Int. Conf. on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2005
M. Papadrakakis, E. Oñate and B. Schrefler (Eds)

COUPLED SIMULATIONS IN THE DESIGN OF ELECTRICAL MACHINES

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Key words: Coupled Problems, Finite Elements, Electrical Machines

Abstract. *The design of electrical machines is an iterative process. To reduce costs and development times, prototyping is more and more replaced by simulations. Single domain Finite Element techniques have reached a high level of precision. However, to fully replace prototyping and measurements, all physical effects have to be regarded accurately. This requires an appropriate model, which can sometimes be huge and therefore computationally expensive. This contribution illustrates the need for coupled simulations in the field of electrical machines, by presenting a panorama of the approaches and methodologies that are applied at our institute to modelize real-life multi-domain applications.*

1 INTRODUCTION

Design of electrical machines is maybe the branch of numerical modelling in industry that features the largest number of coupled phenomena :

- ◇ electromagnetism, field radiation,
- ◇ power electronics, drives, control,
- ◇ mechanical load, motion, deformations,
- ◇ thermics (heating), fluid interactions (cooling),
- ◇ vibrations, acoustics, ...

In their website initiative called *Millennium machines : Multi-domain, Multi-Physics Modelling for Energy Conversion Machines*, Professor S.D. Garvey, School of Mechanical, Materials, Manufacturing Engineering & Management, University of Nottingham and Professor D. Hove, Dept. of Electronic and Electrical Engineering, University of Sheffield, have clearly assessed the situation. They say : “The design of high-performance energy conversion machinery is currently limited by the fact that the majority of the available simulation packages are essentially single-physics. While present day computation capabilities cater well with single-physics modelling problems, the design optimisation of machines *at a system level* can be performed only if various model for predicting different aspects of machine behaviour are intelligently coupled and realistically account for interface conditions. Existing computational hardware is not sufficiently powerful to enable highly-coupled problems to be assembled and solved within acceptable solution times.”

Numerical modelling is hence at a turning point. Now that single-domains techniques have reached maturity, people active in Computational electromagnetics have, in order to go any further, to face numerous important and new questions concerning :

- *Definition of couplings :* How should the interfaces between the different analysis domains be specified ? Is the exchange of solution files between single-domain packages sufficient, or is it necessary to devise specific multi-domain codes and algorithms ?
- *Accuracy :* Does more computational power always lead to a greater real accuracy in typical multi-domains problems ? In which conditions is it worth the investment ? By which techniques can the accuracy still be estimated and controlled ?
- *Computation :* What is the state of advancement of parallelisation techniques, model-reduction techniques, fast solvers (multigrid, fast multipoles) ? Are they worth being implemented and how ?

The purpose of this paper is to partially answer those questions by presenting a panorama of the approaches or methodologies that have been successfully applied at our institute to modelize real-life multi-domain applications. Section 2 deals with the modelling of eddy currents in an induction furnace, with in view the determination of the power transferred by Joule losses to the melt. Section 3 tackles then with the electromagnetic forces and the associated vibrations, which are analysed by means of an eigen value problem. The computation of the deformation of the induction furnace of Section 1 and of the melt are presented, as well as the eigen deformation modes of the stator of an induction motor. Section 4 presents a further coupling, in the case of the same induction motor, in order to estimate the generated acoustic noise. Whereas weak coupling was sufficient in the applications presented in the former, Section 5 deals with situations where the interaction between the different domains is so strong that a strong coupling is required. A dedicated methodology for such system, called staged modelling, is presented, which is applicable to large classes of electrical systems.

2 EDDY-CURRENT SIMULATIONS

An effect that is quite often included in simulations, but not always regarded as a coupling is the interdependence of the magnetic and the electric field. Of course, already for magnetostatic simulation the source imposed is often a current. But it is represented as a known quantity in the right hand side vector of the equation system, thus not being a strong coupling. But, in electrical machines, eddy currents are a very important effect to be included in the simulations. This is achieved through a strong coupling of magnetic and electric quantities. Different approaches exist using different field and/or potential types[1]. Commonly used formulations for the simulation of electrical machines are based on the use of the magnetic vector potential $\vec{A}$, with $\vec{B} = \nabla\times \vec{A}$. In the most simple case, the coupling with the electric field $\vec{E}$ is achieved through the magnetic vector potential with $\vec{E} = -\frac{\partial}{\partial t}\vec{A}$. Other formulations use the electric scalar potential V or the electric vector potential $\vec{T}$ for the coupling[2]. E.g., for a time-harmonic approach with sinusoidal quantities and linear material properties, the equation system in Galerkin form is given by

$$\int_\Omega \nabla\times \vec{\alpha}_i \cdot \nu \,\nabla\times A \,\mathrm{d}\,\Omega + \int_\Omega j\omega\sigma\vec{\alpha}_i \cdot \vec{A} \,\mathrm{d}\,\Omega + \int_\Omega j\omega\sigma\vec{\alpha}_i \cdot \nabla\, V = \int_\Omega \nabla\times \vec{\alpha}_i \cdot \vec{T}_0 \,\mathrm{d}\,\Omega, \quad i = 1, 2, \ldots, n_e, \quad (1)$$

$$\wedge \int_\Omega j\omega\sigma \nabla \alpha_i \cdot \vec{A} \quad \mathrm{d}\Omega + \int_\Omega j\omega\sigma \nabla \alpha_i \nabla V \quad \mathrm{d}\Omega = 0 \quad i = 1, 2, \ldots, n_n, \tag{2}$$

using edge shape functions for $\vec{A}$ and nodal shape functions for V.

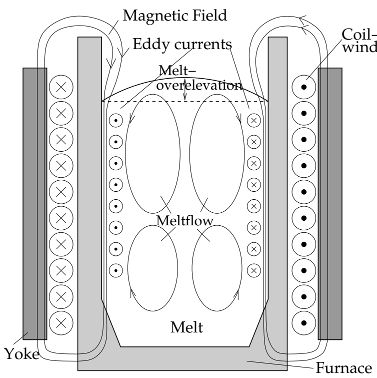
If the effects regarded are non-sinusoidal, or if nonlinear material properties are involved, the abovementioned formulations can be extended to a transient simulation. The time-dependent values are discretized according to the Galerkin scheme[3], resulting e.g. for the magnetic vector potential in

$$\vec{A}(t) = (1-\Theta)\vec{A}_n + \Theta A_{n+1} \tag{3}$$

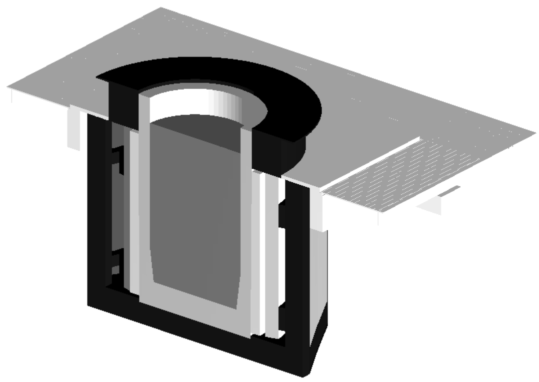
$$\Theta = \frac{t-t_n}{t_{n+1}-t_n} = \frac{t-t_n}{\delta t}; \quad 0 \le \Theta \le 1 \tag{4}$$

with $\Theta = \frac{2}{3}$.

As an example for a time-harmonic simulation, an induction furnace is considered. Fig. 1 shows the operating principle and the geometrical model of the device.



(a) Principle of operation



(b) Geometry

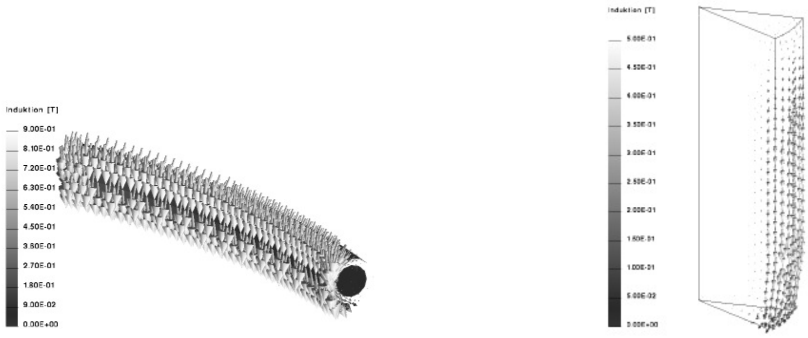
Figure 1: The induction furnace.

An external coil is supplied with a sinusoidal current. The magnetic field penetrates the melt, which acts as a short-circuited secondary coil. Eddy currents produce losses, which then heat up the melt. Figs. 2 and 3 shows the flux density and the current density in the melt and in one coil winding. Due to symmetry, only a 30°-part of the device is modelled.

3 FORCES AND VIBRATIONS

In electrical machines, the most relevant vibrations are those leading to acoustic noise emission, either directly as air-borne sound or as structure-borne sound through the support they are attached to. The forces leading to these vibrations are an effect of the electromagnetic field. They include essentially Lorentz forces on conductors and surface forces at material discontinuities. The force density can be described in general by

$$\vec{f} = \vec{J} \times \vec{B} + \sum_{i=1}^{m} \frac{\partial w}{\partial \alpha_i} \nabla \alpha_i, \tag{5}$$



(a) Flux density in one coil winding.

(b) Flux density in the melt.

Figure 2: Flux density for the induction furnace.

(a) Current density in one coil winding.

(b) Current density in the melt.

Figure 3: Current density for the induction furnace.

with the magnetic energy w and material properties α. In electrical machines, in order to realize the coupling with structure dynamics, one is mainly interested in the surface forces located at the boundaries of the magnetic cores, where the surface force density $\vec{\sigma}$ writes :

$$\vec{\sigma}(t) = \frac{\mu_r - 1}{\mu_r \cdot \mu_0} \cdot \left(B_n(t)^2 - \frac{1}{2} B(t)^2 \right) \cdot \vec{n}_{12}. \tag{6}$$

These forces are input to a structural-dynamic analysis as an excitation. Thus, only a weak coupling is applied. Strongly coupled effects as magnetostriction are not regarded here. The deformation-solver formulation reads[4]:

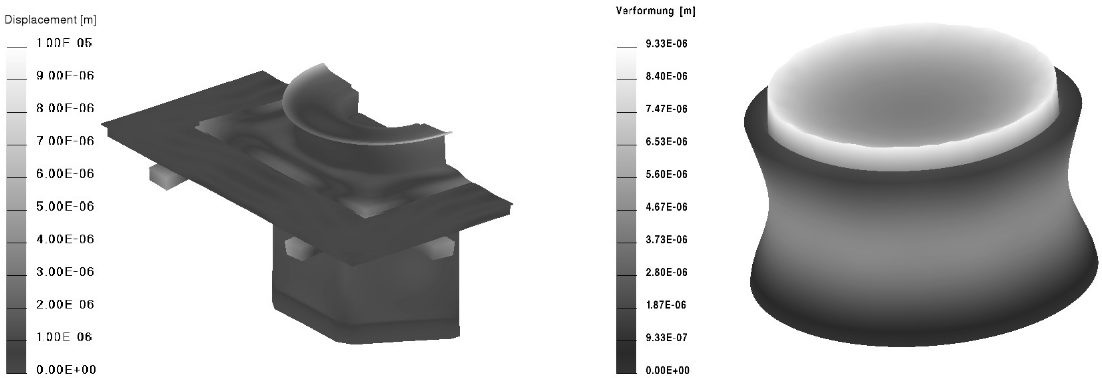
$$\boldsymbol{K} \cdot \boldsymbol{D} + \boldsymbol{F} \cdot \dot{\boldsymbol{D}} + \boldsymbol{M} \cdot \ddot{\boldsymbol{D}} = \boldsymbol{R}. \tag{7}$$

$\boldsymbol{K}$ is the matrix of the stiffness of all elements of the model, $\boldsymbol{M}$ represents the mass, $\boldsymbol{F}$ is the damping, $\boldsymbol{D}$ is the deformation, and $\boldsymbol{R}$ is the exciting force. Due to harmonic analysis, (7) becomes :

$$(\boldsymbol{K} - \omega^2 \cdot \boldsymbol{M} + j\omega \cdot \boldsymbol{F}) \cdot \boldsymbol{D} = \boldsymbol{R}, \tag{8}$$

where $\omega = 2\pi f$ is the angular frequency.

The damping $\boldsymbol{F}$ can be neglected because the induction machnine's stator is built of material with high elastic stress modules. On the contrary, damping can not be neglected for the induction furnace. Equation (7) is solved using a modal analysis. The computed eigenmodes are then superposed in order to obtain the deformation for a given frequency and a given force excitation.



(a) Deformation at 470 Hz.

(b) Deformation of the melt.

Figure 4: Deformation of the structure (a) and the melt (b) at 470 Hz.

Fig. 4(a) shows the deformation (real part) for the induction furnace at a frequency of 470 Hz. The liquid melt cannot be modeled by the above described method. Here, another coupling technique is applied. The melt is modeled as a fluid, and a fluid-structure-interaction is used to couple both parts[5]. Fig. 4(b) shows the deformation (real part) for the melt.

Mechanical models are often more complicated than the corresponding electromagnetic ones, since all relevant constructive parts, which can be neglected in the electromagnetic simulation, have to be modeled. Fig. 5 shows exemplarily an exploded view of the mechanical model of an

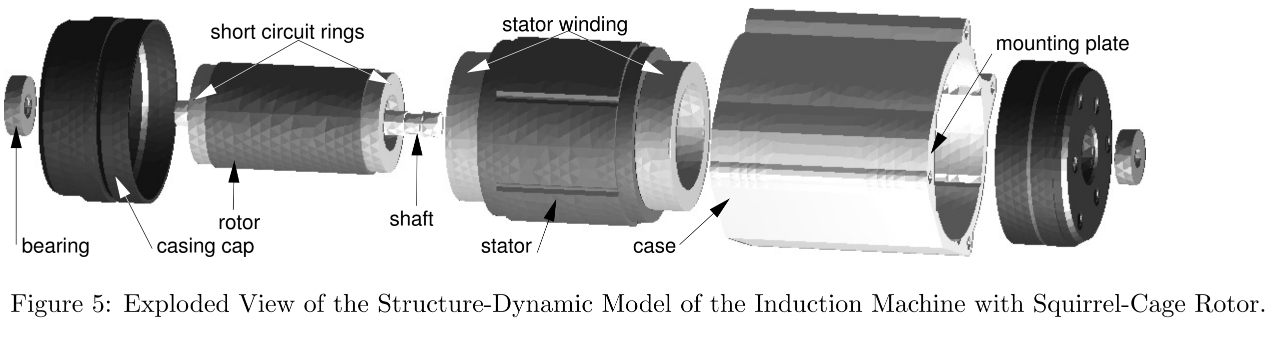


Figure 5: Exploded View of the Structure-Dynamic Model of the Induction Machine with Squirrel-Cage Rotor.

induction machine with squirrel cage rotor. For this machine, the electromagnetic simulation was performed using a transient approach. Through a Fast Fourier Transform, the relevant frequencies present in the force excitation are found. For these, a time-harmonic analysis of the deformation is performed.

In fig. 6 the surface-force distribution on the stator lamination stack for one particular time step is presented.

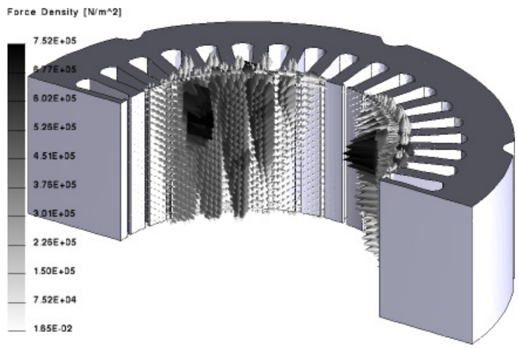
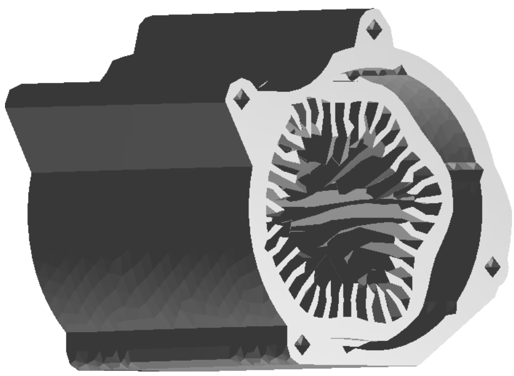
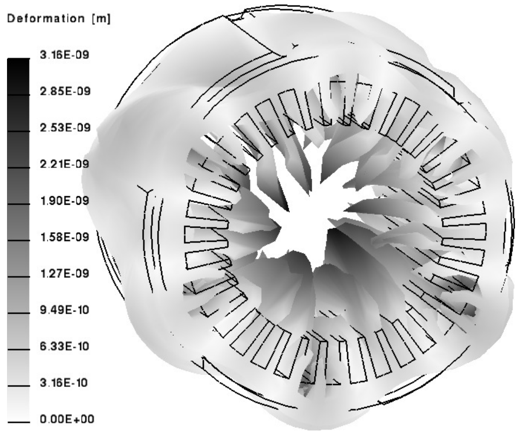


Figure 6: Surface-Force Density Distribution on the Stator Teeth for One Time Step (Only Stator Lamination Shown).



(a) Deformation of Stator and Casing.



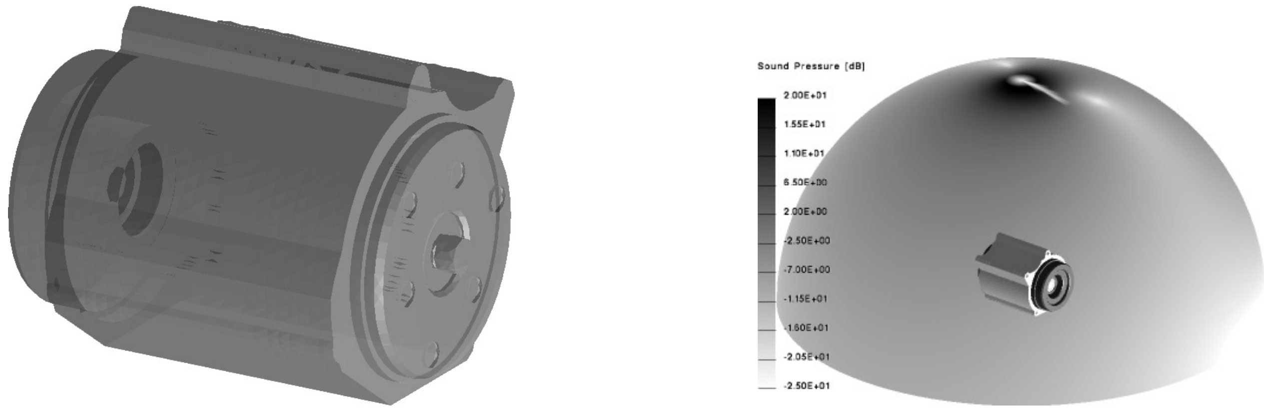
(b) Deformation of Stator.

Figure 7: Real Part of the Deformation for $f = 620\,\mathrm{Hz}$.

The resulting deformation of the stator and the casing for a frequency of 620 Hz are depicted in fig. 7.

4 ACOUSTICS

The vibrations of the mechanical structure, the structure-borne sound, are well suited to describe the behaviour of an electrical machine mounted on a larger, more complex structure. Nevertheless, the air-borne sound is of interest as well. Especially in consumer products or automotive applications, its importance is increasing due to customer's demands. Therefore, an acoustic simulation has to be performed. Although it would possible to use the Finite Element Method, it is not the best approach in practice, as it makes it necessary to discretize a large air region around the regarded device. Here, a Boundary Element Method (BEM) solver is preferably applied. Only the exterior surface of the machine is discretized. Fig. 8(a) shows the acoustic model for the induction machine. The coupling to the mechanical solution is done via the velocity of the surface nodes. The equation system to be solved then reads:



(a) The Acoustic Model of the Machine.

(b) Sound-Pressure Distribution for $f = 520\,\mathrm{Hz}$.

Figure 8: The acoustic model.

$$\boldsymbol{H} \cdot \underline{p} = \boldsymbol{G} \cdot \underline{\vec{v}} \tag{9}$$

where $\underline{p}$ is the complex sound pressure, which is the result of the acoustic simulation, and $\underline{\vec{v}}$ is the complex velocity vector of all nodes of the BEM model.

Post-processing quantities include the sound pressure p on test meshes in the air region (e.g. lines, surfaces, spheres) (Fig. 8(b)). A global quantity is the sound power, which is calculated through the integration of the sound intensity on a surrounding sphere around the regarded device (see Fig. 9). The sound intensity is

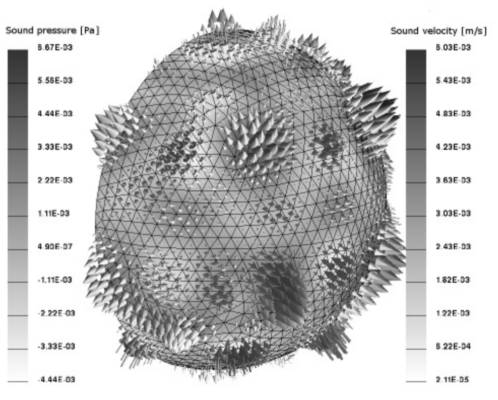


Figure 9: Calculation of the sound power.

$$\vec{I} = \frac{1}{2} \mathrm{Re} \left\{ \underline{p} \cdot \underline{\vec{v}}^{*} \right\}, \tag{10}$$

which leads to the sound power

$$P = \oint_{\partial\Gamma} \vec{I} \cdot \, \mathrm{d}\,\partial\vec{\Gamma}, \tag{11}$$

or expressed as a level in dB:

$$L_P = 10 \cdot \log_{10} \frac{P}{P_0}, \qquad P_0 = 10^{-12} W. \tag{12}$$

Fig. 10 shows the sound pressure distribution on surfaces around the induction furnace.

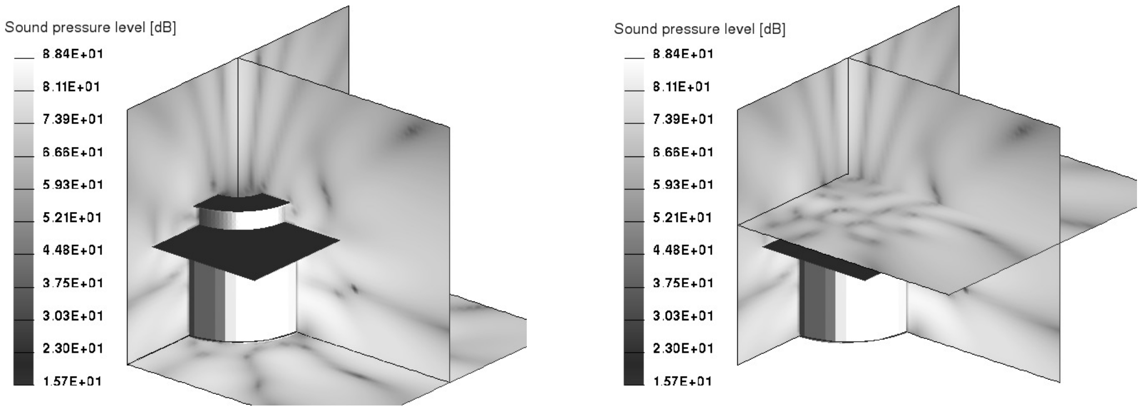


Figure 10: Sound pressure level distribution around the induction furnace.

5 STAGED MODELLING

Whereas *weak coupling* was sufficient in both applications presented in the former sections, this section deals with situations where the interaction between the different domains is so strong that a *strong coupling* of all domains is required. Approaches similar to the one presented so far in this paper, tend to become untractable in such situations. An alternative methodology is therefore presented, which is applicable to large classes of electrical systems.

In general, technical *specifications* of electrical devices concern non-electromagnetic quantities (thermal, mechanical, acoustic), whereas design *parameters* (geometry, winding, magnets, materials, . . .) are electromagnetic quantities. This clearly advocates for a global modelling approach. On the other hand, engineers think, even in the presence of complex systems, in terms of a quite limited set of observable variables. The purpose of *design* is to build (or modify) a system in order to make the relations between those significant variables match predefined specifications. Now, the *model* is the exploratory tool that must help in doing so, i.e. a reliable way to obtain, in a reasonable time, a good picture of the coupled dynamics of the whole system. It should be noted that this picture need not necessarily be from the beginning a highly accurate one, because one is, at the R&D stage, still essentially concerned with quantitative questions, incomplete, not yet fixed or inaccurate data.

The general idea of this approach, which we call *staged modelling*, is to combine the conciseness and intuitive virtues of classical analytical models, with the accuracy of numerical models. For that purpose, the coupled system must be decomposed into a network of sub-systems that interact only through well-defined and controlled channels. Usually, this decomposition implies doing some ad-hoc simplifications, the modelling challenge being to find simplifications that allow a decomposition into meaningful sub-systems, with controllable communication channels, without deteriorating too much the representativity.

Our experience indicates that efficient staged models exist for large classes of applications. This is not surprising if one considers that electrical devices are purposely conceived to convert energy. The energy flow is quite structured in the device and the purpose of the staged modelling is to identify that structure and to match it as closely as possible with the structure of the model. Very often, the effective simplifications are very similar to the ones made to obtain classical analytical models (equilibrated phases, no eccentricity, ...). In the following, the dynamic multi-domain staged models of Brushless DC or Permamnet Magnet machines is described. The proposed approach is more generally applicable to all electrical machines of the synchronous type.

5.1 Interface field/coil : flux plot

The flux plot represents the interface between the magnetic field and the coils, which are connected to the supply. Assuming an equilibrated system, the three phases are equivalent, up to a phase shift, and only one needs then to be analysed, say R. The *flux plot* is by definition the total flux φ_R embraced by the coils of the reference phase, as a function of rotor position θ and stator currents $\{I_R, I_S, I_T\}$.

First approximation : If saturation is neglected, the flux plot can be expressed by

$$\varphi_R(\theta, I_R) = \varphi_{PM}(\theta) + (L_{ph}(\theta) - M_{ph}(\theta))I_R, \tag{13}$$

assuming equilibrated currents. The flux plot is characterised by three functions of θ : the flux φ_{PM} generated in the reference stator phase by the permanent magnets, the self inductance L_{ph} and the mutual inductance M_{ph}. They can be estimated by specific FE computations, or by any other method.

Second approximation : However, in many situations, saturation must be taken into consideration. and the flux plot is then *a priori* a function of 4 parameters. But assuming again equilibrated stator currents, on can adopt the equivalent representation $\{I_R, I_S, I_T\} = \{I, -I/2 + I', -I/2 - I'\}$, of the stator current distribution, and note that the component I' generates a flux in the phases S and T only. That component affects slightly the flux embraced by phase R, only when the core is highly saturated. If one neglects this phenomenon called *cross saturation*, the flux plot depends on only two parameters, θ and I, and takes however most of the saturation effect into consideration. Disregarding cross-saturation is a typical staged modelling assumption.

Represented by a look-up table, the flux plot $\varphi_R(\theta, I)$ (Fig. 11) is associated with a given geometry of the cross section of the machine and is computed by static FE (2D, or 3D if required) computations, imposing $\{I_R, I_S, I_T\} = \{I, -I/2, -I/2\}$. A few side parameters (stack length, number of turns, ...) can be taken into consideration without recomputing the table.

5.2 Time scale splitting

Because of the big difference in scale between one electrical period and the time needed by the motor to reach its nominal speed, a classical transient analysis would require several ten thousands of time steps, and be inefficient. The dynamic analysis is therefore advantageously

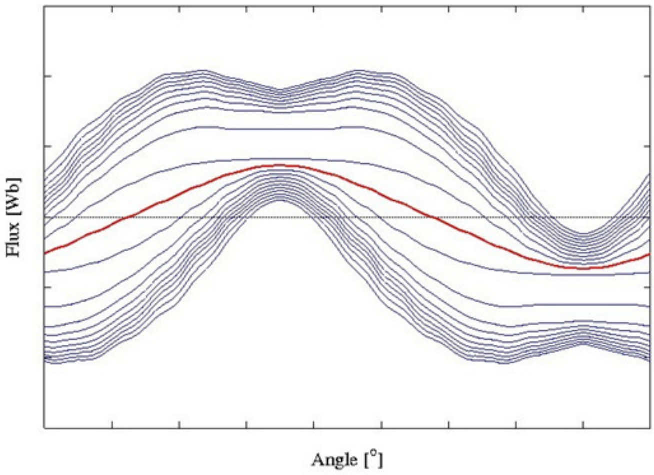


Figure 11: Flux plot $\varphi_R(\theta, I)$ of a PM motor for different values of the stator current parameter I.

split up into two nested loops. The inner loop consists in calculating *'over one electrical period'* the *periodic stationary current and voltage waveshapes* assuming fixed speed ω and fixed temperatures. From this, the mean torque and the losses are estimated. The outer loop is the *transient dynamic and thermal analysis*, with a much larger time step, computing the acceleration and heating up of the machine. This time scale splitting is another typical simplifications of the staged modelling.

For the 'over one electrical period' analysis, the inductances, resistances and back-emf's of the phases (plus possibly extra voltages and resistances due to the electronic components) and the switching strategy of the inverter bridge are taken into consideration. One starts from the relation between the electric quantities associated with the stator phases:

$$V_\xi = R_{ph} I_\xi + \partial_t \varphi_\xi \quad , \quad \xi = R, S, T \tag{14}$$

The *resistance of the phase* R_{ph} is computed analytically. It may be augmented by extra resistance due to the electronic components. The *phase voltage* V_ξ is determined at each instant of time, according to the inverter strategy and knowing the instantaneous values of the phases currents $\{I_R, I_S, I_T\}$. In particular, *flux weakening* strategies can be easily implemented at this level.

Starting from (14), the following explicit time-stepping scheme can now be written,

$$I_\xi(\theta + \Delta\theta) = I_\xi(\theta) + \frac{V_\xi - \omega \; \partial_\theta \varphi_\xi(\theta - \alpha, I_\xi(\theta)) - R_{ph} \; I_\xi(\theta)}{\partial_I \varphi_\xi(\theta - \alpha, I_\xi(\theta)} \Delta\theta \quad , \quad \xi = R, S, T, \tag{15}$$

where α is the flux weakening angle, and implemented in a C++ (for instance) code. The integration scheme is applied until periodic waveshapes over one electrical period τ are obtained, which is usually done in iterating over less than 2 or 3τ.

5.3 Interface field/rotation : mean torque

First approximation : The interface between the electromagnetic quantities and the rigid body motion of the rotor is the *mean torque*. It is expressed by

$$T = 3 \int_0^{2\pi/p} \partial_\theta \varphi_R(\theta - \alpha, I_R(\theta))\; I_R(\theta)\; d\theta. \tag{16}$$

This shows in particular that the torque can be rigorously computed from the flux plot only, without the need of any supplementary field information. The value of the torque can be used directly in the dynamic mechanical equation of the rotor, where the mechanical load of the machine can be taken in consideration.
Second approximation : If one is also interested in torsional vibrations, the *ripple torque* can also be easily computed at this stage by (17).

$$R = \sqrt{\frac{p}{2\pi} \int_0^{2\pi/p} \left\{3\; \partial_\theta \varphi_R(\theta - \alpha, I_R(\theta))\; I_R(\theta) - T\right\}^2 d\theta} \tag{17}$$

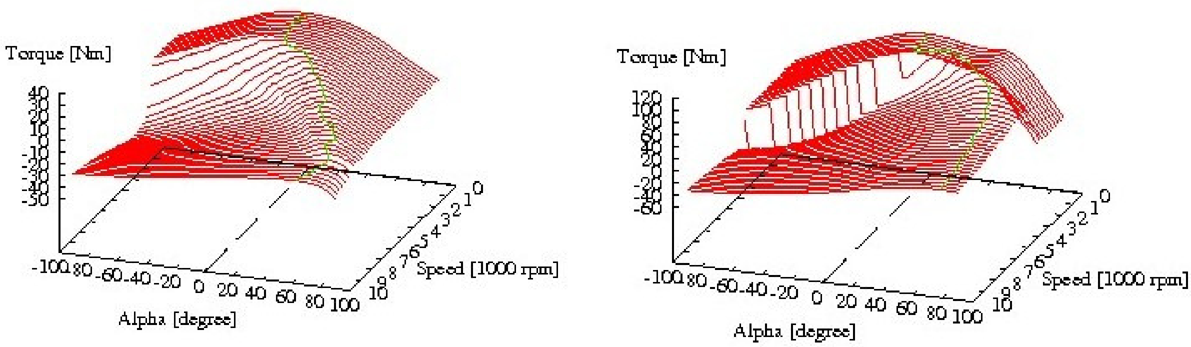


Figure 12: Mean torque as a function of speed and flux weakening angle α, for nominal current (left) and overload current (right).

Fig. 12, shows the computed mean torque as a function of speed and flux weakening angle, for two different reference peak current of the inverter (nominal, overload). Each value of the torque has been computed with the real stator current waveshape computed by a 'over one period' stationary analysis. One may notice the effect of saturation on the shape of the torque plots. The crest line represents, for each speed, the optimal flux-weakening angle.

The generation of those relevant mechanical characteristics of the motor with classical transient computations would have been prohibitive. However, thanks to the staged modelling, the investment that has been done in the computation of the complete flux plot look-up table pays off in enabling to derive at low cost other valuable characteristics of the machine. Another useful characteristic, for instrance, that can be derived directly from the flux plot and a 'over one electrical period' analysis is the *run-time inductance* of the phase :

$$L(\theta) = \partial_I \varphi_R(\theta - \alpha, I_R(\theta)). \tag{18}$$

One sees in Fig. 13 how much more information we have in comparison with the classical $L_d - L_q$ approach.

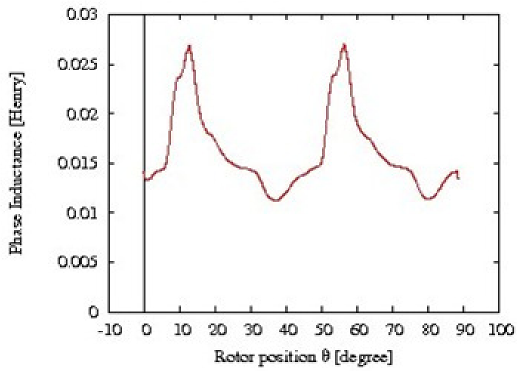


Figure 13: Run-time phase inductance as a function of the rotor position.

5.4 Implementation of the staged model

A staged model is not a computation run. It is rather a growing set of organised information, similar to an expert system. When the sub-system structure has been identified, the interfaces are characterised, and their description stored, under the form of look-up tables in general. The dynamic multi-domain behaviour of the system is then computed by means of a specific programme (C++, Matlab, . . .) that manage with maximum efficiency and versatility the interaction between the sub-systems, performing all required operations (time stepping, integration, etc . . .). The model can so evolve progressively, from the low accuracy/low computational time model of early design, up to a very complete and argumented representation of the system. C++ classes or pieces of code (management of 'over one period' solutions for instance) that have been written for one specific model are usually widely reusable or easily adaptable to other models.

6 CONCLUSIONS

This paper has presented a representative set of state-of-the-art approaches or methodologies that have been successfully applied at our institute to modelize real-life multi-domain applications.

In the cases of the induction furnace and the acoustic radiation of an induction motor, the simplifications that allow to solve the system by weak coupling have been discussed. The coupled model is in such situations a chain of single-physics simulations.

In the case of servo-motors, a strong coupling is required. A dedicated methodology has been proposed, that relies on a decomposition of the system into a set of sub-system that interact only through well-defined and controlled channels. If the sus-bsystems make sense, they give valuable information about internal functioning of the system, they provide intermediary control points in the modelling. Experience shows that, when applicable, staged modelling helps intuitive understanding of a complex system and speeds up development.

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