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Published in IET Science, Measurement and Technology
Received on 8th July 2014
Revised on 5th December 2014
Accepted on 13th November 2014
doi: 10.1049/iet-smt.2014.0201

Special Issue on Computational Electromagnetics (CEM) - Selected papers from the International Conference on Computational Electromagnetics (CEM) 2014

ISSN 1751-8822

Pragmatic two-step homogenisation technique for ferromagnetic laminated cores

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Abstract: Electromagnetic fields and eddy currents in thin electrical steel laminations are governed by the laws of magnetodynamics with hysteresis. If the lateral dimension of the laminations is large with respect to their width, the fields and currents generated under arbitrary excitation inside a lamination can be resolved accurately by solving a one-dimensional finite element magnetodynamic problem across half the lamination thickness. This mesoscopic model is able to produce, by averaging the necessary information, a homogenised laminated core model, to be used in the macroscopic modelling of electrical devices involving ferromagnetic lamination stacks. As each evaluation of the homogenised model at the macroscale implies a finite element simulation at the mesoscale, a monolithic implementation of the homogenisation method would be extremely time-consuming. Hence the idea of this study to use system identification techniques to construct an algebraic approximation of the homogenised model, to be used as a conventional constitutive relationship in two- or three-dimensional macroscale simulations. This pragmatic two-step homogenisation approach turns out to be quite accurate and efficient in practice, and it entails no implementation in the FE code, provided the latter offers enough flexibility in the description of the material laws.

1 Introduction

Modelling iron losses in laminated structures is still an open problem nowadays. This intricate problem is of critical importance for the design of modern electrical drives but also for understanding morphological effects in magnetic properties. The complexity of this question is because of different factors [1, 2]. Iron losses are the macroscopic outcome of a combination of micro- or mesoscopic level physical phenomena [2, 3]: namely, eddy currents, skin effect, saturation and hysteresis. Those phenomena are strongly influenced by the microstructure of the ferromagnetic material [4] and by the laminated structure of the cores, both effects occurring at geometrical scales much smaller than the overall dimensions of real-life applications.

The interplay between magnetic fields and eddy currents in ferromagnetic laminated cores is resolved by solving the laws of magnetodynamics with hysteresis inside individual laminations. The one-dimensional (1D) approximation is reasonable if the lateral dimension of the laminations is large with respect to their width (which is typical in the laminated cores of electrical machines or transformers, and in measurement devices such as Epstein frames or single sheet testers). The response in terms of field, eddy currents and losses of a lamination submitted to an arbitrary excitation can therefore be calculated by means of a transient 1D magnetodynamic finite element (FE) simulation, solved across half the lamination thickness. This FE model is strongly non-linear and should contain a sound hysteresis model. It is able to produce, by averaging the necessary information, a powerful homogenised laminated core model, to be used in macroscale simulations of devices involving ferromagnetic laminated cores.

A homogenisation method is needed to connect the mesoscale model with the macroscale model in an appropriate manner. Among the many available homogenisation techniques, the heterogeneous multiscale method (HMM) [5, 6] is the best adapted to this case. As each evaluation of the homogenised model by the macroscale solver implies a FE simulation at the mesoscale, a monolithic implementation of the HMM would however be extremely time-consuming and would lead to a model too heavy for the everyday desktop work of an electrical engineer. Hence the idea advocated in this paper to use system identification (SI) techniques [7] to construct, as a first step, an algebraic approximation of the homogenised model of the form

$$\mathbf{H} = \mathbf{H}(\mathbf{B}, \dot{\mathbf{B}}, p_k) \text{ or } \mathbf{B}(\mathbf{H}, \dot{\mathbf{H}}, p_k) \tag{1}$$

to be used, as a second step, as a conventional constitutive relationship in two- (2D) or three-dimensional (3D) macroscale simulations. The parametric algebraic expression (1) must be rich enough to represent accurately the macroscopic behaviour of the lamination. It typically

contains saturation terms, and additional dissipative terms to account for Joule losses and hysteresis losses. On the other hand, it must be implementable in the FE code at hand and should therefore not be too involved either.

The splitting of the approach into two independent steps has many practical advantages. The two steps can be completed individually with a conventional FE code, and the identification of the p_k parameters in (1) is implemented readily with standard Python tools. This identification is done region by region in practice, and it can be updated or improved as often as needed. At the limit, the approximation would be done element by element at each time step and one would then recover in principle the original monolithic HMM approach.

The paper is organised as follows. Section 2 describes the 1D mesoscale cross lamination model. The macroscale FE model is presented in Section 3. Only the 2D case is considered in this paper. The principle of the homogenisation is then explained in Section 4 and the algebraic approximation of the homogenised model is discussed in Section 5. A number of examples are then discussed in Section 6 to demonstrate the validity of the two-step approach.

2 1D cross lamination model

The quantitative description of hysteresis and eddy currents inside a ferromagnetic lamination is achieved by solving a 1D magnetodynamic problem with hysteresis [1, 3], which involves the following quantities: the magnetic field strength $\mathbf{h}$, the magnetic flux density $\mathbf{b}$ and the electric field strength $\mathbf{e}$. Considering an individual lamination of thickness $2d$ with an upper surface normal vector $\mathbf{n} = (0, 0, 1)$, the domain of analysis ω is a line parallel to $\mathbf{n}$, across half the thickness and far from the edges, Fig. 1. The boundary condition at the centre of the lamination is $\mathrm{curl}\mathbf{h}(0) \times \mathbf{n} = 0$, whereas a given external field $\mathbf{h}(d)$ is applied at the surface of the lamination.

The $\mathbf{h}$-field formulation is preferred for the 1D model because the magnetic field is the natural driving quantity of a system with magnetic hysteresis. The FE equations are

$$\int_\omega \left(\dot{\mathbf{b}}(\mathbf{h}, \text{history}) \cdot \mathbf{h}' + \sigma^{-1}\ \mathrm{curl}\mathbf{h} \cdot \mathrm{curl}\mathbf{h}'\right) \mathrm{d}\omega = 0 \quad \forall \mathbf{h}' \tag{2}$$

with σ the electric conductivity and $\mathbf{b}(\mathbf{h}, \text{history})$ a non-linear hysteretic magnetic relationship.

The hysteresis behaviour is described by the term $\dot{\mathbf{b}}(\mathbf{h}, \text{history})$ and implemented with the 'BH hysteresis model', developed by Bergqvist [8] and Henrotte [9] and briefly sketched below. This flexible and accurate hysteresis model is based on a thermodynamic approach in terms of an energy density ρ^Ψ and a dissipation potential $\dot{\rho}^Q$. The energy density functional has a stored energy term and an empty space term

$$\rho^\Psi = \rho_{\mathrm{st}}(\mathbf{J}) + \mu_0 \frac{\mathbf{h}^2}{2} \tag{3}$$

with $\mathbf{J}$ is the magnetic polarisation, $\mathbf{b} = \mu_0\mathbf{h} + \mathbf{J}$. The reversible part $\mathbf{h}_r$ of the magnetic field is the derivative of the stored energy

$$\mathbf{h}_r(\mathbf{J}) = \partial_{\mathbf{J}}\rho_{\mathrm{st}} \tag{4}$$

and the 1–1 relationship between $\mathbf{h}_r$ and $\mathbf{J}$ is the anhysteretic curve. It can be inverted and is often represented in the FE model by the magnetic susceptibility χ which is defined by $\mathbf{J} = \chi(|\mathbf{h}_r|^2)\mathbf{h}_r$ and is a scalar if material isotropy is assumed (a tensor otherwise). The non-positive dissipation functional, on the other hand

$$\dot{\rho}^Q = -\kappa|\dot{\mathbf{J}}| = -\mathbf{h}_i \cdot \dot{\mathbf{J}}, \quad \mathbf{h}_i = \kappa\frac{\dot{\mathbf{J}}}{|\dot{\mathbf{J}}|} \tag{5}$$

is the power delivered by the irreversible part $\mathbf{h}_i$ of the magnetic field. The latter, being of constant amplitude and always anti-parallel to $\dot{\mathbf{J}}$, acts analogue to a dry friction force.

Energy conservation

$$0 = \partial_t\rho^\Psi - \dot{\rho}^Q - \mathbf{h} \cdot \dot{\mathbf{b}} = \left\{\mathbf{h}_r + \mathbf{h}_i - \mathbf{h}\right\} \cdot \dot{\mathbf{J}} \tag{6}$$

now implies the vector relationship $\mathbf{h} = \mathbf{h}_r + \mathbf{h}_i(\dot{\mathbf{J}}(\mathbf{h}_r))$ graphically presented in Fig. 2. Knowing $\mathbf{h}$ and the history of the material, this relationship can be solved for $\mathbf{h}_r = \mathbf{h}_r(\mathbf{h}, \text{history})$. Finally, the flux density writes

$$\mathbf{b}(\mathbf{h}, \text{history}) = \mu_0\mathbf{h} + \chi(|\mathbf{h}_r|^2)\mathbf{h}_r \tag{7}$$

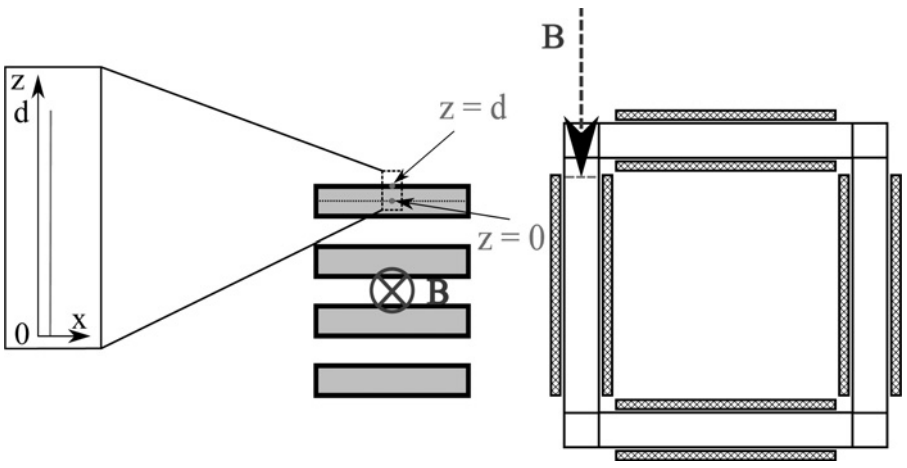

Fig. 1 *Epstein frame and domain of analysis of the 1D cross lamination model*

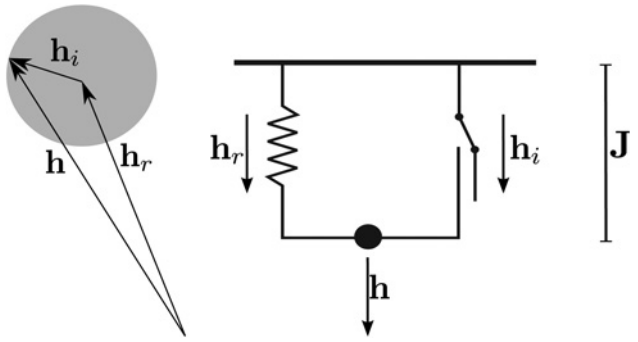

Fig. 2 *Left-hand side: graphical representation of the vector equation* $\mathbf{h} = \mathbf{h}_r + \mathbf{h}_i$. *Right-hand side: Mechanical analogy with a cell composed of a spring parallel connected with a friction lumped element*

and its time derivative in terms of the unknown field $\mathbf{h}$

$$\dot{\mathbf{b}}(\mathbf{h}, \text{history}) = (\mu_0 + (\chi(|\mathbf{h}_r|^2)\mathbb{I} + 2\dot{\chi}(|\mathbf{h}_r|^2)\mathbf{h}_r\mathbf{h}_r)\partial_{\mathbf{h}}\mathbf{h}_r)\dot{\mathbf{h}} \equiv \underline{\mu}^{\partial}\dot{\mathbf{h}}$$

where $\mathbb{I}$ is the identity matrix (note the dyadic product $\mathbf{h}_r\mathbf{h}_r$). This is the term to be substituted in (2) to obtain the mesoscale FE modelling of a ferromagnetic lamination. The non-linear transient formulation is solved with the Newton–Raphson method. A typical discretisation is done with 50 equidistant nodes over the half lamination, and 360 time steps per period over a couple of periods. Power density in the lamination evaluated by the flux of the Poynting vector $\mathbf{e}(d) \times \mathbf{h}(d)$ across the lamination surface, with $\mathbf{e} = \sigma^{-1}\text{curl}\mathbf{h}$.

The hysteresis model is described above in its simplest form, with only one hysteresis parameter κ. This case leads to rectangular hysteresis loops such as those depicted in Fig. 5. The dissipation functional can however be generalised for a more realistic representation of minor loops by connecting in series several cells such as the one depicted in Fig. 2. The presentation of the generalised theory falls out of the scope of this paper. See [10] for a detailed explanation.

Epstein frames are conceived to achieve field homogeneity in the frame, up to minor edge effects that are limited to a minimum by constructional measures. The 1D approximation is hence accurate in those devices. The cross lamination model (2) presented above allows thus identifying the hysteresis parameters with high accuracy by matching Epstein measurements with 1D simulations done in the same conditions. The details of the identification procedure are presented in [11].

3 Macroscale FE model

Fields at the macroscopic scale will be denoted with a capital letter. The most widely used 2D formulation for electrical applications is the magnetic vector potential $\mathbf{A}$ formulation, with the flux density then given by $\mathbf{B} = \text{curl}\mathbf{A}$. The weak formulation of Ampere's law reads

$$\int_\Omega \mathbf{H} \cdot \text{curl}\mathbf{A}' \, \mathrm{d}\Omega = \int_\Omega \mathbf{J}_s \cdot \mathbf{A}' \, \mathrm{d}\Omega, \quad \forall \mathbf{A}' \tag{8}$$

with $\mathbf{J}_s = -\text{grad}V - \partial_t\mathbf{A}$ the current density and V the electric scalar potential. The question is now to determine, on basis of a sound homogenisation theory, a material law of the form (1a) that would represent the homogenised behaviour of a lamination stack. This question is dealt with in the next section.

4 Homogenisation

We begin this section with a remark about the terminology. The term mesoscale is used throughout in this paper to represent the geometrical scale of an individual lamination, in contrast to the geometrical scale of the device containing the laminated core, which we call macroscale. With regard to the mathematical homogenisation theory, mesoscale and microscale are equivalent terms. We simply prefer using the mesoscale here because the thickness of a regular electrical steel lamination cannot be rightly qualified as a microscopic feature.

A formalised homogenisation method is needed to draw a rigorous mathematical link between the mesoscale model and the macroscale model. There exist several families of homogenisation techniques. Algebraic homogenisation techniques rely on an analytical solution of the microscale problem [12] to build effective macroscopic material characteristics. This approach was applied by Giordano to the non-linear homogenisation of magneto-electro-elastic laminated materials [13]. The mathematical developments are rather involved and, the method being based on energy density potentials, it is not applicable to media with irreversibility. Another class of homogenisation technique is that of asymptotic homogenisation methods. It originates in the mechanics of composite materials and specifically deals with heterogeneous media whose governing partial differential equations have periodic and rapidly oscillating coefficients, see for example [14]. The problem we want to solve does not fall into this category.

The HMM is the best adapted to our case. The philosophy of the HMM is to solve the mesoscale model locally in regions where constitutive relationships are missing [5]. Recently, it has been applied successfully by Niyonzima to the case of ferromagnetic laminations [6]. The mathematical relationship between the mesoscale quantities (lower cases) and the macroscale quantities (upper cases) is given by the localisation and the homogenisation relationships (the dependency in time is implicitly assumed)

$$\mathbf{H} = \mathbf{h}(d), \quad \mathbf{B} = \langle \mathbf{b} \rangle = \frac{1}{d}\int_0^d \mathbf{b}(z) \, \mathrm{d}z \tag{9}$$

The macroscopic constitutive relationship of the HMM is the homogenised $\mathbf{B}$–$\mathbf{H}$ relationship associated with the path $\mathbf{H} \rightarrow \mathbf{h} \rightarrow \mathbf{b} \rightarrow \mathbf{B}$ in the diagram depicted in Fig. 3. Practically, it is obtained by evaluating (9) onto the data of a 1D cross lamination simulations (2). As each evaluation implies

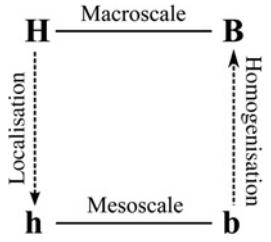

Fig. 3 *Formal representation of the homogenisation concepts*

doi: 10.1049/iet-smt.2014.0201

solving a FE problem (at least one time step) at the mesoscale, a monolithic implementation yields a system of equations whose size is of the order of the size of the macroscale problem multiplied by the size of the mesoscale problem. This is extremely memory and time-consuming, and too heavy a model for the everyday desktop work of an electrical engineer. Hence the idea developed in the next section of constructing an algebraic approximation of the homogenised **B**–**H** relationship that can be used as a conventional constitutive relationship in the laminated core region of the macroscale FE formulation (8). The homogenised macroscale model is then of the same size as the initial non-homogenised problem, and it can be solve with a conventional FE code.

5 Two-step approach

The proposed approach proceeds in two successive steps. The first step consists in building, region by region, algebraic approximations of the homogenised **B**–**H** relationship by solving the mesoscale model (2) and applying then a SI procedure. The second step consists in using these algebraic approximations as constitutive relationships in conventional macroscopic FE simulations.

SI deals with the problem of building mathematical models of dynamical systems based on observed data from the system [7]. It is a very diverse research field in applied mathematics with an abundant literature, a substantial part of which is however devoted to linear systems. In the context of non-linear SI, powerful techniques also exist that usually proceed by assuming a model structure *a priori*, and identifying a number of free model parameters. This identification proceeds by applying excitation signals to the system and determining the free parameters of the parametric model structure by minimisation of the approximation error.

The user has quite a large freedom in the choice of the model structure. It has however to fulfil a number of practical conditions. (i) It must be implementable as a constitutive relationship in the FE formulation used at the macroscale. For a $\mathbf{A}-V$ formulation, it will be of the form $\mathbf{H}(\mathbf{B}, \dot{\mathbf{B}}, p_k)$ and of the form $\mathbf{B}(\mathbf{H}, \dot{\mathbf{H}}, p_k)$ for a $\mathbf{H}-\phi$ or $T-\omega$ formulation. (ii) It must be versatile enough to represent accurately the macroscopic behaviour over a large range of situations. (iii) When physical insight into the system is available, as is the case here, it should be translated into the structure of the model. (iv) Finally, as a macroscale constitutive relationship, it should not deteriorate the convergence of the non-linear iterations. Practically, model structures should be preferred whose Jacobi matrix can be evaluated and implemented, so that a Newton–Raphson approach can be used to solve the macroscale problem. On basis of those arguments, several model structures were tried out and

$$\mathbf{H}(\mathbf{B}, \dot{\mathbf{B}}, p_k) = \left(p_0 + p_1|\mathbf{B}|^{2p_2}\right)\mathbf{B} + \left(p_3 + \frac{p_4}{\sqrt{p_5^2 + |\dot{\mathbf{B}}|^2}}\right)\dot{\mathbf{B}} \tag{10}$$

has finally proven to give satisfactory results. First of all, it parallels the decomposition introduced in Section 2 of the magnetic field into a reversible and an irreversible part. The first term (p_0, p_1 and p_2), which corresponds to the reversible part $\mathbf{h}_r$ of the magnetic field, is a polynomial representation of the anhysteretic saturation curve. A polynomial expression is preferred because polynomials are more stable than tangent-like functions in the identification process. The second term corresponds to the irreversible part $\mathbf{h}_i$ of the magnetic field. It accounts for eddy currents (p_3) and hysteresis (p_4 and p_5). The former is related with the σd^2 terms that appear when solving analytically the eddy currents problem in thin laminations. For large values of $\dot{\mathbf{B}}$, the latter behaves as $p_4\dot{\mathbf{B}}/|\dot{\mathbf{B}}|$, in accordance with (5). The parameters p_k are identified so as to minimise in the least square sense the approximation error for well-chosen excitation signals. For the least-square minimisation, we have used the *scipy.optimize.leastsq* module of Python.

The excitation signals are the macroscale magnetic waveforms $\mathbf{h}(d, t)$ applied as an input to the mesoscale model when identifying the parameters $\{p_k\}$. Their selection is a critical aspect of the two-step method. To better account for local features of the fields (e.g. stator against rotor and teeth against yoke), the parameter identification is done region by region and the excitation signals are chosen similar to the actual waveforms in the real machine in terms of frequency content and wave shape.

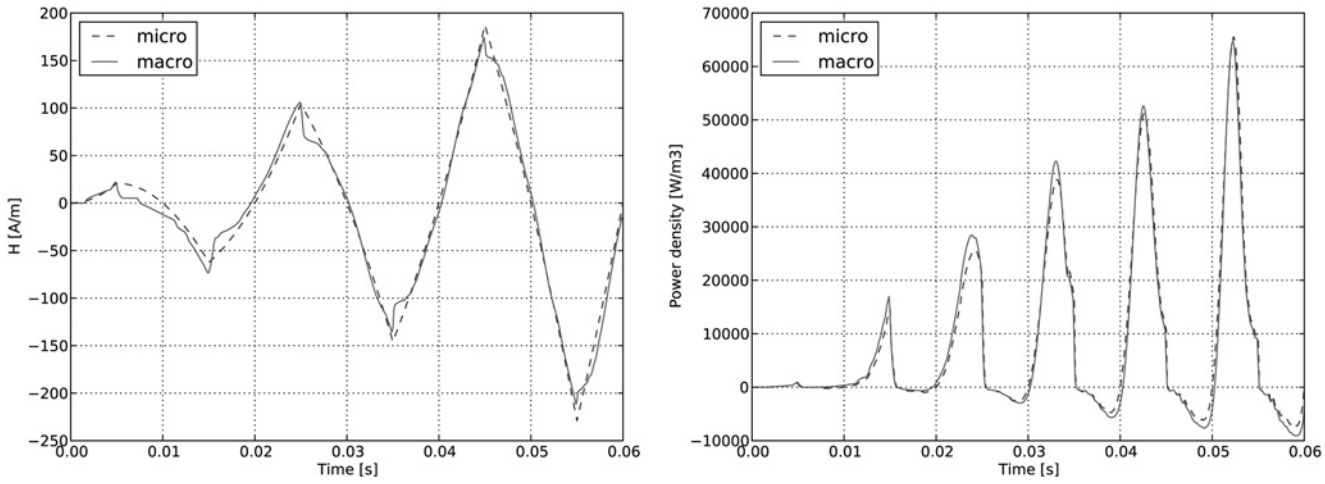

Fig. 4 *Left-hand side: Comparison of the response of the mesoscale model (micro) with its algebraic approximation (macro) for a triangular excitation signal with increasing amplitude. Right-hand side: Comparison of power densities in the same conditions*

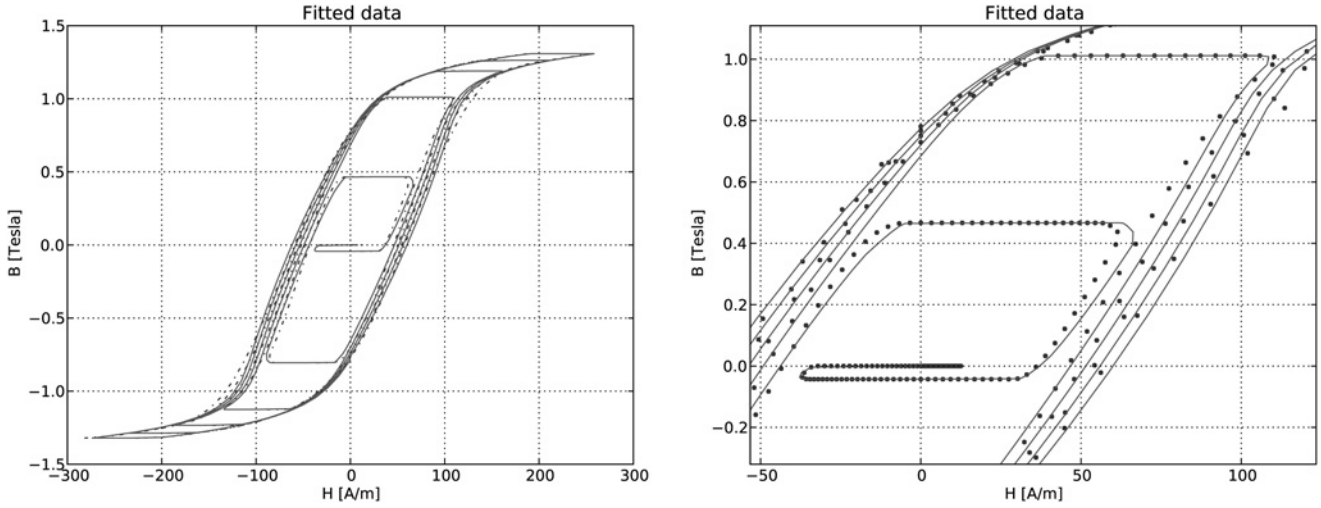

Fig. 5 *Comparison in the* **B**–**H** *plane of the response of the mesoscale model (solid line) with its algebraic approximation (dotted line). On the right-hand side, a zoom-in on the central part of the plot*

In practice, the chosen excitation signals are real magnetic field waveforms evaluated at selected probe points in the machine or they are artificially constructed to excite at once a larger class of system inputs (see Section 6). Once the material parameters p_k are identified, the approximated lamination models (10) are used as a conventional non-linear constitutive relationship in the macroscopic model (8), which is solved at each time step by a classical Newton–Raphson scheme.

For the whole two-step approach to be useful in practice, the mesoscale model needs be robust and reliable. We have used for that a unidirectional implementation of (2) with $\mathbf{h} = (0, h(z), 0)$. This model is unidirectional because it has been developed to identify material parameters from Epstein measurements, which are themselves unidirectional. Despite the complexity of the involved hysteretic material law, this model has proven to have good convergence properties and to be robust. It has been thoroughly validated in a large class of situations. All identifications presented in the next section have been done with this unidirectional model. The identified macroscopic material parameters $\{p_k\}$ are then called to be used in the vector approximation (10). This using of the two-step splitting as an opportunity to vectorise the homogenised constitutive relationship is the second pragmatic assumption of the proposed approach. It is justified by the fact that the $\{p_k\}$ parameters have a physical meaning independent of the field direction.

6 Application

Numerical experiments show that the algebraic approximation (10), with a set of fixed parameters $\{p_k\}$ can be accurate over a quite large field strength range. This is a strong argument in favour of the two-step approach. A triangular waveform at nominal frequency, with an amplitude increasing with time from zero up to the nominal field strength is considered as a first example of excitation signal. Fig. 4 shows that a good match is obtained between the mesoscale model's response and its algebraic approximation, both for the fields and for the power densities. Fig. 5 shows the corresponding results in the **B**–**H**

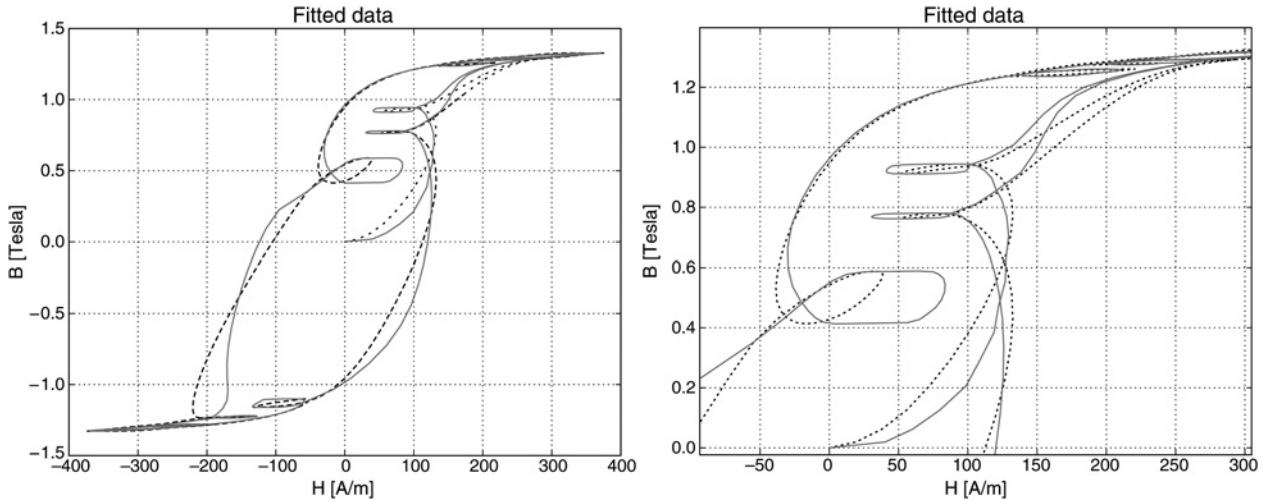

Fig. 6 *Comparison in the* **B**–**H** *plane of the response of the mesoscale model (dotted line) with its algebraic approximation (straight line) in the presence of higher harmonics and minor hysteresis loops. On the right-hand side, a zoom-in on the upper part of the plot*

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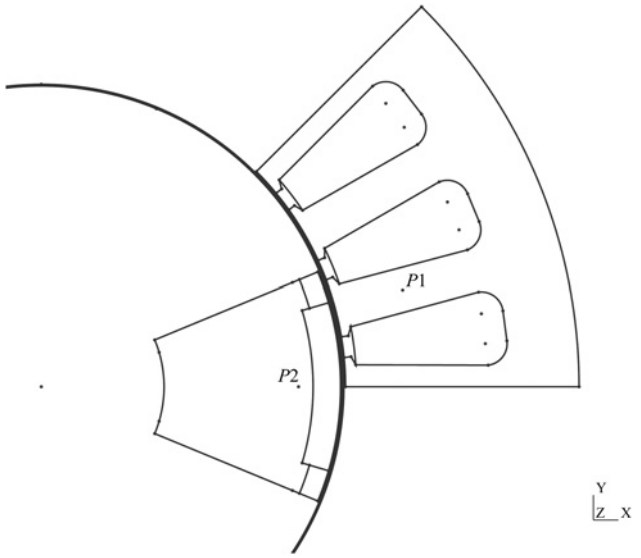

Fig. 7 *Cross section of a four pole pairs PMSM motor. The typical waveforms used for the identification have been evaluated at the points indicated with P1 and P2 in the centre of a stator tooth and below a rotor permanent magnet*

plane. One observes that the algebraic model reproduces accurately the hysteresis loops of the homogenised lamination stack behaviour. Rectangular loops are obtained because the simplest hysteresis model, with only one cell (see Section 2), was used for this identification.

An excitation signal consisting of a fundamental harmonic superposed with a higher harmonic is now considered in order to demonstrate the ability of the algebraic approximation (10) to deal with high frequency components and account for the presence of minor loops. In this case, a more involved hysteresis model with seven cells was used to resolve the minor loops in the mesoscale simulation. Fig. 6 compares in the **B**–**H** plane the response of the mesoscale model with its algebraic approximation. Although the match is not perfect, it is observed that the essential features of the minor looping effect at the mesoscale are satisfactorily brought over to the macroscale.

The triangular waveform used above was however an arbitrary choice. The same analysis could be done with a sinusoidal waveform or with a realistic waveform obtained from the application under analysis. After solving in nominal conditions the four pole pairs permanent magnet synchronous motor (PMSM) whose cross section is depicted in Fig. 7, the magnetic field waveforms has been evaluated at point $P1$ in the middle of a stator tooth. An artificial excitation signal with increasing amplitude has then been created on basis of this waveform. Again, a excellent match is obtained, both for the fields and the power densities, with this time a more realistic waveform, Fig. 8. This shows that the approximate model is accurate, not only at the point $P1$, but also at all places in the stator where the local waveform of the magnetic field is similar to that at point $P1$, irrespective of the amplitude. This represents a more or less important part of the stator cross section, according to the accuracy one wishes to reach. The same operation can then be reproduced with a (small) number of other probe points or the parameters identified at point $P1$ can be pragmatically used all over the stator core.

Magnetic field waveforms at the rotor side, on the other hand, are quite different from those at the stator side in electrical machines. In a PMSM machine, rotor fields are not alternating but fluctuating around an average value that depends on the load of the machine. We take a look into this issue by considering now a typical rotor waveforms evaluated at point $P2$ in Fig. 7 below the permanent magnet. Again, an artificial excitation signal is built on basis of this waveform so as to cover the actual field strength range in the rotor. Fig. 9 shows the match obtained between the mesoscale model and its algebraic approximation.

To summarise, the Table 1 gives a comparison of the p_k parameters identified with all excitation signals considered above. The parameter values depend, as expected, on the excitation signal, but moderately. This again shows that the number of probe points can be quite limited, reaching nonetheless a sufficient accuracy. The distinction between rotor and stator, on the other hand is essential.

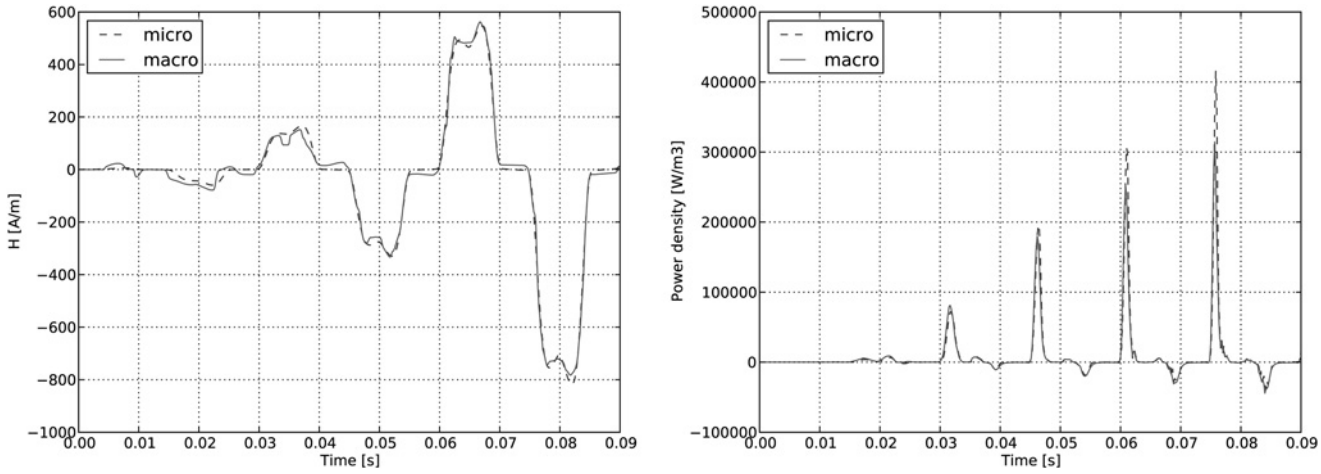

Fig. 8 *Left-hand side: Comparison of the response of the mesoscale model (micro) with its algebraic approximation (macro) for an artificial excitation signal built on the real waveform observed at point P1 in a stator tooth. Right-hand side: Comparison of power densities in the same conditions*

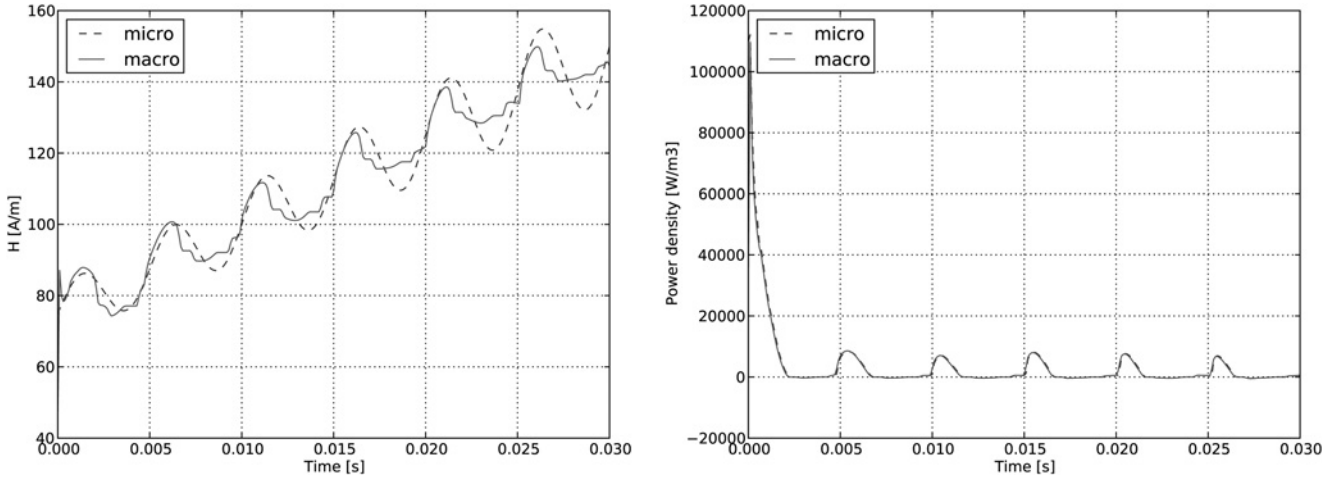

Fig. 9 *Left-hand side: Comparison of the response of the mesoscale model (micro) with its algebraic approximation (macro) for an artificial excitation signal built on the real waveform observed at point P2 in the rotor. Right-hand side: Comparison of power densities in the same conditions*

Table 1 Identified p_k parameters with different excitation signals or in different regions

	p_0	p_1	p_2	p_3	p_4	p_5
triangular	93.3	0.26	11.5	0.035	34.5	8.32
sinusoidal	95.9	0.29	11.4	0.041	28.6	8.03
tooth waveform	77.8	0.80	9.87	0.036	36.7	8.68
rotor waveform	88.6	13.6	3.11	0.053	7.65	9.52

7 Conclusions

Despite an urgent need for such models in industry, there does not exist nowadays a homogenisation method able to account for the complexity of ferromagnetic laminated cores in 2D or 3D macroscopic electrical machine simulations. The papers [6, 15, 16] report on interesting work on this issue, but they say little about the hysteresis model used and, first of all, they propose methods that require an important implementation work and that lead to models one or two orders of magnitude larger than the initial non-homogenised model. The HMM also provides a general theoretical background to achieve the goal with however the same limitations as mentioned above in terms of implementation work and model size.

In this paper, a splitting of the HMM approach into two independent steps has been proposed. This splitting allows introducing two pragmatic simplifications: the multiscale coupling is done region by region, instead of locally, and is taken as an opportunity to vectorise a robust unidirectional mesoscale model. Owing to these simplifications, the two-step approach is an approximation but, in counterpart, it provides a systematic method to construct macroscale constitutive relationships for ferromagnetic laminated core as a function of exact material data and of the main characteristics of the fields at the mesoscale level.

Contrary to homogenisation approaches presented in [17, 18], the two-step approach makes no simplifying assumptions about material properties in the laminations (reversibility and linearity). Contrary to homogenisation approaches presented in [6, 15, 16], it leads to a homogenised model of the same size as the non-homogenised model. This makes the two-step approach a homogenisation method suitable for desktop engineering design.

The two-step approach is non-invasive. It can be used with most FE software and entails no coding. The form (10) of the parametric model structure is an outcome of this paper. It proves accurate, with fixed parameters, over large ranges of excitation signals. The parameters vary moderately with the waveform. They can be given a physical interpretation that justifies their identification based on a unidirectional mesoscale model and their subsequent use, at the macroscale level, in the vector constitutive relationship. The two-step approach opens up the possibility to deal explicitly with the questions of material selection, comparison and optimisation in real-life applications involving electrical steel laminated cores.

8 Acknowledgments

This work was supported in part by the Belgian Science Policy (IAP P7/02), Belgian French Community (ARC 09/14-02) and Walloon Region (WIST3 project 'FEDO'). The work of S. Steentjes is supported by the Deutsche Forschungsgemeinschaft (DFG) as part of the research project 'Improved modelling and characterisation of ferromagnetic materials and their losses'.

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