1-st IEEE Conference
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2016 1st IEEE Conference Advances in Magnetics Proceedings

Energy-Based Ferromagnetic Material Model with Magnetic Anisotropy and Magnetostriction

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***Abstract*—Non-oriented soft magnetic materials are commonly assumed to be magnetically isotropic. However, due to the rolling process a preferred direction exists along the rolling direction. This uniaxial magnetic anisotropy, and the related magnetostriction effect, are critical to the accurate calculation of iron losses and magnetic forces in rotating electrical machines. This paper proposes an extension of an isotropic energy-based vector hysteresis model to account for these two effects.**

***Index Terms*—Hysteresis modeling, magnetic anisotropy, magnetostriction, thermodynamics.**

I. Introduction

Non-oriented soft magnetic materials are widely used as a basic constituent in rotating electrical machines. Although their qualification seems to indicate them to be magnetically isotropic, they exhibit actually, due to the rolling process, a magnetically preferred direction that leads to anisotropy in both the magnetic and elastic behavior [1]. A variety of vector hysteresis models have been developed to simulate the magnetization process under rotational field. Many of them are vector extension of well-established uniaxial scalar models; the vectorization being realized by superposition of a number of scalar models oriented over different directions [2], [3], and [4]. In contrast, the model presented in this paper builds on an isotropic energy-based vector hysteresis model [5], [6], [7], which is inherently a vector model and offers readily a complete theoretical framework to include magnetic anisotropy, magnetostriction, and characteristic features of magnetic hysteresis such as the wiping-out property or rotational hysteresis.

II. Vector Hysteresis Material Model

The ferromagnetic material model follows from the expression of the conservation of energy in the material

$$\dot{u} = \mathbf{h} \cdot \dot{\mathbf{b}} + \dot{Q} \tag{1}$$

where u is the internal energy density functional, $\mathbf{h} \cdot \dot{\mathbf{b}}$ the rate of magnetic work, and $\dot{Q}$ a dissipation functional. The functional u determines the anhysteretic saturation characteristic of the material, which is a 1-1 relationship between the (dual) quantities $\mathbf{h}_r$ and $\mathbf{b}$. The characteristic and its inverse write

$$\mathbf{h}_r = \partial_{\mathbf{b}} u(\mathbf{b}), \quad \mathbf{b} = \mathbf{b}_{\text{an}}(\mathbf{h}_r) \tag{2}$$

Practically, in the saturable isotropic case, the vectors $\mathbf{b}$ and $\mathbf{h}_r$ are co-linear and the characteristics (2) are written in terms of a scalar magnetic reluctivity $\nu(b)$ and a scalar magnetic permeability $\mu(h)$ as

$$\mathbf{h}_r = \nu(|\mathbf{b}|)\mathbf{b} \quad , \quad \mathbf{b} = \mu(|\mathbf{h}_r|)\mathbf{h}_r . \tag{3}$$

In order to describe hysteresis, the ferromagnetic material is regarded as a set of independent abstract domains characterized each by a particular value κ^k of the pinning force. Let ω^k, $\sum_k \omega^k = 1$, be the probability that a magnetic moment in the material belongs to the domain with pinning field κ^k, of which the magnetic state is described by a magnetic polarization $\mathbf{J}^k$. One has $\mathbf{b} = \sum_k \mathbf{J}^k$, and the energy density of the material, and its time derivative, write respectively

$$u = \sum_k \omega^k u\left(\frac{\mathbf{J}^k}{\omega^k}\right) \quad , \quad \dot{u} = \sum_k \mathbf{h}_r\left(\frac{\mathbf{J}^k}{\omega^k}\right) \cdot \dot{\mathbf{J}}^k \tag{4}$$

in terms of the state variables $\mathbf{J}^k$. The dissipation functional, on the other hand, is the algebraic sum of the dissipation in the different domains

$$\dot{Q} = -\sum_k \kappa^k \left|\dot{\mathbf{J}}^k\right| = -\sum_k \mathbf{h}_i^k \cdot \dot{\mathbf{J}}^k \quad \text{with} \quad \mathbf{h}_i^k\left(\dot{\mathbf{J}}^k\right) := \kappa^k \frac{\dot{\mathbf{J}}^k}{\left|\dot{\mathbf{J}}^k\right|}. \tag{5}$$

The terms of (1) can now be summed up to obtain the relationship

$$\left(\mathbf{h} - \mathbf{h}_r\left(\frac{\mathbf{J}^k}{\omega^k}\right) - \mathbf{h}_i^k\left(\dot{\mathbf{J}}^k\right)\right) \cdot \dot{\mathbf{J}}^k = 0, \tag{6}$$

which must hold for arbitrary $\dot{\mathbf{J}}^k$, hence the nonlinear differential equation in $\mathbf{J}^k$

$$\mathbf{h} = \mathbf{h}_r\left(\frac{\mathbf{J}^k}{\omega^k}\right) + \kappa^k \frac{\dot{\mathbf{J}}^k}{\left|\dot{\mathbf{J}}^k\right|} \tag{7}$$

to be solved at each time step in each domain.

III. Magnetic Anisotropy

Soft magnetic steel laminations produced by a rolling process have a crystallographic texture in which the crystals have a preferred orientation. Such materials exhibit at the macroscopic scale a uni-axial kind of anisotropy for which the rolling direction (RD) must be distinguished for the transverse directions (TD). We want to adapt the material model for such materials and seek for an implementation consistent from the point of view of both geometry and energy conservation.

Macroscopic anisotropy has an effect on both the coercive field and the anhysteretic curve. The latter can be accounted for by replacing the scalar magnetic reluctivity $\nu(\mathbf{b})$ with a

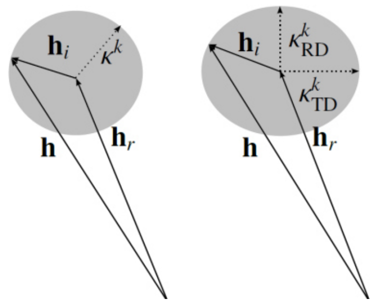

Figure 1: Graphical representation of the vector equation $\mathbf{h} = \mathbf{h}_r + \mathbf{h}_i$ for the perfectly isotropic (left) and anisotropic (right) case.

tensorial magnetic reluctivity $\nu(\mathbf{b})\boldsymbol{G}(\gamma)$ where the tensor (the undotted product of vectors **ab** is the dyadic product)

$$\boldsymbol{G}(\gamma) = \mathbf{e}_{\|}\mathbf{e}_{\|} + \gamma^2 \mathbf{e}_{\perp}\mathbf{e}_{\perp} \tag{8}$$

is defined as a function of a scalar parameter γ, $\gamma > 1$, and the micro-structure-anchored unit vectors $\mathbf{e}_{\|}$ and $\mathbf{e}_{\perp}$ in rolling and transverse direction respectively (they are unitary in the reference configuration). Hence, the anisotropic version of the saturation law (3) writes

$$\mathbf{h}_r = \partial_{\mathbf{b}} u(\mathbf{b}) = \nu(|\mathbf{b}|)\boldsymbol{G}(\gamma)\mathbf{b} \tag{9}$$

$$\mathbf{b} = \mathbf{b}_{\text{an}}\left(\boldsymbol{G}^{-1}(\gamma)\mathbf{h}_r\right) = \mu\left(\left|\boldsymbol{G}^{-1}(\gamma)\mathbf{h}_r\right|\right)\boldsymbol{G}^{-1}(\gamma)\mathbf{h}_r \tag{10}$$

On the other hand, the effect of magnetic anisotropy on the coercivity of the material is represented in the hysteresis model by flattening the spheres of the isotropic model in RD, Fig. 1. The scalar pinning force κ^k does not become a tensor, however, because the magnetic field $\mathbf{h}_i$ must by definition remain colinear with the variation of the magnetic polarization $\dot{\mathbf{J}}$. Anisotropy is represented by having the pinning force κ^k be a function of the angle between $\dot{\mathbf{J}}$ and $\mathbf{e}_{\|}$, e.g.,

$$\kappa^k(\alpha, \gamma') = \kappa^k_{\text{RD}}\left(1 + (\gamma' - 1)\left(1 - \alpha^2\right)\right) \tag{11}$$

where $\alpha = \dot{\mathbf{J}} \cdot \mathbf{e}_{\|}$ is the direction cosine of $\dot{\mathbf{J}}$ in rolling direction.

The anisotropic extension of the material model (7) writes thus

$$\mathbf{h} = \nu\left(\frac{\left|\mathbf{J}^k\right|}{\omega^k}\right)\boldsymbol{G}(\gamma)\frac{\mathbf{J}^k}{\omega^k} + \kappa^k\left(\dot{\mathbf{J}} \cdot \mathbf{e}_{\|}, \gamma'\right)\frac{\dot{\mathbf{J}}^k}{\left|\dot{\mathbf{J}}^k\right|}, \tag{12}$$

for which it is sufficient to identify the scalar parameters γ and γ'. More involved expressions of (8) and (11) with more parameters, or additional unit vectors associated with other significant directions of easy magnetization, can be used if needed, and if there is enough experimental data to identify them. As we are dealing here with the uniaxial anisotropy of standard electric steel laminations, we proceed with anisotropy represented by just these two scalar parameters.

In order to solve (12), it is useful to define the auxiliary quantity, $\mathbf{x} := \boldsymbol{G}^{-1}(\gamma)\mathbf{h}_r\left(\frac{\mathbf{J}^k}{\omega^k}\right) = \nu\left(\frac{\left|\mathbf{J}^k\right|}{\omega^k}\right)\frac{\mathbf{J}^k}{\omega^k}$, in terms of which, using (10), the variation of the magnetic polarization

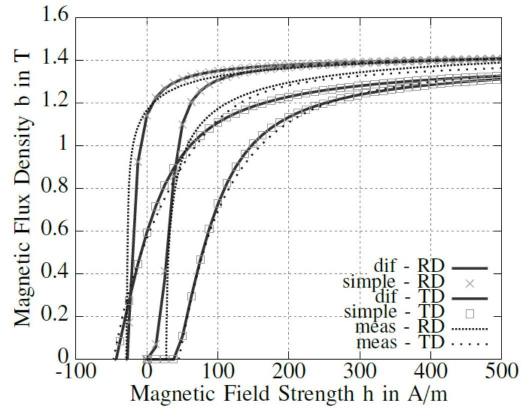

Figure 2: Measured (dotted) and simulated (differential approach: full lines; simple approach: dots) quasi-static major loops in rolling (RD) and transversal direction (TD).

can be linearized as

$$\frac{\dot{\mathbf{J}}^k}{\omega^k} = \partial_t \mathbf{b}_{\text{an}}(\mathbf{x}) = \partial_t\left\{\mu(|\mathbf{x}|)\mathbf{x}\right\}$$
$$= \left\{\mu(|\mathbf{x}|)\boldsymbol{I} + 2\partial_{h^2}\mu(|\mathbf{x}|)\mathbf{x}\mathbf{x}\right\}\dot{\mathbf{x}} := \mu^{\partial}(\mathbf{x})\dot{\mathbf{x}},$$

and then, at first order,

$$\frac{\dot{\mathbf{J}}^k}{\left|\dot{\mathbf{J}}^k\right|} = \frac{\mu^{\partial}(\mathbf{x})\,\dot{\mathbf{x}}}{\left|\mu^{\partial}(\mathbf{x})\,\dot{\mathbf{x}}\right|} \approx \frac{\mu^{\partial}(\mathbf{x}_p)(\mathbf{x} - \mathbf{x}_p)}{\left|\mu^{\partial}(\mathbf{x}_p)(\mathbf{x} - \mathbf{x}_p)\right|} \tag{13}$$

with $\mathbf{x}_p$ the value of $\mathbf{x}$ at the previous time step. Equation (12) rewrites a nonlinear implicit differential equation in $\mathbf{x}$

$$\mathbf{h} = \boldsymbol{G}(\gamma)\mathbf{x} + \kappa^k\left(\dot{\mathbf{J}} \cdot \mathbf{e}_{\|}, \gamma'\right)\frac{\mu^{\partial}(\mathbf{x}_p)(\mathbf{x} - \mathbf{x}_p)}{\left|\mu^{\partial}(\mathbf{x}_p)(\mathbf{x} - \mathbf{x}_p)\right|} \tag{14}$$

which can be solved efficiently. Comparison of the anisotropic model with measurements done on a M235-35A steel lamination is presented in Fig. 2.

IV. Magnetostriction

Once the magnetic energy functional $u(\mathbf{b})$ involves material vectors like $\mathbf{e}_{\|}$, additional terms appear in the algebraic expression of the variation of energy that involve the strain tensor [8]. This is the fundamental geometric origin of magnetostriction, which will be developed in the full paper.

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