

Simulation of a three-phase transformer using an improved anisotropy model

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Abstract— The numerical modelling of electromagnetic devices with non-linear and anisotropic materials requires the knowledge of the reluctivity tensor. Its entries generally depend both on the magnitude and the direction of the flux density. They can be obtained from measurements with a single sheet tester by considering an improved anisotropy model. The paper discusses how this model can be integrated in the finite element method. It is successfully applied for simulating a three-phase transformer.

I. INTRODUCTION

NON-LINEAR magnetostatic systems are described by the equation

$$\text{curl}(\nu \text{curl}A) = J, \quad (1)$$

where A is the vector potential [Vs/m], ν the reluctivity [Am/Vs] and J the applied current density vector [A/m²]. The reluctivity is represented by a symmetric second-rank tensor. It relates the flux density B [Vs/m²] to the field strength H [A/m] according to $H = \nu B$. In its principal coordinate system {PQ}, all off-diagonal entries are zero. In order to obtain the tensor in the global coordinate system {XY}, the following transformation rule is used [1]:

$$\begin{pmatrix} \nu_{xx} & \nu_{xy} \\ \nu_{yx} & \nu_{yy} \end{pmatrix} = \mathbf{T}^{-1} \begin{pmatrix} \nu_p & 0 \\ 0 & \nu_q \end{pmatrix} \mathbf{T}, \quad (2)$$

with

$$\mathbf{T} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \quad (3)$$

the matrix of direction cosines and ϕ the angle between the X-axis and the P-axis. In the general case of non-linear anisotropic materials, the diagonal entries are not equal and depend on the magnitude and the direction of B . This behaviour can be analyzed by the improved anisotropy model presented in [2]. This paper shows how this hybrid model can be integrated in the finite element method. It is demonstrated for the simulation of a three-phase transformer.

II. IMPROVED ANISOTROPY MODEL

Silicon steels that are widely used in transformers exhibit a considerable non-linear anisotropy, originating from their Goss-texture [3], [4]. Physical considerations and measurements show that B and H are aligned if the applied field is directed along the rolling direction (RD) or transverse direction (TD). Hence, these directions are the principal axes of the

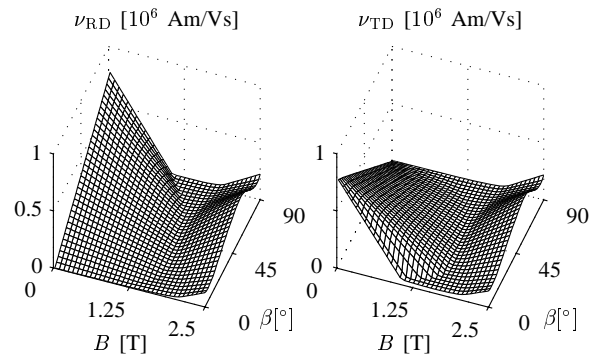


Fig. 1. The tensor reluctivity components of a Goss-textured ferromagnetic material as a function of the magnitude and the direction of B .

reluctivity tensor. By combining the laws of ferromagnetism with measurements from a single sheet tester, it is possible to determine the reluctivity in the RD (ν_{RD}) and TD (ν_{TD}), as a function of the magnitude of B and the angle β it makes with the RD [2]. Their shape is plotted in Fig. 1. These characteristics reveal that the hardest magnetization occurs for a certain angle between 0° and 90° (Theoretically at 54.7°). At lower flux density levels, this improved anisotropy model yields an area in the $B - \beta$ plane in which ν_{RD} and ν_{TD} are undefined. The corresponding B -vectors are never reached during the measurements because B tends to stay close to the RD or TD. As the finite element method may require to evaluate the reluctivity in this area, it is supposed that $\nu_{RD}(0, 90^\circ)$ and $\nu_{TD}(0, 0^\circ)$ equal the reluctivity of air, and that the reluctivity components increase linearly towards this value over the empty region.

III. NON-LINEAR COMPUTATION

Convergence is not easily achieved when simulating devices with non-linear anisotropic material characteristics such as the ones presented in Fig. 1. Several simulations have shown that reliable solutions are obtained by applying a fixed-point iteration (successive substitution), in which the reluctivity tensor is initialized as linear isotropic

$$\nu_{RD} = \nu_{TD} = \text{constant}, \quad (4)$$

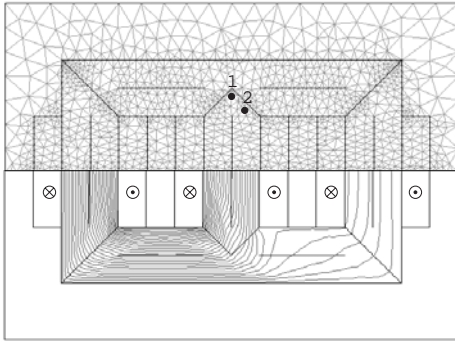


Fig. 2. The mesh and a field solution, for a particular instant of time, obtained by applying the improved anisotropy model. The RD is indicated by lines in the ferromagnetic parts.

followed by a smooth transformation into non-linear isotropic

$$\nu_{RD} = \nu_{TD} = f(\|B\|) \quad (5)$$

and eventually into non-linear anisotropic.

$$\nu_{RD} = f_1(\|B\|, \beta) \quad \nu_{TD} = f_2(\|B\|, \beta) \quad (6)$$

For the applied finite element analysis, the mathematical software libraries PETSc (Portable Extensible Toolkit for Scientific Computing) and TAO (Toolkit for Advanced Optimization) have been used [5], [6].

IV. EXAMPLE

The hybrid model is used for simulating the three-phase transformer of Fig. 2. Obviously, the bending of the flux lines is located close to the joints. This is not observed with isotropic materials. In order to demonstrate the improvement which can be achieved, when compared to anisotropy models of the type

$$\nu_{RD} = f_1(\|B\|) \quad \nu_{TD} = f_2(\|B\|) \quad (7)$$

as e.g. discussed in [7], the B -loci in points 1 and 2 of Fig. 2 are plotted in Fig. 3. The applied current varies sinusoidally over one period. For a fair comparison, $f_1(\|B\|, 0^\circ)$ and $f_2(\|B\|, 90^\circ)$ of Fig. 1 have been used. The smooth loci obtained with the latter model are in contrast with the complicated loci of the improved anisotropy model. Both models show a preferred orientation of the flux density in the rolling direction. However, the improved model additionally accounts for the more difficult magnetization along the hard axis of the material ($\beta = 54.7^\circ$). Similar B -loci have been measured and presented in [8].

V. CONCLUSIONS

Anisotropy of magnetic materials can be implemented in numerical simulations by means of a reluctivity tensor. Its diagonal entries generally depend on both the magnitude and

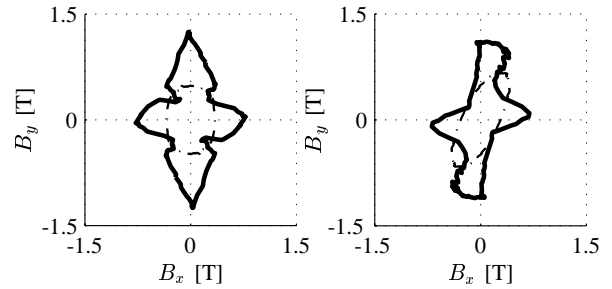


Fig. 3. The B -loci obtained by applying an improved (solid) and a traditional (dashdot) anisotropy model, at points 1 (left) and 2 (right) indicated in Fig. 2.

the direction of the flux density. This dependency can be obtained from an improved anisotropy model. However, the shape of the resulting reluctivity surfaces impedes the convergence of the numerical simulation method. Therefore, a fixed-point method, which gradually increases the non-linearity and the anisotropy, is adopted. By the simulation of a three-phase transformer it is shown that the improved anisotropy model yields a solution which better corresponds with the physical phenomenon, compared to the solution obtained by applying an anisotropy model which ignores the angle dependency of the tensor entries.

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