related with the electro-magnetic functioning of the motor. This separation advocates clearly to adopting a global modelling approach, i.e. a model that involves not only the motor, as is usually the case, but also the supply (inverter) and the load (turbine).

In this context, a classical *finite element (FE) analysis* is not the most appropriate approach. Indeed, even if one manages to solve all equations together (magneto-dynamic, thermal, power electronics and mechanical), the model would anyhow be so heavy and slow that it would be of no practical use for design, because designing means exploring thoroughly a large domain of parameters. On the other hand, a simplified analysis based on an *analytical model* of the motor would fail to provide a sufficiently accurate description of the system, because of the critical importance of losses and saturation in this application.

This paper presents a methodology which, by doing a limited number of simplifications, allows to combine the *conciseness* of *analytical models* with the *accuracy* of *discrete models* (finite element models, time stepping in circuit equations, . . .). The provided model is exactly what is needed at the design stage.

2. Towards staged modelling

A. Finite element modelling

The reasons of the success of the finite element method are probably its wide applicability and flexibility, as well as the fact that the discretisation error can be theoretically made as small as wished. In view of the development of computers and numerical techniques in the recent years, one might feel free to imagine that there exist no limit to the power of representation of FE models and that all the various aspects of any system could be taken at once into consideration. Therefore, FE modelling is sometimes seen as the end of *empirical modelling*, which is the kind of modelling that consists in making simplifications and approximations, in carefully selecting the representative quantities of the system and finding out the relationships between them.

But the major trends in FE modelling of electrical devices (i.e. multiphysics, coupling with electronic circuits, 3D geometries, accurate material modelling, etc.) have remained unchanged for quite a time now (Hameyer, 1999). The development and validation of large FE models of complex electrical systems has become a common academic exercise, which requires months and which is nowadays a typical PhD thesis subject. Experience shows however that, even if such ambitious projects turn out to be successful at the end, the developed models lack generality, and they are very heavy and hardly reusable. Moreover, their accuracy can not be controlled, unless by experimental means in a limited set of particular situations.

B. Design of electrical systems

Let us consider the problem from another point of view. Engineers think in terms of a limited set of variables, even in presence of complex systems. The purpose of *design* is to build or to modify a system in order to make the interrelations between those variables match predefined technical specifications. Now, the *model* is the exploratory tool that should help in doing so, i.e. a thing one may interrogate at will instead of doing measurements.

But at R&D stage, designers are still essentially concerned with qualitative questions and they must usually work on basis of incomplete, not yet fixed or inaccurate data sets.

On the other hand, in order to be able to play a significant role in a modern design process, the model has to account for all important aspects of the system, including interactions with its input and output systems. It must also be flexible and open to gradual enrichment and improvements as the development progresses. Finally, its computation should require a reasonable amount of time and its accuracy should always stay under control. These are principles of what we call *staged modelling* : an intellectual construction, based on well-argued simplifications, and intended to help answering pre-defined technical questions. Although FE computations play an important role in building staged models, they are not themselves the kind of model we are looking for.

The staged modelling of the system under consideration in this paper is described in the following sections. A set of *preliminary computations* are first presented in section 3. The *'over one period stationary' analysis* is then presented in section 4. and the *transient analysis* in section 5. As this is essentially a paper about methodology, rather than an application paper, the data and parameters might be incomplete at some places. The authors apologizes for that.

3. Preliminary computations

A. The flux plot

The key point in the realisation of a staged model is to define carefully the interface quantities between the different components of the system. Between the inverter and the motor, the interfacing quantities are the electrical quantities associated with the three stator phases. In particular, to represent the *back-emf* of the phase (Miller, 1989), we will pay attention to the so-called *flux plot*

$$\phi(\theta, I) = \int_{S_{ph}} b(\theta, I) \cdot \vec{n}\, dS \qquad (1)$$

which gives the total flux embraced by one stator phase as a function of the angular position of the rotor θ and the current in that phase I. If the motor works in an equilibrate state, the flux plot of one phase is sufficient, as the flux plot for the other phases are obtained by a simple angle shift. If the machine works off saturation, the dependence on I can be neglected. The flux plot gathers in one table all the needed information concerning the motor, seen from the point of view of the inverter.

B. Core losses parameters

In a PM motor, core losses are mostly located in the stator core. At high speeds, they may override copper losses and need therefore to be estimated carefully over a wide range of frequencies. A difficulty in the calculation of core losses is that the magnetic flux density not only varies in time but also varies widely in space (Hendershot, 1994). One may distinguish *hysteresis losses* which are proportional to the frequency and *eddy current losses* which are proportional to the square of the frequency. One will therefore assume that ore losses can be expressed accurately by the formula

$$Q_{core}(I, \omega) = C_1(I)\, \omega + C_2(I)\, \omega^2 \qquad (2)$$

where the *frequency independent* constants $C_1(I)$ and $C_2(I)$ are expressed by

$$\begin{aligned} C_1(I) &= \int_{stator} \left(\int_0^{2\pi} h\, \partial_\theta b(\theta, I)\, d\theta \right), && (3) \\ C_2(I) &= \int_{stator} \left(\frac{C_{eddy}}{2\pi} \int_0^{2\pi} |\partial_\theta b(\theta, I)|^2 d\theta \right). && (4) \end{aligned}$$

This assumption is one of the simplifications of the staged modelling.

One sees that the flux plot (1) as well as the core losses parameters (3) and (4) depend on $b(\theta, I)$ and $\partial_\theta b(\theta, I)$. They can therefore be computed beforehand for a given geometry of the motor, by postprocessing adequately a series of 2D static FE computations, and then stored into tables.

TABLE I
COEFFICIENT FOR THE DISTRIBUTION OF THE BATTERY VOLTAGE OVER THE DIFFERENT PHASES (WYE-CONNECTION).

phase A	phase B	phase C
$+1 \to +1$	$-1 \to 0$	$0 \to -1$
0	OFF	0
1/2	OFF	-1/2
-1/3	2/3	-1/3
1/3	1/3	-2/3
$+1 \to 0$	$0 \to +1$	$-1 \to -1$
OFF	0	0
OFF	1/2	-1/2
-2/3	1/3	1/3
-1/3	2/3	-1/3
$0 \to -1$	$+1 \to +1$	$-1 \to 0$
0	0	OFF
-1/2	1/2	OFF
-1/3	-1/3	2/3
-2/3	1/3	1/3
$-1 \to -1$	$+1 \to 0$	$0 \to +1$
0	OFF	0
-1/2	OFF	1/2
1/3	-2/3	1/3
-1/3	-1/3	2/3
$-1 \to 0$	$0 \to -1$	$+1 \to +1$
OFF	0	0
OFF	-1/2	1/2
2/3	-1/3	-1/3
1/3	-2/3	1/3
$0 \to +1$	$-1 \to -1$	$+1 \to 0$
0	0	OFF
1/2	-1/2	OFF
1/3	1/3	-2/3
2/3	-1/3	-1/3

4. The 'over one period stationary' analysis

Because of the big difference in scale between one electrical period and the time needed by the motor to reach its nominal speed, a classical transient analysis would require several ten thousands of time steps. The transient analysis is therefore split up into two parts. The first part consists in calculating *'over one period'* the *stationary current waveshapes* for a fixed speed ω, a fixed commutation angle α and fixed temperatures. The second part is the *transient dynamic and thermal analysis*. This splitting is one of the simplifications of the staged modelling.

During the 'over one period stationary' analysis, the inductances, resistances and back-emf's of the phases, the extra voltages and resistances due to the electronic components and the switching strategy of the inverter bridge are all taken into consideration for the computation of the waveshapes of the phase currents and voltages.

One starts from the relation between the electric quantities associated with each stator phase :

$$V_\xi = R_\xi I_\xi + \partial_t \phi_\xi \quad , \quad \xi = A, B, C \tag{5}$$

where ϕ_ξ is given by (1). The *total resistance of the phase* R_ξ is computed analytically. It may be augmented by extra resistance due to the electronic components. The *total phase voltage* V_ξ is equal to the battery voltage $V_{battery}$ distributed over the different phases, according to the particular topology at each instant of time of the inverter and to the connection type (here a Wye-connection) (Hendershot, 1994). One writes

$$V_\xi = \lambda V_{battery} \tag{6}$$

where λ is the coefficient taken from Tab. I.

The table has six sections, which correspond to the six periods between the switching-on of two successive phases. For instance, let us consider the last group of four lines in the table. In the considered period, the phase A has to be switched on ($0 \to +1$) and the phase C has to be switched off ($+1 \to 0$), the phase B remaining in the same state ($-1 \to -1$). Immediately after sending the switch-on signal to phase A, all three phases are conducting. The inverter topologies corresponding with the last two lines of the group are then used. In both of them, the voltage is positive in the switched on phase (i.e. phase A), the voltage is negative in the switched off phase (i.e. phase C), and the voltage is either positive or negative in phase B, which allows to control the current in that phase. After a while, the current in phase C reaches zero and the phase ceases to be conducting. From that instant on, the first two lines of the group are used, the first one to let the current decrease (*freewheeling*) and the second one to make it increase in the loop formed by the phases A and B.

Starting from (5), the following explicit time-stepping scheme can be written :

$$I_\xi(\theta + \Delta\theta) = I_\xi(\theta) + \frac{V_\xi - \omega \partial_\theta \phi_\xi - R_\xi I_\xi(\theta)}{\omega \partial_I \phi_\xi} \Delta\theta \tag{7}$$

and applied until the achievement of a stationary waveshape $I_\xi(\theta)$ over one period τ.

Fig. 1 and Fig. 2 show examples of the computed stationary waveshapes of the phase currents and of the voltages that are the different terms of (5).

The mean torque is now computed by

$$T(\omega, \alpha) = \frac{3}{2\pi} \int_0^{2\pi} -\partial_\theta \phi(\theta - \alpha, 0) I(\theta)\, d\theta - \frac{Q_{core}(I, \omega)}{\omega}. \tag{8}$$

with Q_{core} given by (2). This first part of the staged model can then be seen as a procedure that gives the torque for given speed, commutation angle and temperatures.

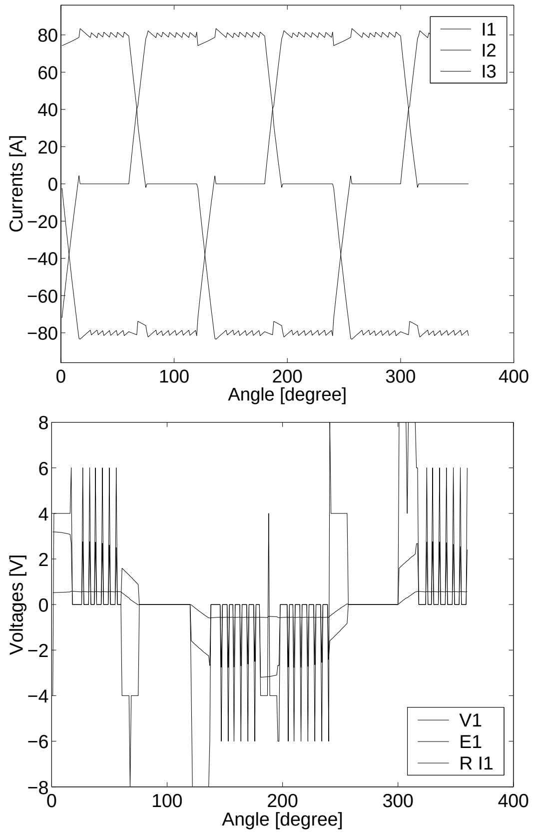

Fig. 1. Stationary waveshapes of the phase currents (in all phases) and of the voltages (in one phase) at low speed ($\omega = 1000\ [rad/s]$) and with $\alpha = 0$.

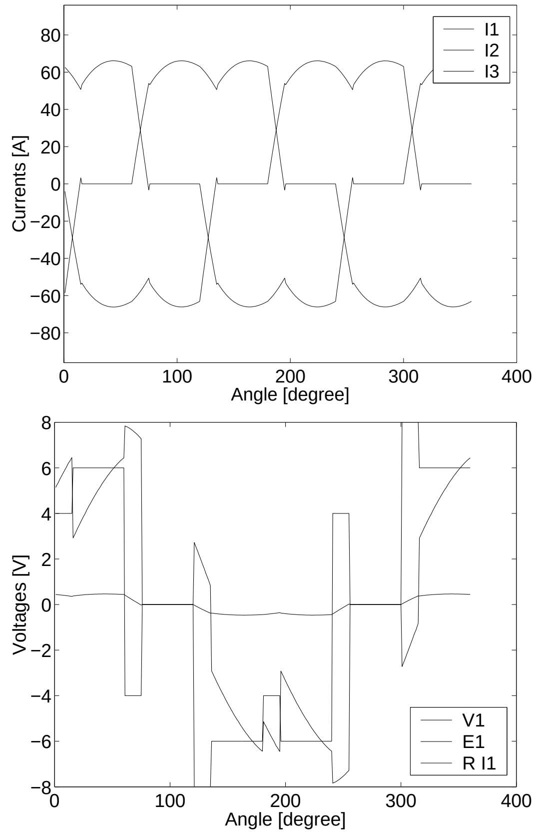

Fig. 2. Stationary waveshapes of the phase currents (in all phases) and of the voltages (in one phase) at a higher speed ($\omega = 2500\ [rad/s]$) and with $\alpha = 55°$

5. The transient analysis

The transient analysis of the system can now be described. Actually, two transient analysis are carried over in parallel. One for the temperatures and one for the speed.

A. The thermal analysis

Temperature has a strong influence on the behaviour of PM motors (through the temperature dependency of the magnet strengths and the electrical conductivity). Knowing the different losses (Joule losses in the stator coils, stator iron losses and power dissipated by the power electronics components) and their location in the different regions of the system, an elementary transient thermal model can be built up in order to estimate the temperature level of the different parts of the machine (Hamdi, 1994). The thermal model (Fig. 3) associates one node with each region of the model (e.g. coils, yoke, cooling fluid, power electronics components, ambient air around the motor, etc.). Each node is associated with a temperature T_I, a heat source P_I and a thermal capacity C_I. The nodes are linked with branches. Each branch is associated with a heat flux Q_{IJ} and a thermal conductivity K_{IJ}. All those parameters are estimated analytically or by means of dedicated thermal FE analysis.

One can now write evolutionary equations for the temperatures :

$$\partial_t(C_I T_I) + \sum_J Q_{IJ} = P_I, \qquad (9)$$

$$Q_{IJ} = -K_{IJ}(T_J - T_I). \qquad (10)$$

B. The dynamic analysis

The second part of the transient analysis is the *transient dynamic analysis* of the speed of the motor, according to the ordinary differential equation in time

$$J\partial_t\omega + f(\omega) = T(\omega, \alpha) \qquad (11)$$

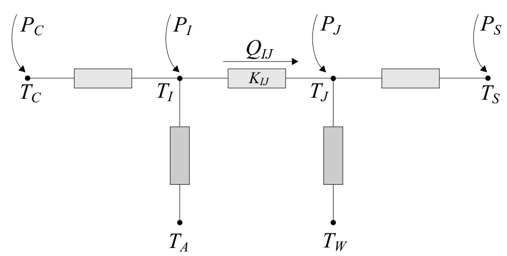

Fig. 3. Sketch of a thermal model with six nodes.

where J is the inertia of the rotating part, $f(\omega)$ is the reaction torque exerted by the turbine and friction forces and where $T(\omega, \alpha)$ is given by (8).

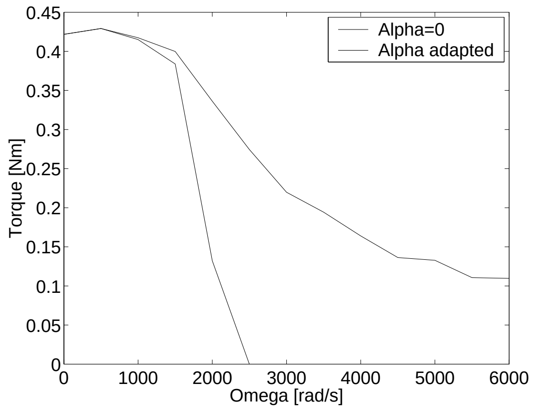

Fig. 4. Plot of the torque vs speed characteristic, with and without flux weakening.

Different strategies of *flux weakening*, i.e. modification of α as a function of ω, can now be analysed easily, Fig. 4 and Fig. 5.

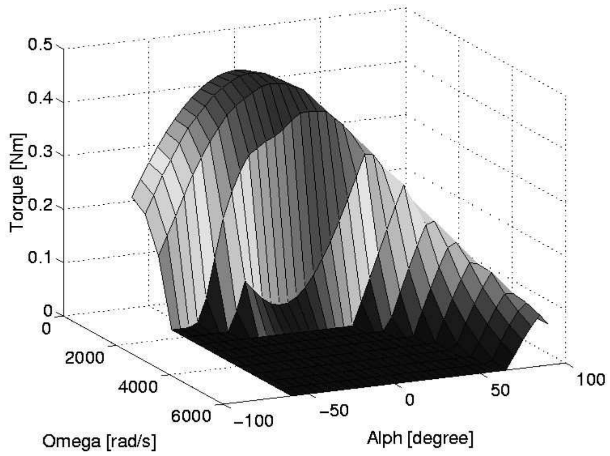

Fig. 5. Plot of the torque as a function of the speed ω and the commutation angle α. Flux weakening consists in selecting, at each speed ω, the commutation angle α that maximizes the torque.

C. Combination with the 'over one period stationary' analysis

At each time step of the transient analysis, one 'over one period stationary' analysis is done with the current values of the speed ω, of the commutation angle α and of the temperatures T_I (which affect the values of all resistances and of the flux plot). The outcome of the 'over one period stationary' analysis is the value of the torque $T(\omega, \alpha)$ and the values of all losses P_I in the different regions of the system. With this values, the speed ω and the temperatures T_I are updated by the explicit schemes described in the previous sections.

Conclusion

Building the staged model has implied doing simplifications. In counterpart, the staged model has the big advantage of providing at a reasonable computational cost a faithful dynamic description of the overall system. As the development goes along, the staged model is open to gradual enrichment and improvements, thanks to further FE investigations aiming at determining its different parameters or at estimating the influence of the simplifications that have been done.

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