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TRANSIENT NON-CONFORMING SLIDING INTERFACES FOR MOTION IN EDDY CURRENT CALCULATIONS

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***Abstract* – Non-conforming sliding interfaces can be used in finite element (FE) simulations for the flexible implementation of the relative motion between stator and rotor of e.g. electrical machines. Lagrange multipliers are applied to ensure the continuity of the field across the non-conforming interface. A previously proposed approach is extended by time stepping methods to consider eddy-currents. In this paper, the formulation is presented and applied to a benchmark problem to validate the approach.**

Introduction

Numerical simulation of electrical machines by FE methods requires a flexible implementation of the rotor position in the model. Describing the relative motion by Eulerian variables is feasible only if the studied problem is invariant in the direction of motion regarding excitation and material properties. In addition the convective term can lead to numerical instabilities. Hence, Lagrangian variables are chosen to describe the motion between stator and rotor. Common methods to connect the moving and stationary parts of the considered domain require re-meshing of specific areas or additional constraints for the discretization. For two-dimensional problems the Moving-Band [7] approach can be employed where an annulus-shaped band in the air gap between rotor and stator of the electrical machine is re-meshed at every time step. But re-meshing is especially in three-dimensional problems a computationally expensive task and yields supplementary discretization errors whereas additional constraints for the discretization circumvent an arbitrary rotor position. To avoid this issues the mortar element method with Lagrange multiplier [2], [5] is applied as described in [1], [8] and extended to consider eddy-currents.

Methodology

Let Ω be a two-dimensional domain which is the cross section of a three-dimensional device e.g. electrical machine. The quasi-static electromagnetic field problem can be described by the Maxwell equations with neglected displacement currents:

$$\mathbf{curl}\,\boldsymbol{e} = \mathrm{x} - \frac{\partial \boldsymbol{b}}{\partial t}, \qquad \mathbf{curl}\,\boldsymbol{h} = \sigma\boldsymbol{e} + \boldsymbol{j_0}$$
$$\mathbf{div}\,\boldsymbol{b} = 0, \qquad \mathbf{div}(\sigma\boldsymbol{e} + \boldsymbol{j}_0) = 0$$
$$\boldsymbol{b} = \mu \;\; \boldsymbol{h}$$

with $\boldsymbol{e}$ the electric field, $\boldsymbol{h}$ the magnetic field, $\boldsymbol{b}$ the magnetic flux density, σ the electrical conductivity, μ the magnetic permeability and $\boldsymbol{j}_0$ the source current density. By addition of the corresponding boundary conditions the problem gets a unique solution. The introduction of the magnetic scalar potential $\boldsymbol{a}$ with $= \mathbf{curl}\,\boldsymbol{a}$, $\boldsymbol{a} = (\mathbf{0}, \mathbf{0}, \mathrm{a_z})^{\boldsymbol{T}}, \boldsymbol{j}_0 = (\mathbf{0}, \mathbf{0}, j_0)^{\boldsymbol{T}}$ yields

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$$\sigma \frac{\partial a_z}{\partial t} - \mathbf{div}(\nu\, \mathbf{grad}\, a_z) = \mathrm{j}_0 \tag{1}$$

where ν denotes the magnetic reluctivity $\nu = \mu^{-1}$.

Discrete Formulation

Let us first consider the spatial discretization of the considered domain Ω. To handle the relative motion between stator and rotor by a conforming mesh approach it is necessary to change the mesh for the spatial discretization at every time step which e.g. is done by the Moving-Band method. Instead we apply a non-conforming mesh approach by dividing the domain Ω into two complementary domains Ω^{m} and Ω^{s} called mortar and slave, e.g. the stator and rotor of an electrical machine, $\Omega = \Omega^{\mathrm{m}} \cup \Omega^{\mathrm{s}}$. Let $\Gamma^{\mathrm{m}} \subset \partial\Omega^{\mathrm{m}}$ and $\Gamma^{\mathrm{s}} \subset \partial\Omega^{\mathrm{s}}$ be the corresponding non-conforming interface. The relative motion between rotor and stator is described by the mapping $R: \Gamma^{\mathrm{s}} \to \Gamma^{\mathrm{m}}$. The continuity of the field quantities across the non-conforming interface is ensured by extending problem (1) with the following boundary conditions which are weakly imposed by means of the mortar element method with Lagrange multiplier $\boldsymbol{\lambda}$:

$$\begin{aligned} \boldsymbol{h}^s \times \boldsymbol{n}^s &= \boldsymbol{\lambda} && \text{on } \Gamma^{\mathrm{s}}, \\ \boldsymbol{h}^m \times \boldsymbol{n}^m &= -\boldsymbol{\lambda} \circ R^{-1} && \text{on } \Gamma^{\mathrm{m}}, \\ \boldsymbol{a}^s &= \boldsymbol{a}^m \circ R. && \text{on } \Gamma^{\mathrm{s}}. \end{aligned} \tag{2}$$

Discretization in time is done for all unknowns, which are denoted by u in the following, by linear interpolation between the time steps with $\theta \in [0,1]$:

$$u(t) = u_{n+1} \cdot \theta + u_n \cdot (1-\theta),\ \frac{\partial\, u(\mathrm{t})}{\partial t} = \frac{u_{n+1} - u_n}{\Delta t}\ , \Delta t = t_{n+1} - t_n \tag{3}$$

We propose to use an implicit Euler scheme for the time stepping ($\theta = 1$) but the presented approach also allows the use of any other time stepping scheme including adaptive time stepping.

The application of the standard Galerkin method transforms the weak formulation into a linear equation system. In contrast to the equation system resulting from the conforming formulation this equation system is not positive definite and weakly conditioned [6] but can be solved by a generalized minimal residual algorithm [3]. To preserve the numerical properties of the conforming formulation the saddle-point problem is transferred to a positive definite problem which can be solved by standard conjugate gradient methods. The first step is to split the unknowns of the master and slave domain into two blocks each. The first block $\boldsymbol{u}_\Gamma^k$ with $k \in \{m, s\}$ contains all unknowns lying on the sliding interface Γ^k and the second one $\boldsymbol{u}_i^k$ all other interior unknowns of the domain Ω^k:

$$\boldsymbol{u}^m = \begin{pmatrix} \boldsymbol{u}_i^m \\ \boldsymbol{u}_\Gamma^m \end{pmatrix},\ \boldsymbol{u}^s = \begin{pmatrix} \boldsymbol{u}_i^s \\ \boldsymbol{u}_\Gamma^s \end{pmatrix}. \tag{4}$$

The resulting saddle-point problem at time step t reads:

$$\begin{pmatrix} \boldsymbol{S}_{i,i}^m & \boldsymbol{S}_{\mathrm{i},\Gamma}^m & 0 & 0 & 0 \\ \boldsymbol{S}_{\Gamma,i}^m & \boldsymbol{S}_{\Gamma,\Gamma}^m & 0 & 0 & -\boldsymbol{M}^T \\ 0 & 0 & \boldsymbol{S}_{\Gamma,\Gamma}^s & \boldsymbol{S}_{\Gamma,\mathrm{i}}^s & \boldsymbol{D}^T \\ 0 & 0 & \boldsymbol{S}_{\mathrm{i},\Gamma}^s & \boldsymbol{S}_{\mathrm{i},i}^s & 0 \\ 0 & -\boldsymbol{M} & \boldsymbol{D} & 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_\mathrm{i}^m \\ \boldsymbol{u}_\Gamma^m \\ \boldsymbol{u}_\Gamma^s \\ \boldsymbol{u}_\mathrm{i}^s \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_\mathrm{i}^m \\ 0 \\ 0 \\ \boldsymbol{b}_\mathrm{i}^s \\ 0 \end{pmatrix} \tag{5}$$

where the standard stiffness matrix is denoted by $\boldsymbol{S}$ and $\boldsymbol{M}$, $\boldsymbol{D}$ denote the coupling matrices which link the Lagrange multiplier to the magnetic vector potential. The unknowns of the vector potential are denoted by $\boldsymbol{u}$ and $\boldsymbol{b}_\mathrm{i}^k$ denotes the excitation of the field problem:

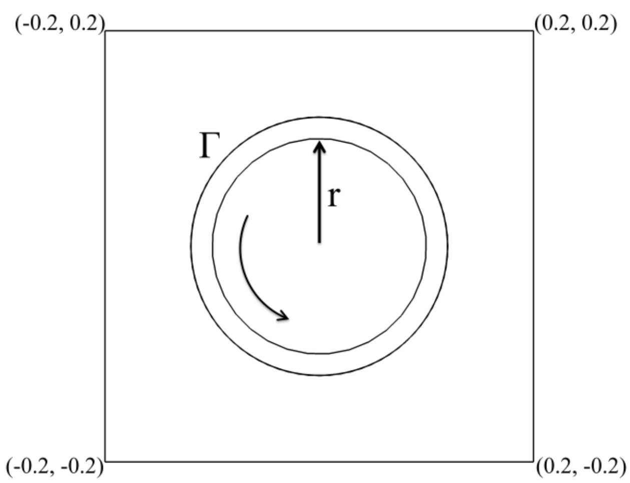

Fig.1: Benchmark problem.

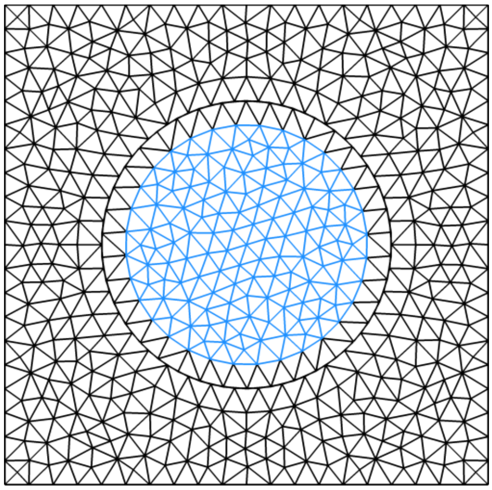

Fig.2: Coarse Discretization.

$$S_{ln}^{k} = \int_{\Omega^k} \nu \,\mathrm{grad}\, \boldsymbol{\alpha}_l^k \cdot \mathrm{grad}\, \boldsymbol{\alpha}_n^k + \sigma \frac{1}{\Delta t} \boldsymbol{\alpha}_l^k \boldsymbol{\alpha}_n^k \mathrm{d}\Omega \tag{6}$$

$$M_{jl} = \int_{\Gamma^s} \boldsymbol{\mu}_j \cdot \boldsymbol{\alpha}_l^m \circ R \,\mathrm{d}\Gamma, \qquad D_{jl} = \int_{\Gamma^s} \boldsymbol{\mu}_j \cdot \boldsymbol{\alpha}_l^s \,\mathrm{d}\Gamma \tag{7}$$

$$b_l^k = \int_{\Omega^k} \boldsymbol{j}_0^k \cdot \boldsymbol{\alpha}_l^k \,\mathrm{d}\Gamma \tag{8}$$

In order to transform this equation system into a positive definite system the unknowns $\boldsymbol{u}_\Gamma^s$ associated with the non-mortar interface Γ^s are replaced by a linear combination of the unknowns $\boldsymbol{u}_\Gamma^m$ derived from the last line of the saddle-point problem (5):

$$\begin{aligned} & \boldsymbol{D}\boldsymbol{u}_\Gamma^s - \boldsymbol{M}\boldsymbol{u}_\Gamma^m = 0 \\ \Leftrightarrow \quad & \boldsymbol{u}_\Gamma^s = \boldsymbol{D}^{-1}\boldsymbol{M}\boldsymbol{u}_\Gamma^m = \boldsymbol{Q}\boldsymbol{u}_\Gamma^m \end{aligned} \tag{9}$$

With the discrete projection operator between the mortar and non-mortar interface:

$$\boldsymbol{Q} = \boldsymbol{D}^{-1}\boldsymbol{M} = (\boldsymbol{M}^{\boldsymbol{T}}\boldsymbol{D}^{-\boldsymbol{T}})^{\boldsymbol{T}} \tag{10}$$

In addition the Lagrange multiplier is extracted from the third line of (5):

$$\boldsymbol{\lambda} = -\boldsymbol{D}^{-\boldsymbol{T}}\boldsymbol{S}_{\Gamma,\Gamma}^{s}\boldsymbol{D}^{-1}\boldsymbol{M}\boldsymbol{u}_\Gamma^m - \boldsymbol{D}^{-\boldsymbol{T}}\boldsymbol{S}_{\Gamma,\mathrm{i}}^{s}\boldsymbol{u}_\mathrm{i}^s \tag{11}$$

By replacing $\boldsymbol{u}_\Gamma^s$ and $\boldsymbol{\lambda}$ in (5) the Lagrange multiplier $\boldsymbol{\lambda}$ is eliminated and the resulting system of equations is positive definite and can be solved by standard conjugate gradient methods:

$$\begin{pmatrix} \boldsymbol{S}_{\mathrm{i},i}^{m} & \boldsymbol{S}_{\mathrm{i},\Gamma}^{m} & 0 \\ \boldsymbol{S}_{\Gamma,i}^{m} & \boldsymbol{S}_{\Gamma,\Gamma}^{m} + \boldsymbol{Q}^{\boldsymbol{T}}\boldsymbol{S}_{\Gamma,\Gamma}^{s}\boldsymbol{Q} & \boldsymbol{Q}^{\boldsymbol{T}}\boldsymbol{S}_{\Gamma,\mathrm{i}}^{s} \\ 0 & 0 & \boldsymbol{S}_{\mathrm{i},i}^{s} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_\mathrm{i}^m \\ \boldsymbol{u}_\Gamma^m \\ \boldsymbol{u}_\mathrm{i}^s \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_\mathrm{i}^m \\ 0 \\ \boldsymbol{b}_\mathrm{i}^s \end{pmatrix} \tag{12}$$

To assemble the system matrix in the shown way it is necessary to compute the inverse of the coupling matrix $\boldsymbol{D}$ which results in an additional computational effort dependent on the number of unknowns of the non-mortar interface Γ^s and prevents the commonly used element wise assembly of the system matrix. The structure of $\boldsymbol{D}$ results from the shape functions $\boldsymbol{\mu}$ of the Lagrange multiplier and has to be computed in advance before the system matrix is assembled. This is not necessary if the matrix $\boldsymbol{D}$ exhibits a diagonal structure which inversion is trivial.

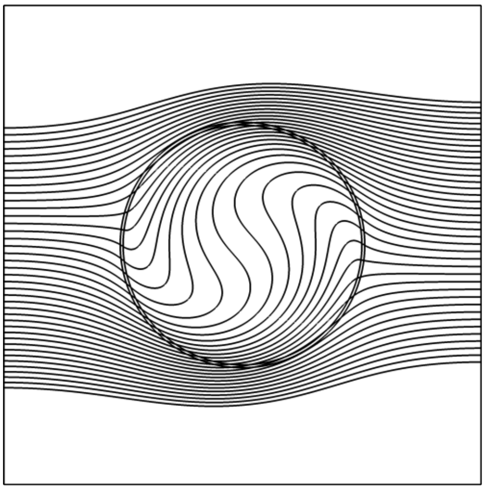

Fig.3: Field solution.

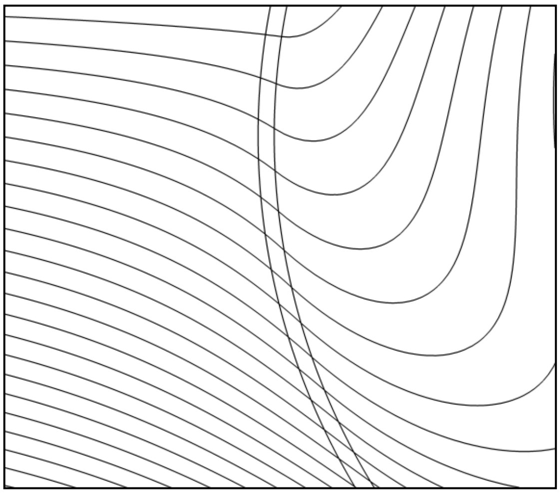

Fig.4: Field solution at the interface Γ.

Therefore the function space of $\boldsymbol{\mu}$ is chosen in a way that the diagonal structure of $\boldsymbol{D}$ is ensured by the utilization of bi-orthogonal shape functions [1] and dual Lagrange multiplier spaces respectively [4], [5] so that the following relation is verified:

$$D_{jl} = \int_{\Gamma^s} \boldsymbol{\mu}_j \cdot \boldsymbol{\alpha}_l^s \, \mathrm{d}\Gamma = c_i \delta_{ij} \int_{\Gamma^s} \boldsymbol{\alpha}_l^s \, \mathrm{d}\Gamma. \tag{13}$$

Where δ_{ij} denotes the Kronecker-Delta and $c_i \neq 0$ is a constant value which is dependent on the used basis functions on the non-mortar boundary Γ^s.

The presented approach has been implemented in the institute's in-house FE-package ***i***MOOSE [www.iem.rwth-aachen.de] and is applied to a benchmark problem in the next section to analyze the accuracy.

Numerical Results

To validate the implemented formulation the benchmark problem shown in Fig. 1 is considered. The problem consist of a conducting cylinder with radius r = 0.1m which is surrounded by air. The cylinder is centered at the origin of the xy-plane and the surrounding air is modeled as square with an edge length equal to 0.4m. The cylinder rotates with a velocity v, has a conductivity of 10 MS/m and its permeability equals air. Excitation of the field problem is represented by two dirichlet boundary conditions at the top (y = 0.2m) and the bottom (y = -0.2m) with values of $1Vs/m$ and $-1Vs/m$. The left and right boundaries of the domain correspond to the natural boundary condition. Fig. 2 shows an example of a coarse discretization of the described geometry and the non-conforming interface Γ can be observed which is located in the air one layer of elements away from the conducting cylinder.

In Fig. 3 the resulting field solution at a velocity of v = 22 Hz is presented for a fine discretization. The mesh consists of approximately 290,000 triangles and the solution shows the resulting field after 50 time steps with $\Delta t = 2.5ms$. The isopotential lines of the magnetic vector potential are drawn for values in the range of $-0.5Vs/m$ up to $0.5Vs/m$. As depicted in Fig. 4 one can clearly observe that the field lines are continuous across the interface Γ regarding the normal component as well as the tangential component of the field as expected with identical magnetic reluctivity on the mortar and non-mortar side.

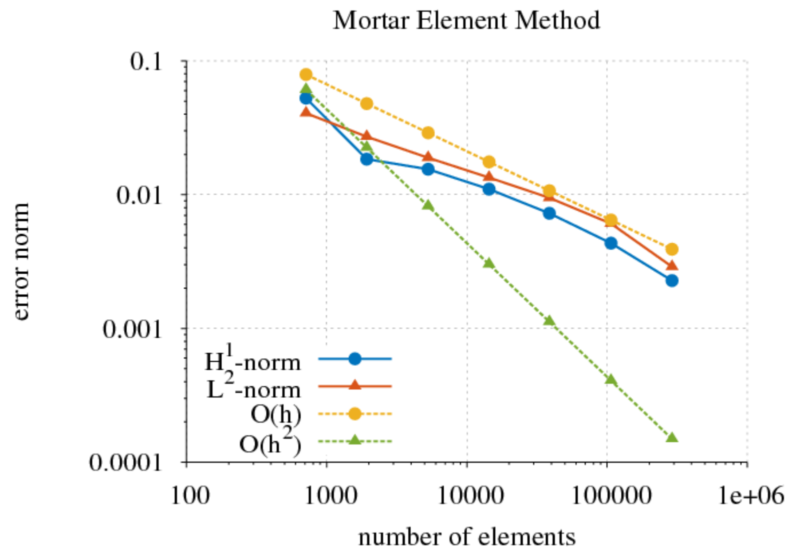

Fig.5: Convergence behavior benchmark problem (Mortar).

In the following the convergence of the proposed approach is evaluated. For this purpose we generate eight different meshes for the benchmark problem with varying discretization ranging from ~700 to ~800,000 numbers of elements. The field solution resulting from the finest discretization is used as reference solution to compute deviation in the energy norm H^1 as well as in the L^2-norm for the different meshes. Linear elements are used in the benchmark problem so that a discretization error of order O(h) for the H^1-norm and order $O(h^2)$ for the L^2-norm is expected. The computed error norms for the presented non-conforming mortar method approach are given in Fig. 5 along with the theoretical order as a function of the number of elements. The energy norm is in good agreement with the expected order but the L^2-norm shows a convergence far below the expected order $O(h^2)$.

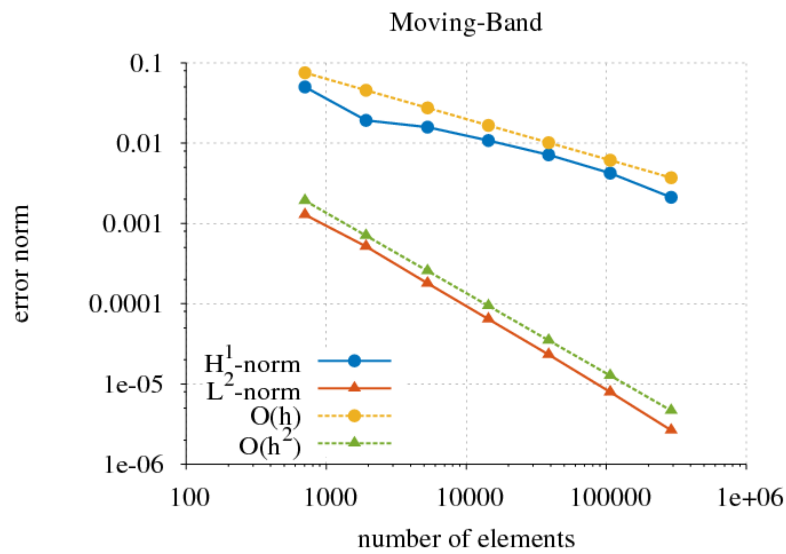

Fig.6: Convergence behavior benchmark problem (Moving-Band).

To further analyze this behavior we compare the convergence of the presented approach to a conforming method. We utilize the Moving-Band method [7] where an annulus-shaped band in the air gap between rotor and stator of the electrical machine is re-meshed at every time step. We choose the band for re-meshing in such a way that all elements located at the outside of the former non-conforming and now conforming interface Γ are contained in it and hence one distinct mesh is generated at every time step of the simulation. The same benchmark problem is computed by the conforming approach and the results are shown in Fig. 6. In contrast to the previous results the L^2-norm shows the expected convergence behavior and corresponds to the theoretical order $O(h^2)$. Following this outcome an error in the computation of the norms or the discretization can be excluded so that the observed convergence behavior needs additional investigations.

Conclusions

This paper presents an approach to handle the relative motion between rotor and stator of electrical machines by non-conforming sliding interfaces and take eddy currents into account. Non-conforming sliding interfaces overcome the issues of re-meshing methods to handle the relative motion in FE analysis of electrical machines. The approach is based on the mortar element method with Lagrange multiplier with bi-orthogonal shape functions to ensure a positive definite system matrix. Application to a benchmark problem shows that the continuity of the field quantities regarding the normal component of the magnetic flux density and the tangential component of the magnetic field strength is fulfilled at the non-conforming interface. Analysis of the convergence behavior indicates that the implementation of the presented approach does not reach the theoretical predicted order. Future work will therefore include additional studies on this approach.

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