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DETERMINATION OF A DYNAMIC RADIAL ACTIVE MAGNETIC BEARING MODEL USING THE FINITE ELEMENT METHOD

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***Abstract* – The dynamic model of radial active magnetic bearings, which is based on the current and position dependent partial derivatives of flux linkages and radial force characteristics, is determined using the finite element method. In this way magnetic nonlinearities and cross-coupling effects are considered more completely than in similar dynamic models. The presented results show that magnetic nonlinearities and cross-coupling effects can change the electromotive forces considerably. These disturbing effects have been determined and can be incorporated into the real-time realization of nonlinear control in order to achieve cross-coupling compensations.**

Introduction

Active magnetic bearings are a system of controlled electromagnets, which enable contact-less suspension of a rotor [1]. Two radial bearings and one axial bearing are used to control the five degrees of freedom of the rotor, while an independent driving motor is used to control the sixth degree of freedom. No friction, no lubrication, precise position control and vibration damping make Active Magnetic Bearings (AMB's) particularly appropriate and desirable in high-speed rotating machines. Technical applications include compressors, centrifuges and precise machine tools.

The electromagnets of the discussed radial AMB's are placed on the common iron core [2]. Their behavior is, therefore, magnetically nonlinear. Moreover, the individual electromagnets are magnetically coupled. An extended dynamic AMB model is determined in this paper using the Finite Element Method (FEM). The parametrization coupling model of the discussed radial AMB's is derived in this way. The presented dynamic AMB model is based on partial derivatives of flux linkages and radial force characteristics and, therefore, describes magnetic nonlinearities and cross-coupling effects more completely than similar dynamic AMB models [2], [3]. Moreover, it is appropriate for nonlinear control design and is compact and fast enough for the real-time realization.

FEM-computed force is compared with the measured force, while the flux linkages were not measured due to mechanical problems with rotor fixation. The current and position dependent partial derivatives of flux linkages are calculated by analytical derivations of the continuous approximation functions of the FEM-computed flux linkages. The impact of magnetic nonlinearities and cross-coupling effects on the properties of the discussed radial AMB's is then evaluated based on the performed calculations.

Dynamic AMB Model

The dynamic AMB model is according to the circuit model presented in Fig.1 given by (1) and (2), where u_1, u_2, u_3 and u_4 are the supply voltages, I_0 is the constant bias current, $i_{x\Delta}$ and $i_{y\Delta}$ are the control currents in the x- and in y-axis. ψ_1, ψ_2, ψ_3 and ψ_4 are the flux linkages of the corresponding electromagnets. R stands for the coil resistances. F_x and F_y are the radial force components in the x- and in y-axis, m is the mass of the rotor.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = R\begin{bmatrix} I_0 + i_{x\Delta} \\ I_0 - i_{x\Delta} \\ I_0 + i_{y\Delta} \\ I_0 - i_{y\Delta} \end{bmatrix} + 2\begin{bmatrix} \frac{\partial\psi_1}{\partial i_{x\Delta}} & \frac{\partial\psi_1}{\partial i_{y\Delta}} \\ \frac{\partial\psi_2}{\partial i_{x\Delta}} & \frac{\partial\psi_2}{\partial i_{y\Delta}} \\ \frac{\partial\psi_3}{\partial i_{x\Delta}} & \frac{\partial\psi_3}{\partial i_{y\Delta}} \\ \frac{\partial\psi_4}{\partial i_{x\Delta}} & \frac{\partial\psi_4}{\partial i_{y\Delta}} \end{bmatrix}\begin{bmatrix} \frac{di_{x\Delta}}{dt} \\ \frac{di_{y\Delta}}{dt} \end{bmatrix} + \begin{bmatrix} \frac{\partial\psi_1}{\partial x} & \frac{\partial\psi_1}{\partial y} \\ \frac{\partial\psi_2}{\partial x} & \frac{\partial\psi_2}{\partial y} \\ \frac{\partial\psi_3}{\partial x} & \frac{\partial\psi_3}{\partial y} \\ \frac{\partial\psi_4}{\partial x} & \frac{\partial\psi_4}{\partial y} \end{bmatrix}\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} F_x(i_{x\Delta}, x) \\ F_y(i_{y\Delta}, y) \end{bmatrix} \tag{2}$$

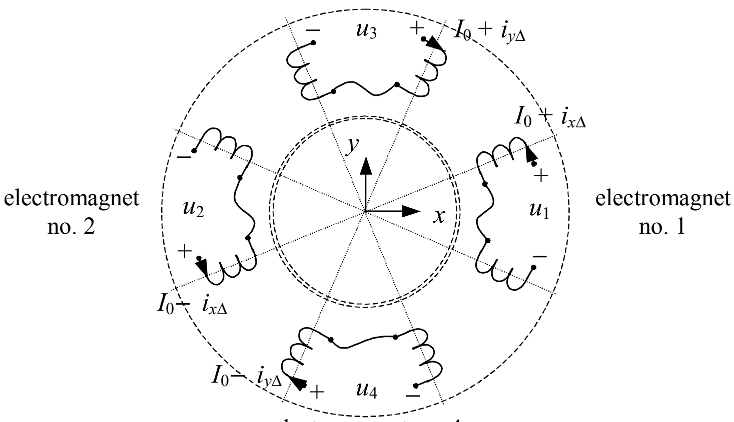

Fig.1 The circuit AMB model.

The current and position dependent partial derivatives of the flux linkages required in (1) are calculated by analytical derivations of the continuous approximation functions of FEM-computed flux linkages. The force characteristics $F_x(i_{x\Delta},x)$ and $F_x(i_{y\Delta},y)$ required in (2) are determined by FEM. In this way the obtained dynamic AMB model (1), (2) is described in terms of parametrization coupling.

When considering the symmetry in geometry (Fig.2), and the differential driving mode of currents $i_1 = I_0 + i_{x\Delta}$, $i_2 = I_0 - i_{x\Delta}$, $i_3 = I_0 + i_{y\Delta}$ and $i_4 = I_0 - i_{y\Delta}$, the interaction between electromagnets in the x-axis (no. 1 and no. 2) and electromagnets in the y-axis (no. 3 and no. 4) can be expressed as (3) and (4).

$$\frac{\partial\psi_1}{\partial i_{x\Delta}} = \frac{\partial\psi_3}{\partial i_{y\Delta}},\ \frac{\partial\psi_1}{\partial i_{y\Delta}} = \frac{\partial\psi_3}{\partial i_{x\Delta}},\ \frac{\partial\psi_2}{\partial i_{x\Delta}} = \frac{\partial\psi_4}{\partial i_{y\Delta}},\ \frac{\partial\psi_2}{\partial i_{y\Delta}} = \frac{\partial\psi_4}{\partial i_{x\Delta}} \tag{3}$$

$$\frac{\partial\psi_1}{\partial x} = -\frac{\partial\psi_2}{\partial x} = \frac{\partial\psi_3}{\partial y} = -\frac{\partial\psi_4}{\partial y},\ \frac{\partial\psi_1}{\partial y} = -\frac{\partial\psi_2}{\partial y} = \frac{\partial\psi_3}{\partial x} = -\frac{\partial\psi_4}{\partial x} \tag{4}$$

The ElectroMotive Forces (EMF's) due to the magnetic nonlinearities are reflected in terms like $\frac{\partial \psi_3}{\partial i_{y\Delta}}$ and $\frac{\partial \psi_3}{\partial y}$, which are normally given as constant inductance and speed coefficient respectively [1]. In [3] magnetic nonlinearities are partially considered with dynamic inductance. However the EMF's due to cross-coupling effects, which are reflected in terms like $\frac{\partial \psi_1}{\partial i_{y\Delta}}$ and $\frac{\partial \psi_1}{\partial y}$, are neglected in [3]. The dynamic AMB model (1), (2), therefore, describes magnetic nonlinearities and cross-coupling effects more completely than similar dynamic models. Furthermore, it is appropriate for nonlinear control design and is compact and fast enough for the real-time realization.

FEM Computation of Flux Linkage and Radial Force Characteristics

Magneto-static computation was performed by 2D FEM. The geometry and magnetic field distribution of the discussed radial AMB's is shown in Fig.2. The flux linkage characteristics $\psi_1(i_{x\Delta},i_{y\Delta},x,y)$, $\psi_2(i_{x\Delta},i_{y\Delta},x,y)$, $\psi_3(i_{x\Delta},i_{y\Delta},x,y)$ and $\psi_4(i_{x\Delta},i_{y\Delta},x,y)$ were calculated in the entire operating range from the average values of the magnetic vector potential in the stator coils. The radial force characteristics $F_x(i_{x\Delta},x)$ and $F_y(i_{y\Delta},y)$ were also calculated in the entire operating range by the Maxwell's stress tensor method, where integration was performed over a contour placed along the middle layer of the three-layer air gap mesh. The obtained results were incorporated into the extended dynamic AMB model (1) and (2). The parametrization coupling model is derived in this way.

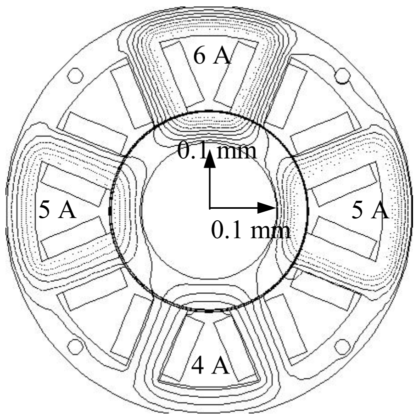

Fig.2 The geometry and field distribution of the discussed radial AMB's.

Results

The magnetic properties of the rotor surface changed due to the manufacturing process of the rotor steel sheets. Therefore, the magnetic air gap became larger than the geometric one. In order to obtain good agreement between the calculated and measured forces in the linear region, the air gap was increased in the FEM computation from 0.4 mm to 0.45 mm. The increase in the air gap of 0.05 mm can be compared with the findings in [3].

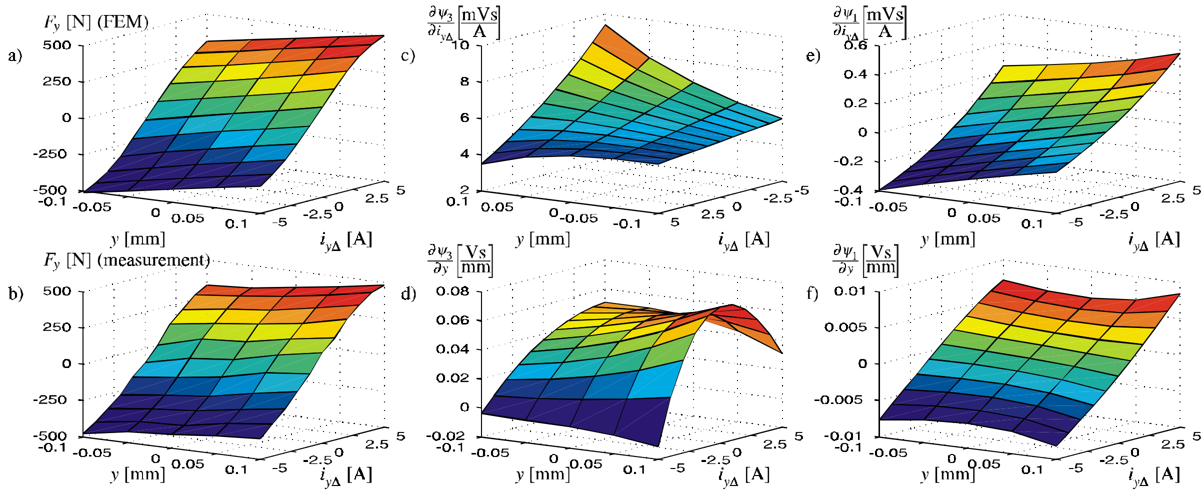

Fig.3 Results for the case: x = 0 mm, $i_{x\Delta}$ = 0 A and I_0 = 5 A: calculated and measured force (a and b) and flux linkage partial derivatives (c, d, e and f).

The good agreement between the FEM-computed and the measured radial force characteristics can be seen in Figs.3a,b). The current and position dependent partial derivatives of flux linkages, shown in Fig.3c) - Fig.3f), were calculated by analytical derivations of the continuous approximation functions. In the results shown in Figs.3c,d) the influence of magnetic nonlinearities can be seen, while the influence of magnetic cross-coupling effects can be seen in the results shown in Figs.3e,f). Based on the obtained results the ratio $\frac{\partial\psi_1}{\partial i_{y\Delta}} \Big/ \frac{\partial\psi_3}{\partial i_{y\Delta}}$, as well as the ratio $\frac{\partial\psi_1}{\partial y} \Big/ \frac{\partial\psi_3}{\partial y}$ was calculated inside the operating range. From the performed comparison it is established, that due to magnetic nonlinearities and cross-coupling effects the EMF's can vary in a range of up to 12 %.

Conclusion

The extended dynamic AMB model is presented in this paper. It is based on the FEM-computed current and position dependent partial derivatives of flux linkages and radial force characteristics. The parametrization coupling model of the discussed radial AMB's is derived in this way. The obtained dynamic AMB model, therefore, considers magnetic nonlinearities and cross-coupling effects more completely than similar dynamic AMB models. The results of the performed calculations show, that inside the operating range of the discussed radial AMB's, the EMF's can vary due to magnetic nonlinearities and cross-coupling effects in a range of up to 12 %. These disturbing effects deteriorate the static and dynamic performances of the overall system. In order to improve the system dynamics, the obtained results have to be incorporated into the real-time realization of nonlinear control.

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