

Modeling the influence of varying magnetic properties in soft magnetic materials on the hysteresis shape using the flux tube approach

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[Modeling the influence of varying magnetic properties in soft magnetic](http://dx.doi.org/10.1063/1.4906956) [materials on the hysteresis shape using the flux tube approach](http://dx.doi.org/10.1063/1.4906956)

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Magnetic properties can vary significantly inside soft magnetic steel sheets (SMSSs), both due to mechanical stresses and structural changes originating from different manufacturing processes. The integral consideration, i.e. averaging these effects over the SMSS, leads to a strong simplification of the underlying mechanisms. Such simplification is often inadequate when considering the influence of the varying magnetic properties on the hysteresis loop shape and its dynamic behavior. This paper presents a new approach to model irregular hysteresis loops of non-oriented SMSSs using the flux tube approach, where the SMSS is divided into several flux tubes having different magnetic properties. This enables to model non-homogeneous distributions of the magnetic flux and irregular hysteresis loops subject to varying magnetic properties. © 2015 AIP Publishing LLC. [\http://dx.doi.org/10.1063/1.4906956]

I. INTRODUCTION

It is well known that the manufacturing processes of soft magnetic steel sheets (SMSSs) and also magnetic cores of electromagnetic devices cause mechanical stresses and structural changes in the microstructure of the soft magnetic material, which influence and alter magnetic properties of the final product.^{1–[9](#page-4-0)} Such stresses and structural changes are introduced in many different manufacturing steps, such as rolling, cutting, grinding, stacking, bending, winding, sticking, welding, drilling, and riveting, of SMSSs and deteriorate the magnetic properties thereof, which are reflected in modified shapes of hysteresis loops. $1-4$ When measuring hysteresis loops of processed SMSSs or assembled magnetic components, various loop shapes can be obtained that can heavily deviate from the hysteresis loop shapes of unprocessed soft magnetic material. In addition, the altered loops often show irregular shapes, which are difficult to reproduce using well established hysteresis models such as the Jiles-Atherton (J-A) model. The knowledge of underlying mechanisms and actual magnetic properties are therefore indispensable for modeling and designing of magnetic components.

The main objective of this paper is to present an approach to model irregular static hysteresis loops of nonoriented SMSSs (NO SMSSs) that were exposed to mechani-cal treatment.^{1–4,[6,7,9](#page-4-0)} Such loops can be modeled using the flux tube approach, $14-16$ $14-16$ $14-16$ where the soft magnetic material is modeled by an adequate magnetic equivalent circuit (MEC). The performed analysis shows that the proposed methodology is very promising when modeling SMSSs with nonhomogeneously distributed magnetic properties.

II. THEORETICAL BACKGROUND

A. Local changes of magnetic properties inside SMSS

The deterioration effect generated during the manufacturing process is mostly taken into account based on empiri-cal knowledge, i.e., using so-called building factors.^{[1](#page-4-0)} More detailed consideration of the discussed effects represent vari-ous magneto-mechanical models.^{[10–13](#page-4-0)} These models, however, deal with the whole material with homogeneously altered magnetic properties, which can be on one side useful for engineering applications, but on the other side do not represent the background of the underlying physical mechanisms correctly. As it is observed, validated and discussed by many authors, the deterioration effects are not induced homogeneously in the whole magnetic material, but rather locally. $1-4,6,7,9$ Therefore, the magnetic properties (and deteriorations) of processed SMSSs are a function of geometry and can change remarkably when observing individual zones of the SMSSs that are affected by the mechanical or thermal treatment. In contrast to this, in the less affected zones, the magnetic properties change only slightly. It is worthwhile to note that the measured magnetic properties of the whole SMSS represent an integral picture of the locally varying magnetic properties. Furthermore, the local deteriorations and the geometric properties of their influenced zones depend heavily on the type of the manufacturing treatment. Manufacturing treatments like cutting cause distinct deterioration zones that can be often considered independent of one or two spatial coordinates.^{[1,4,7,9](#page-4-0)} However, some steps like drilling and welding can induce deteriorations in small zones, where it is difficult to apply aforementioned simplifications. In this paper, the emphasis is (without limitation of generality) on considering the deteriorations due to cutting.^{[1](#page-4-0)} The proposed concepts can, in general, also be applied to other types of local deteriorations.

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B. Varying Magnetic Properties - Flux Tube Approach

Cutting of SMSSs is inevitable in the manufacturing process of magnetic cores. SMSSs can be cut using different techniques, e.g., mechanical cutting or laser cutting, where different techniques affect the structural features of the SMSS in a different way.^{[1,2](#page-4-0),[9](#page-4-0)} Using mechanical cutting, plastic deformations appear in the close proximity of the cutting line, where the affected zone can go up to some mm into the magnetic material depending on the material thickness. Laser cutting induces thermal stresses that alter magnetic properties as well, although the affected zone is not as concentrated as if mechanical cutting is applied.¹ When considering mechanical cutting, the deterioration effect is strongest in the close proximity of the cutting line and fades in respect to the coordinate x towards the center of the SMSS (as experimentally veri-fied^{[1](#page-4-0)}), which is schematically represented in Fig. 1.

Such a SMSS with non-homogeneously distributed magnetic properties can be approximated using the flux tube approach, $14-16$ $14-16$ $14-16$ where the SMSS is discretized in N parallel flux tubes, as shown in Fig. $2(a)$. The magnetic properties inside individual flux tubes are best described using average magnetic properties of the observed zone of the SMSS. In this way, a piece-wise constant distribution of magnetic properties as a function of geometry can be considered, where the whole SMSS can be expressed as a MEC as shown in Fig. 2(b). The reluctances R_{mi} of individual elements $i \in [1, N]$ in the MEC describe magnetic properties of individual zones i inside the SMSS, where Θ represents the magneto-motive force generated by the current i_e in the excitation winding with N_e turns, Φ_i is the magnetic flux inside the flux tube *i*, and Φ_m is the integral (average) magnetic flux inside the whole SMSS.

C. Solving the Magnetic Equivalent Circuit of a SMSS

The obtained magnetic circuit for a SMSS can be solved in different ways. $14-\frac{16}{16}$ In this paper, the incremental perme-ability approach^{[14](#page-4-0)} is used, as it is very flexible and enables calculation of transient magnetic states under arbitrary excitation waveforms. This approach is based on the determination of the differential permeabilities μ_{di} of individual flux tubes i in each calculation step, which can be determined either using adequate non-linear characteristics or adequate hysteresis models.^{[14](#page-4-0)} Based on μ_{di} , instantaneous differential magnetic reluctances $R_{\text{mid}} = \frac{l_{\text{mi}}}{\mu_{\text{mid}} A_{\text{mi}}}$ of individual flux tubes are calculated, where l_{mi} represents the mean magnetic length and A_{mi} is the cross section of the observed flux tube.

FIG. 2. The flux tube approach: (a) dividing the SMSS into parallel flux tubes with different magnetic properties and (b) corresponding MEC.

The instantaneous differential magnetic reluctances R_{mid} enable the calculation of magnetic fluxes $\mathbf{\Phi} = [\Phi_{\rm m}, \ \Phi_1, \ \Phi_2, \ \ldots, \ \Phi_N]^{\rm T}$ inside individual flux tubes, which can be expressed using the graph theory, 14 in general, matrix form by

$$
\frac{\mathrm{d}}{\mathrm{d}t}\Phi = \mathbf{C}^{\mathrm{T}} \left[\mathbf{C} \mathbf{R}_{\mathrm{md}} \mathbf{C}^{\mathrm{T}} \right]^{-1} \mathbf{C} \mathbf{N}_{\mathrm{e}} \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathrm{e}}.
$$
 (1)

In Eq. (1), C represents the incidence matrix of the MEC, whereas $\mathbf{R}_{\text{md}} = \text{diag}(0, R_{\text{md1}}, R_{\text{md2}}, ..., R_{\text{mdN}})$ and $N_e = [N_e, 0, 0, ..., 0]^T$ represent the matrix of magnetic reluctances, and the vector of number of turns of the excitation windings, respectively. The time derivatives of the magnetic fluxes Φ calculated with (1) are used to determine the differential permeabilities μ_{di} for the next calculation step in com-bination with an adequate hysteresis model.^{[14](#page-4-0)}

The excitation current i_e is calculated based on (2), where u_e is the applied voltage, R_e is the resistance, and L_e is the leakage inductance of the excitation winding, whereas L_d represents the incremental inductance

$$
\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{e}} = \frac{u_{\mathrm{e}} - R_{\mathrm{e}}i_{\mathrm{e}}}{(L_{\mathrm{e}} + L_{\mathrm{d}})}; \quad L_{\mathrm{d}} = \mathbf{N}_{\mathrm{e}}^{\mathrm{T}} \mathbf{C}^{\mathrm{T}} \left[\mathbf{C} \mathbf{R}_{\mathrm{md}} \mathbf{C}^{\mathrm{T}} \right]^{-1} \mathbf{C} \mathbf{N}_{\mathrm{e}}.
$$
 (2)

D. MEC model identification and complexity

The choice of the hysteresis model type can be arbitrary, although inverse hysteresis models are more suitable for implementation. However, a choice of a parametric hysteresis model seems to be convenient, as the parameters of such a model could be correlated with the locally varying magnetic properties. Therefore the inverse J-A hysteresis model is applied.^{[17](#page-4-0),[18](#page-4-0)} Finally, the major task is the discretization of the SMSS (number and size of the flux tubes) and determination of corresponding magnetic properties (local hysteresis loops). This poses a complicated, expensive and often inaccessible task. Lacking experimental data, the identification could be based on the geometric properties, the hysteresis loops of the unprocessed and processed material and general knowledge of the deterioration effects, e.g., Refs. [1–9](#page-4-0). It is practical to obtain a final model that is as simple as possible and describes the SMSS's behavior desirably accurate. Consequently, the modeling process should start using less (minimum 2) flux tubes and the complexity (more flux tubes) should be raised if the desired accuracy using the previous model is not reached.

When modeling wide SMSSs processed by mechanical FIG. 1. Schematic presentation of the deterioration effect due to cutting. cutting, there is a distinct distribution of magnetic properties as shown in Fig. [1](#page-4-0). In Refs. 1 and 6 , it is shown that the flux distribution with respect to the x coordinate of the SMSS has a parabolic shape, which relates to the locally varying magnetic properties. The simplest model of such a SMSS is obtained, when 2 flux tubes are used where symmetry of distribution of magnetic properties is assumed. The first flux tube represents the material in the center of the SMSS, where the magnetic properties are assumed to be unaffected, whereas the second flux tube represents the material in the near proximity of both cutting lines at each side, where the deterioration effect is most severe. Such a model may not be the most accurate, but it is very convenient for the presentation and understanding of the proposed modeling approach. When modeling a wide SMSS, it is safe to assume that the magnetic properties in the center of such a SMSS are almost unaffected by the mechanical cutting, hence the hysteresis loop of unprocessed magnetic material is a good reference to describe the magnetic properties of the first flux tube. The cross sections of both flux tubes and the hysteresis loop of the second flux tube can be determined in the next step based on the shape of the final hysteresis loop. For this purpose, e.g., an optimization algorithm can be applied, $17,18$ where the objective function represents the deviation between the integral (average) model hysteresis loop and the measured hysteresis loop of processed SMSS. Such a procedure can be applied also for more complex models with more flux tubes where values of parameters of hysteresis models of individual flux tubes are restricted in such a way that they correspond to the physical background of the deterioration effect. For a successful identification of more complex models also measured hysteresis loops of processed samples with different widths¹ are helpful giving additional information about the deterioration processes related to a specific cutting technique.

III. RESULTS

A model with two flux tubes describing varying magnetic properties was used to analyze proposed modeling concepts without limitation of generality. Using such a simple example, the presentation of basic concepts is clear where unnecessary complexity is avoided. The J-A hysteresis model parameters of the first flux tube were determined using measurements of $a = 30$ mm wide and $b = 0.5$ mm thick samples of grade M400-50A NO 3.2% Fe-Si SMSSs that were thermally processed (annealed) to eliminate the stresses induced by cutting. The experimental setup consisted of an Epstein frame within a computer-aided setup in accordance with the international standard IEC 60404-2, where the hysteresis loops were evaluated for quasi-static conditions. Based on the measured

TABLE I. J-A model parameters describing magnetic properties of flux tube 1.

Parameter	Quantity	Value
$M_{\rm s}$	Magnetization saturation	1.2×10^6 A/m
\overline{A}	Hysteretic parameter	12.47 A/m
\boldsymbol{k}	Domain wall-pinning parameter	38.69 A/m
α	Mean-field parameter	34.42×10^{-6}
	Domain wall flexing parameter	0.035

hysteresis loop for $B_{\text{max}} = 1.6 \text{ T}$, the J-A model parameters were determined using differential evolution¹⁸ (Table I). Although the hysteresis shape of the second flux tube could not be determined experimentally, two J-A model parameters $(A = 191.25$ A/m and $k = 58.69$ A/m) of the second flux tube were adjusted manually to consider the deteriorated hysteresis loop for the presented analysis. These adjustments are based on experimental results and conclusions for mechanical cutting observing SMSSs with different widths $a₁¹$ $a₁¹$ $a₁¹$ where the increased coercive magnetic field strength H_c and decreased residual magnetic field density B_r for deteriorated SMSSs are taken into account approximately.^{[1](#page-4-0)}

The presented model was analyzed in such a way that the cross sections of affected zones were varied, which is comparable with the experimental analysis based on evaluat-ing SMSSs of different widths.^{[1](#page-4-0)} When assuming that the severely deteriorated zone is 1 mm wide, different proportions of flux tube cross sections can be obtained. For a $a = 30$ mm wide SMSS, the unaffected zone amounts for 93.3% and the affected zone amounts for 6.7% [Fig. 3, curve a], for $a = 20$ mm the proportions are 90%–10% [Fig. 3, curve b], for $a = 10$ mm the proportions are 80%–20% [Fig. 3, curve c], for $a = 5$ mm the proportions are 60% and 40% [Fig. 3, curve d], and for $a = 3.33$ mm the flux tube cross section proportions are 33.3%–66.7% [Fig. 3, curve e] of the effective cross section of the SMSS.

The obtained calculated integral (average) hysteresis loops in Fig. 3 show remarkably similar behavior as the ex-perimental results^{[1](#page-4-0)} despite the simplicity of the model. From the presented analysis, it is clearly visible that the proportion of flux tube cross sections dictates the shape of the calculated integral hysteresis loop: in the cases of less damaged SMSS, the integral hysteresis loop shape is more similar to the hysteresis loop shape of the first flux tube, where in the cases of heavily damaged SMSS, the integral hysteresis loop shape becomes more similar to the hysteresis loop shape of the second flux tube. When using models with more flux tubes, such conclusions are not as straight forward. It is also

FIG. 3. Hysteresis loops used to describe the magnetic properties of both flux tubes along with calculated average hysteresis loops for SMSSs that are geometrically affected to different degrees.

FIG. 4. Calculated magnetic variables inside a 20% deteriorated SMSS: (a) magnetic field strength H , (b) magnetic flux densities B , and (c) time change rates of magnetic flux densities dB/dt.

important to note that when proposed modeling approach is applied also very non-standard shapes of integral hysteresis loops can be obtained, which are often observed measuring processed SMSSs and are, e.g., impossible to reproduce using standard J-A hysteresis models.

In addition, Fig. 4 shows that the magnetic flux distribution inside the SMSS varies with time depending on the conditions inside the observed SMSS. The densities of magnetic flux inside individual flux tubes can deviate significantly from the average density B_m that is dictated by the applied sinusoidal voltage u_e in the excitation winding. The calculated results show that the magnetic field inside deteriorated zones starts to build up rapidly when the unaffected zone saturates, which is a rough representation of phenomena inside a SMSS where the same effect is only more distributed. Consequently, also the change rates of magnetic field vary significantly inside different zones. This is of great importance when considering also dynamical effects (induced eddy currents) inside such SMSS and their influence on the losses and shape of dynamic hysteresis loops, which is out of the scope of this paper, but represents a very interesting subject for further research. The obtained results also show a potential weakness of majority of the developed static hysteresis models and their dynamic extensions that take the deterioration effect into account integrally, i.e., consider the whole material with homogeneously altered magnetic properties.

IV. CONCLUSION

In this paper, an alternative approach for modeling of SMSSs with locally varying magnetic properties is proposed. Based on the presented analysis, several interesting results were obtained, which can explain the origin of distorted static hysteresis loop shapes of processed SMSSs as well as put in question some of modeling approaches that take the deterioration effects into account integrally. The proposed modeling approach is very promising for further development as it is very flexible, where the complexity of the model can be adjusted to account for different types of deteriorations of magnetic properties. The MEC can be, e.g., upgraded, where also serial reluctances can be taken into account, which represent deteriorations of different origins (e.g., due to bending of SMSSs). The identification of more complex models represents, however, a subject for further research, where the parametric basis of the individual hysteresis models enables to link parameters of these models to the microstructure of deteriorated SMSSs. Future work will focus also on extending the modeling concepts for modeling of dynamic hysteresis loops, where the influence of induced eddy currents is taken into account.

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