

# MSFEM with Network Coupling for the Eddy Current Problem of a Toroidal Transformer

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**Abstract**—The nonlinear eddy current problem with network coupling is solved for a toroidal transformer. The problem is assumed to be axially symmetric to allow calculations in two dimensions. Furthermore the fine structure of the laminated core is not resolved in the finite element mesh. Instead a MSFEM (multiscale finite element method) ansatz is used which allows one to treat the core as a bulk material. The local behavior is recovered by enriching the finite element space with additional micro-shape functions. Measurement data are used to evaluate the quality of the simulation.

**Index Terms**—Eddy currents, multiscale FEM, network coupling, nonlinear material.

## I. PROBLEM SETTING

The nonlinear eddy current problem with network coupling reads as: For a given frequency  $f$  and voltage  $u(t) = U_{max} \cos(2\pi ft)$  find the current  $i(t)$  and the magnetic vector potential  $\mathbf{A}(t) \in H(\text{curl})$  so that

$$\int_{\Omega} \mu^{-1}(\mathbf{A}) \text{curl} \mathbf{A} \text{curl} \mathbf{v} + \frac{\partial}{\partial t} \sigma \mathbf{A} \mathbf{v} \, d\Omega = \int_{\Gamma} i \frac{N}{2r\pi} \mathbf{v} \mathbf{w} \, d\Gamma$$

$$iR + \int_{\Gamma} \frac{\partial}{\partial t} \mathbf{A} \frac{N}{2r\pi} \mathbf{w} \, d\Gamma = u \quad (1)$$

for all  $\mathbf{v} \in H(\text{curl})$ . The coefficients in (1) are the magnetic permeability  $\mu$ , the electric conductivity  $\sigma$ , the number of windings  $N$  and the electrical resistance  $R$ .

A three dimensional FE model of the laminated core with inner diameter of 48 mm and outer diameter of 60 mm consisting of ten 0.5 mm thick sheets is shown in Fig. 1.

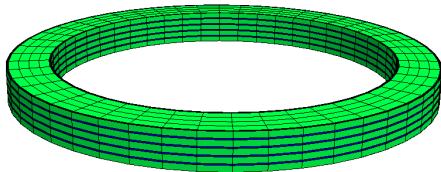


Fig. 1. FE model of the iron core (green) with air gaps (blue). For the sake of visibility only 5 of the 10 sheets are shown and the air gap is drawn disproportionately.

## II. IMPLEMENTATION

The problem is solved in two dimensions utilizing cylindrical coordinates.

To further reduce the required degrees of freedom, the problem is solved on a mesh which does not resolve the single iron sheets and the multiscale ansatz

$$\mathbf{A} = \mathbf{A}_0 + \phi \begin{pmatrix} A_1 \\ 0 \end{pmatrix} + \nabla(\phi w) \quad (2)$$

is used with  $\mathbf{A}_0 \in H(\text{curl})$ ,  $A_1 \in L^2$  and  $w \in H^1$ . The piecewise linear, periodic micro-shape function with a period length equal to one iron sheet thickness plus one air gap is denoted as  $\phi$ , for details see [1].

The nonlinearity in the coefficient  $\mu^{-1}(\mathbf{A})$  in (1) is considered as a piecewise linear interpolation of measurement data of the magnetization curve. In each time step the nonlinear system is solved using a damped Newton iteration. At each integration point a local average of the curl of the multiscale solution is used to calculate a mean coefficient for the permeability and the differential permeability which is used in the assembly of the system.

## III. RESULTS

To test the two dimensional multiscale ansatz, a reference solution has been computed on a mesh which resolves each iron sheet. Figure 2 shows an excellent agreement between the currents calculated by the multiscale method and the reference solution. Comparison with measurement data shows a good agreement of amplitude and phase shift of the current. Note that (1) does not incorporate hysteresis effects, which is a possible cause of the visible differences.

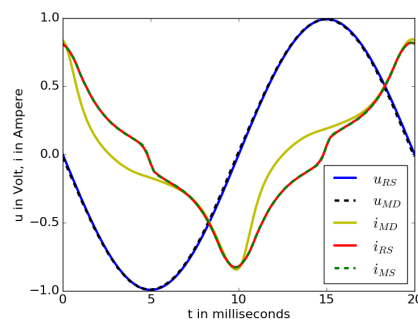


Fig. 2. Voltage  $u$  and current  $i$  over one period for the measurement data (MD), the reference solution (RS) and the multiscale solution (MS) at a peak average magnetic flux density of 1.3T.

## IV. ACKNOWLEDGMENT

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## REFERENCES

- [1] K. Hollaus and J. Schöberl, "A higher order multi-scale FEM with A for 2-D eddy current problems in laminated iron," IEEE Transactions on Magnetics, Volume 51, Issue 3, March 2015