Sensitivity Analysis on Tolerance Induced Torque Fluctuation of a Synchronous Machine

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Abstract **— The manufacturing process of electrical machines influences the geometric dimensions and material properties, e.g. the yoke thickness. These influences occur by statistical variation as manufacturing tolerances. The effect of these tolerances and their potential impact on the mechanical torque output is not fully studied up to now. This paper conducts a sensitivity analysis for geometric and material parameters. For the general approach these parameters are varied uniformly in a range of 10 %. Two dimensional finite element analysis is used to simulate the influences at three characteristic operating points. The studied object is an internal permanent magnet machine in the 100 kW range used for hybrid drive applications. The results show a significant dependency on the rotational speed. The general validity is studied by using boundary condition variations and two further machine designs. This procedure offers the comparison of matching qualitative results for small quantitative deviations. For detecting the impact of the manufacturing process realistic tolerance ranges are used. This investigation identifies the airgap and magnet remanence induction as the main parameters for potential torque fluctuation.**

Keywords — Electrical Machines; Sensitivity Analysis; Tolerance range; Parameter Variation; Parameter Uncertainty

I. INTRODUCTION

Possible tolerance influences are often neglected during the electromagnetic design phase. Nevertheless tolerance induced variations can lead to significant torque fluctuation in maximum and continuous output. These fluctuations are unwanted, lead to rejects and the risk of losing the certification of conformity [1]. To avoid this, influential parameters should be identified and taken into consideration by machine designers. The reproducibility of the intended mechanical output is increased by defining narrow tolerance ranges where needed. The result is a decrease of deviations which enables the reduction of safety factors used in the machines design. Thereby a higher utilization factor can be achieved. Tolerance ranges of parameters with low impact can be widened, which also reduces manufacturing costs.

The number of produced electric drives rises due to the increasing electrification of the drivetrain. As a result possible tolerance influences attract particular attention. This is supported by the rising number of publications which study possible influences of manufacturing imperfections. Considered are different machine types, sizes, power ratings and parameters. To identify the effect of tolerances [2] investigated 10 asynchronous drive machines and their torque deviations at different operating points. The standard deviations for the torque output range from approx. 0.7 % at maximum torque to approx. 2.5 % at maximum power. At least 1 % of these deviations are because of production uncertainty. This demonstrates the presence of torque fluctuation between basically identical machines. Furthermore it shows the dependency on the considered operating point. Also the air gaps vary in a range of 3.2 %. Reference [3] investigates three parameters regarding their influence on the torque output. It is shown that the air gap is a highly influential parameter. A more in depth sensitivity analysis of a surface mounted permanent magnet machine is conducted by [4]. Six parameters are studied in the context of a performance optimization. Five of these parameters are relevant since the pole number is not affected by tolerances. The magnet width and core length emerge as the most influential ones followed by magnet height and air gap. Reference [5] presents similar results considering different geometrical parameters in the context of a robust design sensitivity analysis. The investigation of static and dynamic rotor eccentricity shows no significant influence, neither for a permanent magnet synchronous machine [6] nor an asynchronous machine [7]. Despite the fact that torque fluctuation is a critical factor for type approval, most of the recent publications study vibro-accoustic effects. In this regard, tolerance influences have a high impact on cogging torque and torque ripple. The number of studied parameters is considerably higher [8] and mostly focused on the permanent magnet [9], [10]. Previous works also take into account different magnet configurations regarding magnetization faults [11] or magnet misplacement [12], [13]. Also rotor misplacement in form of static and dynamic eccentricity is investigated [14]. Likewise researched are manufacturing influences on the magnetic properties of electrical steel. The punching effect [15] as well as global stresses, e.g. through press fit connections [16], are analyzed.

The investigation of tolerance induced influences is particularly challenging, because of the quantity of parameters to consider. Additionally it should include possible interdependencies between the parameters. The complexity of this topic is further increased by the dependency on the operating point. Most studies use a conventional and time consuming approach. For this reason they limit the number of parameters reviewed. They use different tolerance ranges for the parameters, without respecting possible interdependencies.

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This limits the generality and transferability of their findings. The studies mostly concentrate on vibro-accoustic instead of performance impacts. In contrast, this paper takes into account geometric dimensions and material properties. It also identifies interdependencies with additional impact on the torque output. The approach considers boundary conditions, different machine designs as well as three characteristic operating points. To simulate the variations a two dimensional finite element analysis (FEA) combined with a time efficient design of experiments (DOE) is used. The DOE consists of a single parameter variation coupled with a fractional experimental design to classify the parameters impact, i.e. the sensitivity. To put the results of the general approach into perspective, realistic tolerance ranges are applied. The paper provides extensive results useful to define appropriate tolerance ranges in order to minimize torque fluctuation.

II. Investigated Parameters and Methodology

The investigations are conducted using three different machine models, to determine the generality of the researched sensitivities. They are of the type internal permanent magnet machine with concentrated winding, featuring an identical number of slots per pole. An overview of the studied machine models is given in Table 1. Based on the reference model, a geometrically scaled machine with identical detail geometries of tooth, permanent magnet and rotor is used. It is differentiated by using more poles and slots to realize a 52 % larger outer diameter. The third machine is optimized featuring similar radial dimensions as the reference machine but has 27 % shorter axial length. It exhibits different detail geometries, including tooth and slot geometry as well as a smaller bridge thickness. The permanent magnet is optimized and offers 17 % higher remanence at the same coercivity, e.g. by using the grain boundary diffusion process. The machine uses thinner electrical steel, which is optimized for reduced losses. It has similar permeability and saturation polarization and therefore no significant effect on the torque output. Despite a higher conductor per slot ratio the copper fill factor is comparable due to a thinner wire diameter. All machines are simulated without skew.

TABLE I. Overview of the Investigated Machine Models

	Reference	Scaled	Optimized
Outer Diameter	100 %	152 %	103 %
Detail Geometry	Base	Base	Optimized*
Stack Length	100 %	100 %	73 %
PM Remanence	100 %	100 %	117 %
DC Voltage mot.	320 V	320 V	350 V
Supply Current	300 A_{rms}	300 A_{rms}	320 A_{rms}
No. of Turns	63	63	78

*Details mentioned in text

Furthermore the influence of boundary condition variations on the sensitivities is examined. Varied boundary conditions are supply current and voltage as well as machine temperature. The reviewed geometric parameters are pictured in Fig. 1. All selected parameters are tolerance influenced in a real machine.

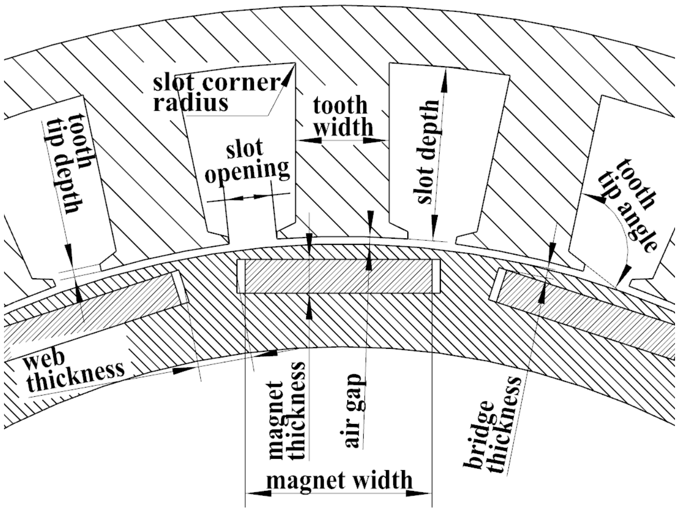

Fig. 1. Overview of the investigated geometric parameters of stator core, rotor core and permanent magnet.

The slot depth varies the models yoke thickness due to the parametrization of the model. Magnet width and web thickness both influence the tangential space between magnet and web. A variation of the airgap results in a change of the rotors outer diameter. This also has a slight influence on the tangential space between magnet and web. For every machine model the respective id/iq vectors are specified for the nominal values. These vectors are used unchanged for each variation to replicate the machines behavior without modifications in the machines control.

The proposed general approach begins with a single parameter variation to assess the parameter sensitivities. This part not only provides information about the maximum impact of a parameter but also about the linearity of its influence. To offer fundamental benefit, the parameters are varied detached from realistic tolerance ranges. For this reason, all sensitivities are determined by uniformly altering the tolerances in the range of ± 5 %. The incremental step for the simulations is 2.5 %, resulting in five calculations per parameter. Thus the assessed sensitivities are comparable and serializable. Possible interdependencies are identified following the single parameter variation. Therefor a fractional design of experiments is used. This DOE is able to assess interdependencies up to second degree, e.g. air gap variation combined with magnet remanence variation. The influence of the interdependencies is quantified using the factorial design. The thirteen most important are identified and further detailed with ‘two parameter at a time’ variations. This approach, illustrated in Fig. 2, reduces the number of required simulations significantly, compared to a conventional factorial DOE to about 0.5 ‰. In contrast to robust design approaches, no control factors have to be classified. This enables the investigation of the control factors which are likewise tolerance influenced.

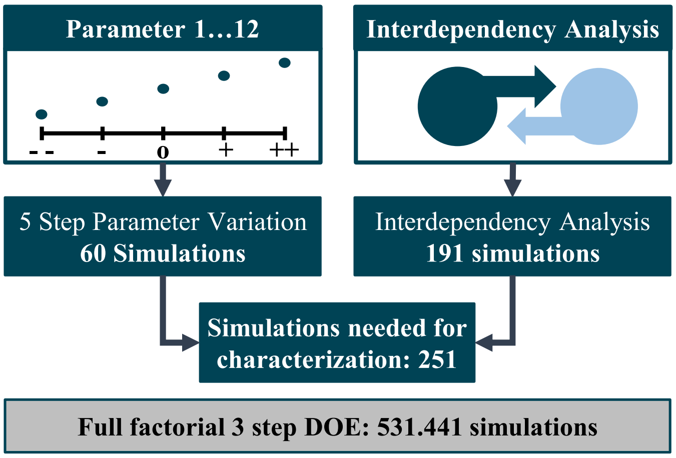

Fig. 2. Methodology as combination of single parameter variation and fractional design of experiments compared to a conventional approach.

For every parameter variation the simulation is conducted using three different operating points: maximum torque at 200 revolutions per minute (rpm), maximum torque at the point of maximum power and maximum torque at maximum speed. The observed point of maximum power is specified by the nominal machine model and kept identical for each parameter variation. A possible shift of the examined point to lower or higher rotational speeds is neglected with respect to the testing regulatory for the certification of conformity, where each unit is tested at the primarily specified point of maximum power. The maximum speed is specified at 7000 rpm. These operating points are used to determine the influence of each parameter x on the different parts of the characteristic curve. The sensitivities are quantified by calculating a sensitivity value S. It represents the relative sum of deviations according to

$$S_x = \sum_{i=1}^{n} \frac{|T_n - T_0|}{T_0} \quad (1)$$

with T_i as the calculated average dq torque for n variation steps and T_0 being the torque value of the nominal machine model. The calculation of the sum of relative deviations is visualized in Fig. 3. Calculating the sensitivity using (1) takes into account nonlinear behavior. Therefore it enables a more realistic representation of the parameters influence instead of e.g. the maximum deviation of a parameter or the maximum slope of a linear regression.

Fig. 3. Visualization of the calculation of the sum of relative deviations from the nominal value to quantify the influence of a parameter deviation.

III. Results and Discussion

In this section the results for the aforementioned parameter and boundary condition variations are presented. The interdependencies as well as the realistic tolerance ranges are discussed. Every variation is conducted on the three specified operating points.

A. *Influence of Single Parameter Variation*

First of all the sensitivities for the reference machine are studied. Each parameter is varied ± 5 % and the sensitivities are calculated as the sum of relative deviations following (1). The results are presented in Fig. 4.

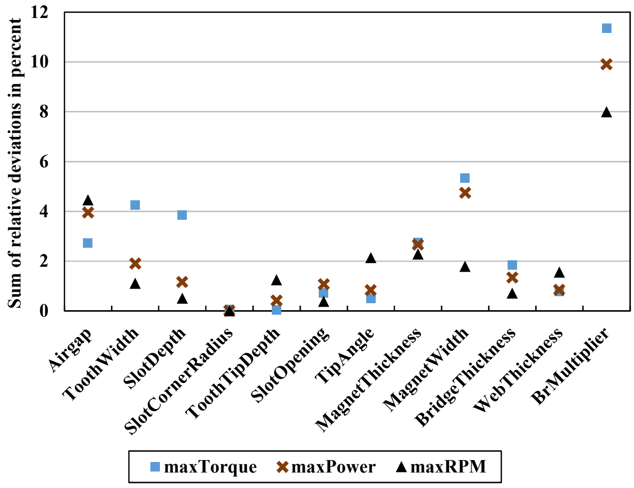

Fig. 4. Sensitivites as the sum of relative deviations of the reference machine at three operating points.

The shown sensitivities offer direct comparability between the parameters, because of the uniform variation. They display a distinct speed dependency and thereby a varying order for each operating point. Magnet remanence induction, referred to as Br Multiplier in Fig. 4, is most influential regardless of the operating point. Its sensitivity decreases with increasing speed due to the decreasing d-axis current. Magnet width, magnet height, tooth width and slot depth behave accordingly as they directly influence the magnetic flux. The airgap gets more sensitive with increasing speed since the parameter influences the flux weakening capabilities. Slot depth influences the yoke thickness and exhibits a strong nonlinear behavior. In case of high saturation a reduction influences the magnetic circuit far more than a yoke enlargement. The slot corner radius has nearly no influence, expressed by the sensitivity value close to zero.

B. *Influence of Boundary Conditions*

To specify the sensitivities persistency against boundary condition changes, variations of supply current, supply voltage and machine temperature are investigated. Fig. 5 pictures the sensitivities for a current variation of ± 5 % in contrast to the reference models (300 A) sensitivities. To conduct this analysis new id/iq vectors had to be generated for each current level.

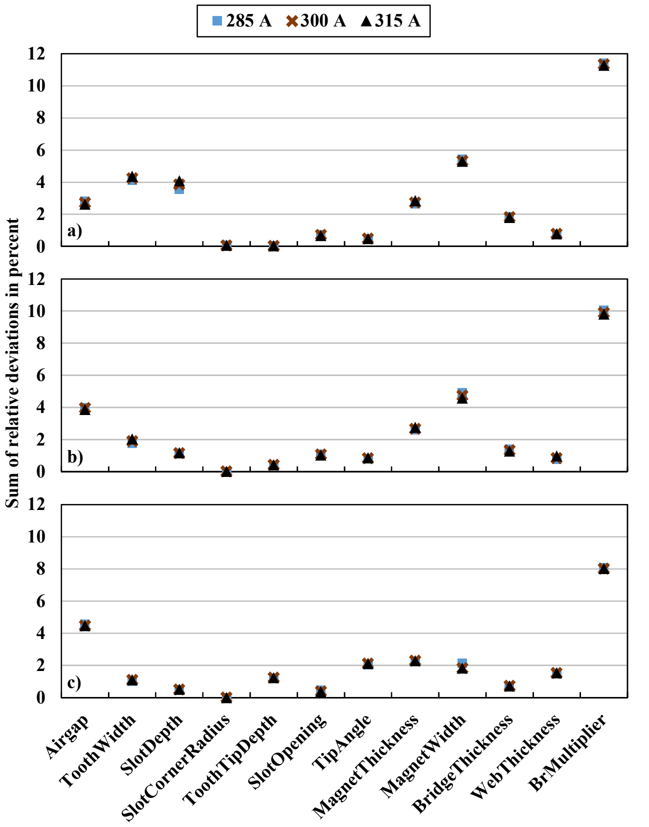

Fig. 5. Sensitivities for the reference machine under the influence of current variation at a) max Torque, b) max Power and c) max RPM.

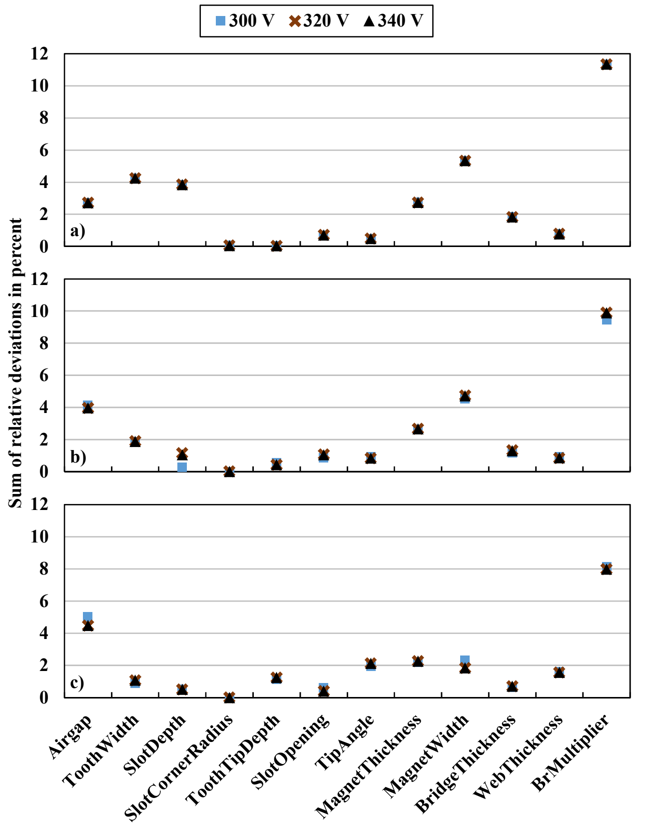

Fig. 6. Sensitivities for the reference machine under the influence of voltage variation at a) max Torque, b) max Power and c) max RPM.

The sensitivities show only small deviations, restricted to the main magnetic parameters influencing the magnetic flux. They are mostly visible at slot depth in Fig. 5 a). Lower currents lead to less saturation in the yoke and therefore to a decreased sensitivity of a yoke thickness variation.

In Fig. 6 a voltage alteration ± 20 V is pictured with no significant influence at operating point one. For higher speeds a slight influence is visible for slot depth, magnet remanence induction, airgap and magnet width. Voltage variation leads to a scaling of the characteristic torque curve, so a different point on the curve is analyzed. Therefore Fig. 6 b) exhibits slightly changed sensitivities for maximum power. For operating point three the sensitivities of airgap and magnet width increase because of their influence on the field weakening capabilities.

No significant influence on the sensitivities is visible for the temperature variation in the same range of ± 5 %. Significant effects are visible for higher temperature variations due to the temperature dependency of the magnet remanence induction. All examined condition variations exhibit absolute torque changes while the quantitative sensitivity values are almost unchanged. As a result the relative influences of the parameters have to be the same.

C. *Interdependencies between investigated parameters*

The proposed approach takes into account interdependencies up to second degree. The thirteen most important are analyzed in-depth by altering both parameters at the same time. The resulting sensitivities are shown in Fig. 7.

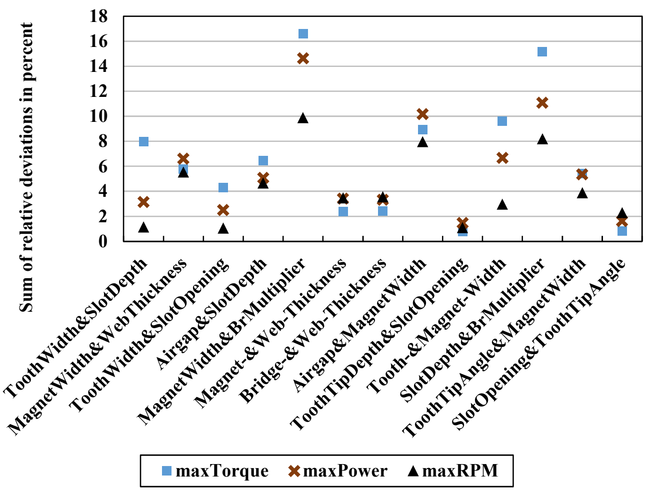

Fig. 7. Sensitivities as the sum of relative deviations of the interdependencies of the reference machine at three operating points.

The combinations including the magnet remanence induction are the interdependencies with the highest sensitivity. The combination of airgap and magnet width stands out since the sensitivities do not change linear over rotational speed. This occurs due to the significantly decreased influence of magnet width at high speeds. The decrease is not compensated by the airgap parameter.

To identify the important interdependencies for a machine designer, a different approach is applied. Therefor a sensitivity for the linear addition is calculated and compared to the interdependency sensitivity. It evaluates, how well the interdependency is described by a linear addition of the parameters. The differences between these two sensitivities are calculated and shown in Fig. 8.

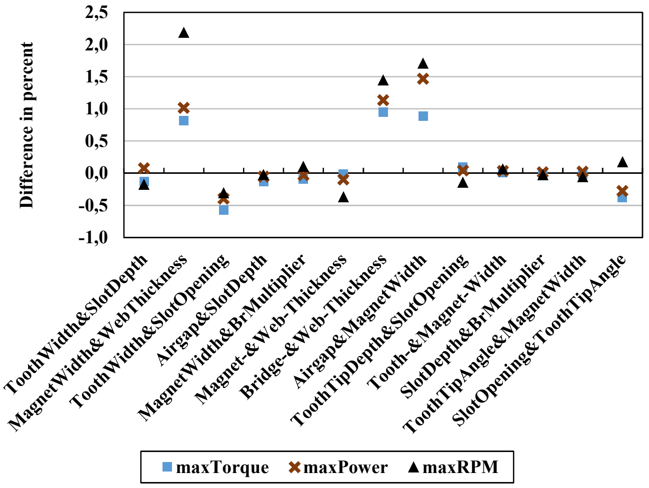

Fig. 8. Deviation between interdependency and linear addition sensitivities of the reference machine at three operating points.

The higher the deviation from the linear addition, the more severe an interdependency effect is in form of a gain (positive values) or reduction (negative values). Many interdependencies show little deviation. For most parameters the influence increases with rotational speed. The most important interdependencies, i.e. not being properly described by the linear addition, are magnet width and web thickness, bridge and web thickness as well as the combination of airgap and magnet width. The deviation of the latter combination consists of two parameters influencing the airgap flux. The combination of bridge- and web-thickness may influence the stray flux. For this reason their combined influence differs from the linear addition. Magnet width as well as web thickness change the air filled space between magnet and web and seem to interact nonlinear when varied simultaneously.

D. Considering Real Tolerance Ranges

The used uniform variation of each parameter was necessary, to get comparable and serializable results. Looking at real manufacturing processes the individual tolerance ranges vary and are considerably smaller. Therefore realistic tolerance ranges are assumed for each parameter. The resulting sensitivities are shown in Fig. 9. This investigation indicates two important parameters for series manufacturing.

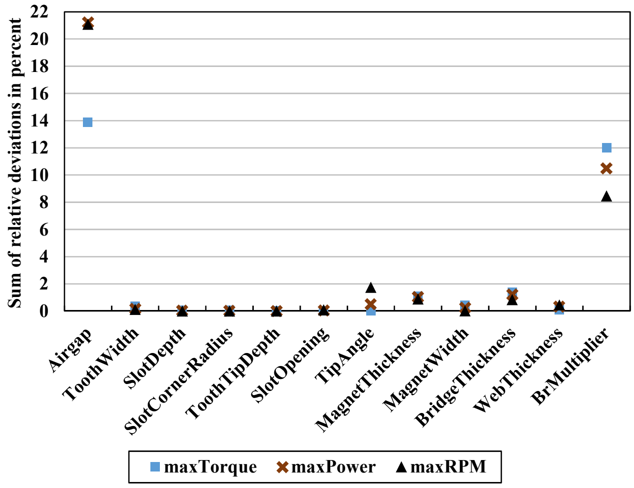

Fig. 9. Sensitivities of the reference machine with assumed tolerance ranges for each parameter at three operating points.

Typically the material properties of permanent magnets, in this case ± 5 %, have higher deviations than their geometric counterparts. The geometric tolerances of the permanent magnet are comparably small, leading to their reduced influence. Likewise the sensitivities of the iron core parameters decrease, due to the small tolerance ranges of only hundreds of millimeters feasible in a stamping process. A noticeable shift in influence appears at the airgap, where a comparably wide geometric tolerance range (0.375 mm) is assumed. The reasons for this wide range are various geometric tolerances influencing the airgap. As a result the airgap is the parameter with the highest effect, although its sensitivity is significantly lower compared to the magnet remanence induction. The effect also increases with speeds beyond the armature-setting range because of field weakening capabilities.

IV. Investigation of General Validity

Having established the negligible influence of boundary conditions, the general validity of the sensitivities is investigated. Used are two differently designed machines with the described alterations.

A. Geometrically Scaled Machine

To identify possible effects of a geometrical alteration, the aforementioned scaled machine is studied. Fig. 10 pictures the sensitivities for the scaled and the reference machine. The sensitivities of tooth width and slot depth are significantly lower due to decreased flux density in the core. This occurs due to a reduced induction since the scaled model has more coils but is operated at the same current. This also accounts for the differences in the permanent magnet sensitivities because the flux emitted by the magnet has a higher influence. Also the coupling between rotor and stator gets more important, expressed by the increased airgap sensitivity. This effect intensifies for higher speeds with airgap, slot opening and magnet width sensitivities rising significantly. All of them influence the so called effective airgap, what leads to the mutual increase.

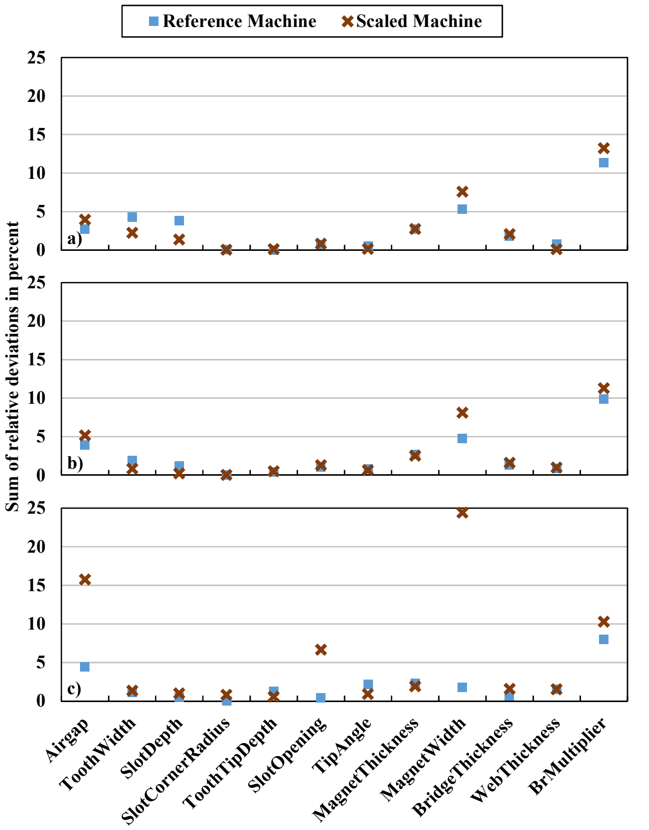

Fig. 10. Comparison of sensitivities for the reference and scale model at a) max Torque, b) max Power and c) max RPM.

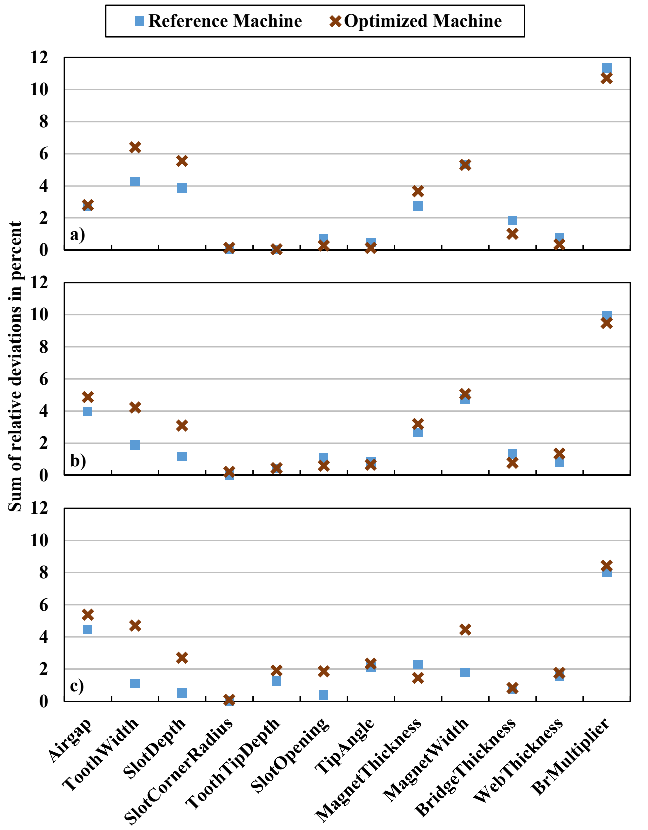

Fig. 11. Comparison of sensitivities for the reference and optimized machine at a) max Torque, b) max Power and c) max RPM.

The machines design with its large diameter is predestined for high torque, but not for high speed applications. To reach operating point three high field weakening capabilities are needed. This is reflected in the effective airgap parameters. The differences between the reference and scaled machine are calculated as the sum of relative deviations so the absolute difference for each variation point is smaller.

B. Optimized Machine using a Different Design

The optimized machine, as described in section II, has comparable radial dimensions while it is shorter, has different detail geometries, different materials and also different supply values. Its sensitivities, in comparison to the reference machines, are pictured in Fig. 11 for the three operating points.

Contrarious to the scaling influence, the optimized machine shows increased sensitivities for most parameters, especially for the stator core ones. These overall increased sensitivities are not due to the increased supply values. This has been established with the voltage and current variation of the reference machine in III.B. The increases reflect the design of the optimized machine. It exhibits decreased robustness in exchange for a higher utilization factor. It achieves similar torque output as the reference machine despite being 27 % shorter.

V. Conclusions

The effect of tolerance induced torque fluctuation on an internal permanent magnet machine was investigated by varying geometric as well as material parameters. It was shown that the sensitivities, calculated as the sum of relative deviations, of most parameters change with rotational speed. Additionally the influence of boundary conditions was studied. Changes of these conditions exhibit only a slight influence on the calculated sensitivities. To validate the generality two further machine designs were studied. Both designs exhibit quantitative deviations in the sensitivities without changing the qualitative results. The magnet remanence induction is established as the most influential parameter for the general approach. Furthermore it was demonstrated that most parameters are well controlled when assuming realistic tolerance ranges. Not adequately restricted are magnet remanence induction and airgap. Especially the airgap sensitivity in combination with the assumed tolerance range is a source of possible torque fluctuation. With the ambition to achieve higher utilization factors the risk of tolerance induced torque fluctuation is increased. For this reason the presented investigations offer important results for future design optimizations.

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