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Robust Predictive Current Control for Performance Improvement of Induction Motors with Parameter Variation

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***Abstract*—In this paper a robust predictive current control approach is developed to tackle the impacts of parameter variation in induction motors on the control performance. The robustness can be achieved in general model predictive control (MPC) by integrating the disturbance into the prediction model, which is estimated by an observer and whose effect is compensated to the system input or output. In such a way the controller is robust against those uncertainties and the system can converge on the required available operating point. However, due to the variation, a non neglectable overshooting appears inevitably, which is critical for systems with hard constraints. In contrast a robust predictive control based on linear matrix inequalities (LMIs) implemented in this paper shows a significant improvement of current control in induction motors. For the real-time application the approximated multiparametric semidefinite programming (mp-SDP) with orthogonal partitioning is approached.**

I. Introduction

Induction motors (IM) are widely used in high performance electric drive systems because of their simple and reliable structure as well as low manufacturing cost. In order to improve the dynamic of the drive systems, the optimization based control strategy, called Model Predictive Control (MPC), is increasingly discussed and proposed for electric drives in the last decades [1]. When compared to PI controllers, the system constraints and uncertainties can be incorporated systematically into the MPC design [2] in such a way, that they are included in the optimization problem formulation.

Traditionally, the system uncertainties are extended in the prediction model as disburbance in MPC, which is estimated by an observer. Their effects are fed back to the controller as input or output disburbance. However, the current overshooting can occur due to parameter mismatch, particularly inductance mismatch caused by saturation in high current level, which makes the problem more critical. To deal with this problem, an off-line min-max control approach based on LMIs is implemented for the current loop in this work.

The robust MPC with explicit incorporated system uncertainties based on min-max optimization, also called min-max MPC, was firstly introduced 1996 in [3]. The plant is described as a linear time-varying system with polytopic uncertainties. The objective function is formulated as a quadratic Lyapunov function in infinite horizon and minimized by solving the LMIs on-line. Robust MPC in finite horizon was introduced 2004 in [4]. The parameter-dependent Lyapunov function is applied, which is divided into two parts: the first $N-1$ steps and the terminal step. In [5] the performance of the robust MPC in finite horizon was further improved by defining Lyapunov function for each prediction step.

To reduce the computational burden caused by on-line optimization and to ensure the robust MPC applicable for the dynamic systems with high sampling rate, the optimization should be moved off-line. In [6] and [7] the concept of asymptotically invariant ellipsoids describing the time-varying terminal constraint set was introduced. A sequence of explicit control laws corresponding to these ellipsoids is constructed off-line by solving semidefinite programming (SDP). According to the current state the ellipsoid and the control law are searched on-line. To improve the feasibility and optimality of the min-max control, the control law of the large ellipsoid is obtained in a backward manner with the knowledge of the control laws associated with included ellipsoids [8]. In [9] and [10] the further performance improvement of the invariant ellipsoid-based off-line min-max control was given.

Instead of solving SDP off-line according to the invariant ellipsoids, the original optimization problem can be formulated as multi-parametric or rather as approximated multi-parametric semidefinite program (mp-SDP). [11] gave a solution of the mp-SDP. However, the finite termination property cannot be garanteed by mp-SDP due to the variation of the optimal solution of a SDP problem according to the parameter vector [12]. Therefore, in [13] and [14] the algorithms based on approximated mp-SDP were introduced, in which instead of ellipsoids the state space is partitioned into triangle regions. The binary search tree algorithm is used for such approaches. To improve the on-line search efficiency the orthogonal search tree was applied in [15] for multi-parametric quadratic program (mp-QP) by superseding the triangle regions by quadratic ones. The similar idea using k-d tree was approached to solve approximated mp-SDP in [16] and [17]. In [18] and [19] the applications of robust MPC in dynamic systems can be found.

In this work the algorithm in [5] in combination with approximated mp-SDP and quadratic search tree is applied for the current control in IMs. This paper is organized as follows. Section II introduces the background of robust MPC, both the on-line solution and the off-line algorithm used in this work. In

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section III a robust current control approach based on min-max optimization is designed. The performance of the general MPC with Kalman Filter (KF) extended with disturbance model and of the proposed controller in simulation will be compared in section IV. Furthermore, the both approaches are appied to a testbench and the experimental results will be discussed in this section. In section V the conclusions are given.

II. Robust Model Predictive Control

A. On-line optimization

The system uncertainties can be incorporated into the robust MPC in such a way, that the plant is described either with polytopic or structured uncertainties. The former is applied in this work as follows:

$$\begin{aligned} x(k+1) &= A(\theta)x(k) + B(\theta)u(k) \\ y(k) &= Cx(k) \\ [A(\theta), B(\theta)] &\in \Omega, \end{aligned} \tag{1}$$

where Ω is the polytopic set defined by

$$\Omega = Co\{[A_1, B_1], [A_2, B_2], ...[A_L, B_L]\}. \tag{2}$$

The symbol Co represents the convex hull and $[A_l, B_l]$ the vertices with $l \in L$. Therefore, for any $[A(\theta), B(\theta)] \in \Omega$ exists

$$[A(\theta), B(\theta)] = \sum_{l=1}^{L} \theta_l [A_l, B_l], \quad \forall \theta_l \in \Theta, \tag{3}$$

where Θ is the unit simplex with $\Theta = \{\sum_{l=1}^{L} \theta_l = 1, \theta_l \geq 0\}$. The cost function for finite horizon control is defined as:

$$J_k(x, U) = \|x_{k+N|k}\|_P^2 + \sum_{i=0}^{N-1} \|x_{k+i|k}\|_Q^2 + \|u_{k+i|k}\|_R^2. \tag{4}$$

The optimal control laws of robust MPC are obtained by minimizing this cost function for the worst case:

$$J_k^*(x(k)) = \min_{U=\{u_k..u_{k+N-1}\}} \max_{\theta \in \Theta} J_k(x(k), U, \theta), \tag{5}$$

wherefore it is also called min-max control.

In [3] it was proved that if the following inequality is satisfied

$$\begin{aligned} &V(x(k+i+1|k)) - V(x(k+i|k)) \\ &\leq -x(k+i|k)^T Q x(k+i|k) - u(k+i|k)^T R u(k+i|k) \end{aligned} \tag{6}$$

for any $[A(k+i),\ B(k+i)] \in \Omega$ and all $x(k+i|k)$, $u(k+i|k)$ at time k, the cost function of infinite horizon robust MPC has an upper bound

$$\max_{[A(k+i),\, B(k+i)] \in \Omega,\, i \geq 0} J_\infty(k) \leq V(x(k|k)), \tag{7}$$

where $V(x(k|k)) = x^T P x$ describing a quadratic Lyapunov function. Therefore, (5) can be represented by:

$$\min_{\gamma,\, S} \quad \gamma \tag{8}$$

$$s.t. \begin{bmatrix} 1 & x_k^T \\ x_k & S \end{bmatrix} \geq 0 \tag{9}$$

with $S = \gamma P^{-1}$. The inequality (6) can be replaced by the LMI

$$\begin{bmatrix} S & (A_l S + B_l Y)^T & S Q^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_l S + B_l Y & S & 0 & 0 \\ Q^{\frac{1}{2}} S & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \quad l = 1, 2...L, \tag{10}$$

where $Y = FS$ and $u(k+i|k) = Fx(k+i|k)$. Thus, the unconstrainted min-max problem can be transformed into the optimization of a linear objective (8) subject to LMIs (9) and (10). If the system constraints should also be considered, the LMIs

$$\begin{bmatrix} X & Y \\ Y^T & S \end{bmatrix} \geq 0, \quad X_{jj} \leq u_{j,max}^2, j = 1, 2...n_u \tag{11}$$

for input constraint and LMIs

$$\begin{aligned} &\begin{bmatrix} S & (A_l S + B_l Y)^T \\ A_l S + B_l Y & \Gamma_x \end{bmatrix} \geq 0, \\ &(\Gamma_x)_{mm} \leq x_{m,max}^2,\ m = 1...n_x,\ l = 1...L \end{aligned} \tag{12}$$

$$\begin{aligned} &\begin{bmatrix} S & (A_l S + B_l Y)^T C^T \\ C(A_l S + B_l Y) & \Gamma_y \end{bmatrix} \geq 0, \\ &(\Gamma_y)_{nn} \leq y_{n,max}^2,\ n = 1...n_y,\ l = 1...L \end{aligned} \tag{13}$$

for state and output constraints are introduced to the optimization problem. Therefore, the constrained min-max problem based on LMIs is defined as a semi-definite program (SDP)

$$\begin{aligned} &\min_{\gamma, S, Y, X, \Gamma} \quad \gamma \\ &\text{s.t.} \quad (9),\ (10),\ (11),\ (12),\ (13)\,. \end{aligned} \tag{14}$$

In [5] and [12] the optimality of finite horizon robust MPC was improved by defining independent upper bounds for each prediction step. The worst case of the cost function (4) in finite horizon can be formulated by:

$$\begin{aligned} \max_{\theta \in \Theta} J_k(x, U) = &\max_{[A_{k+i}, B_{k+i}] \in \Omega,\ i=1...N} \|x_{k+N|k}\|_P^2 \\ &+ \sum_{i=0}^{N-1} \|x_{k+i|k}\|_Q^2 + \|u_{k+i|k}\|_R^2 \\ = &\|x_k\|_Q^2 + \|u_k\|_R^2 \\ &+ \max_{x_{k+N|k} \in E_{k+N|k}} \|x_{k+N|k}\|_P^2 \\ &+ \sum_{i=1}^{N-1} \max_{x_{k+i|k} \in E_{k+i|k}} \|x_{k+i|k}\|_Q^2 + \|F_{k+i} x_{k+i|k}\|_R^2, \end{aligned} \tag{15}$$

where $E_{k+i|k}$ represents the invariant ellipsoids from (9), in which all feasible state variables at time $k+i$ predicted at time k are located. According to (15) the upper bound of the cost function in the worst case γ can be superseded by γ_0, γ_N as well as $\sum_{i=1}^{N-1} \gamma_i$. The formulation of the new SDP problem based on this concept is similar to (14) and therefore will not be described in this paper. For more information please refer to [12].

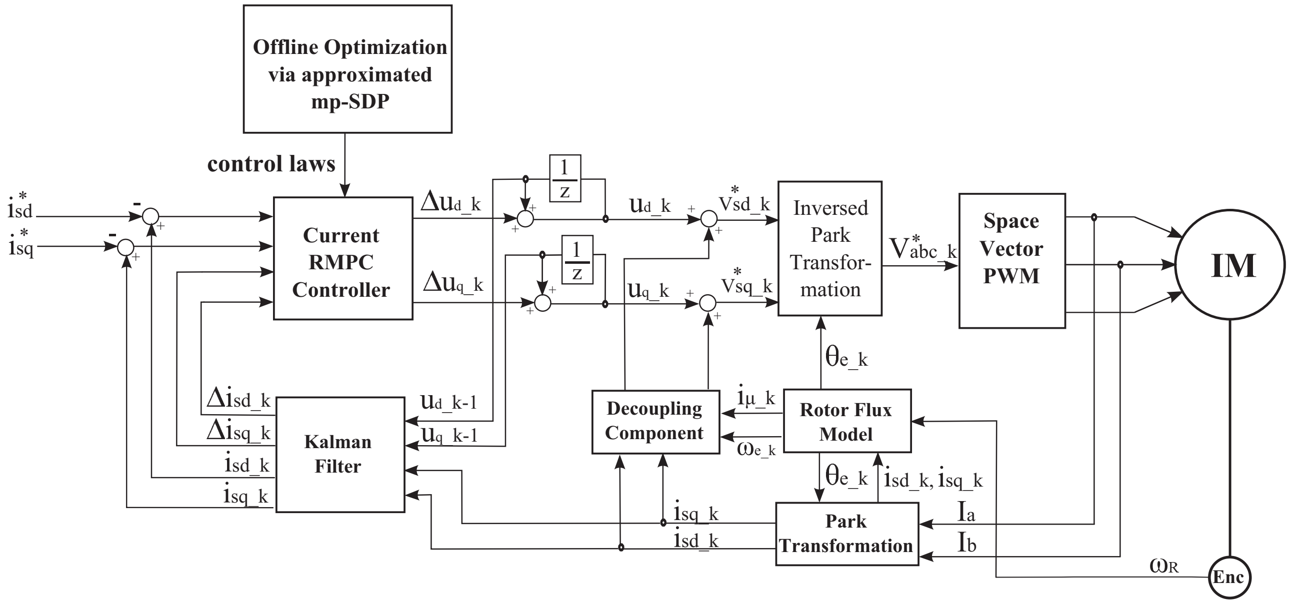


Fig. 1: Block diagram of the control structure with robust MPC.

B. Approximated mp-SDP

In order to apply the min-max control approach for dynamic systems with high sampling rate such as current control of IM, the optimization of SDP problem has to be moved off-line. As mentioned before, it can be realized by both invariant ellipsoid and approximated mp-SDP concepts. In this work an approach based on approximated mp-SDP is implemented.

A standard mp-SDP problem is formulated as follows:

$$\begin{aligned} J^*(x) =& \min_z \quad J(z,x) \\ \text{s.t.} \quad & F(z,x) \geq 0, \end{aligned} \tag{16}$$

where z denotes the new variable for the optimization problem to be solved and x the state variable. Either the upper bounds of the cost functions or the actuating variable can be approximated for the application of off-line optimization. In this work the actuating variable approximation is taken into account and therefore $z(x) = u_k$. The fundamental idea of approximated mp-SDP is to construct a piecewise affine function of z according to the parameter vectors. The state space is partitioned into several regions, in which a control law in linear form is defined respectively. The regions are defined in this work as quadrat:

$$\begin{aligned} \mathcal{B}_r &= \{x \in \mathbb{R}^n \ : \ H_r x \leq d_r\} \\ H_r &= \begin{bmatrix} I \\ -I \end{bmatrix}, d_r = \begin{bmatrix} h_u \\ -h_l \end{bmatrix}, \quad \forall r \in \mathbb{I} \end{aligned} \tag{17}$$

$\mathcal{B}_r$ describes the unique quadrat, $I \in \mathbb{R}^{n\times n}$ the unit matrix, h_u, h_l the upper and lower boundaries. $\mathbb{I}$ is the set of the indices of the feasible quadrats. In order to obtain the approximated linear solution in each quadrat

$$\hat{z}(x) = \hat{K}_r x + \hat{g}_r, \quad \forall x \in \mathcal{B}, \tag{18}$$

the vertices of the quadrat $V = \{v_1, v_2 ... v_M\}$ with $M = 2^n$ are in use. The factor $\hat{K}_r$ and $\hat{g}_r$ of the approximated solution for each quadrat, which contains a control law, can be then computed by solving the following optimization problem

$$\begin{aligned} &\min_{\hat{K}_r,\hat{g}_r} \quad \sum_{i=1}^{M} (z^*(v_i) - (\hat{K}_r v_i + \hat{g}_r))^T H_1 (z^*(v_i) - (\hat{K}_r v_i + \hat{g}_r)) \\ &\text{s.t.} \quad F((\hat{K}_r v_i + \hat{g}_r), v_i) \geq 0, \quad i \in \{1, 2 ... M\} \end{aligned} \tag{19}$$

Furthermore, to garantee the accuracy of the approximation approach the absolute and relative error tolerances should be defined for the optimization. In this paper the definition and computation of these error tolerances will not be introduced. For more information please refer to [12].

III. Control Synthesis

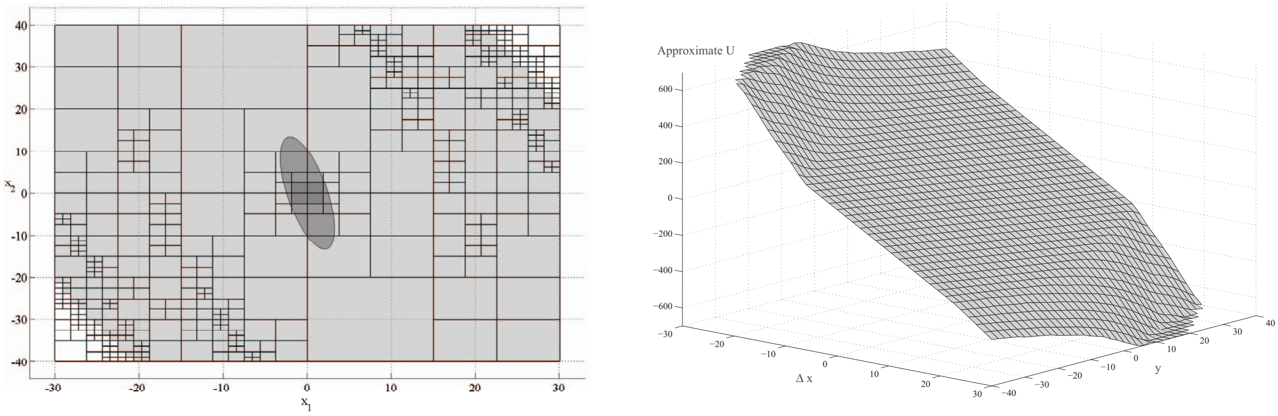
The nominal model of IM for the current control loop is given as follows:

$$u_{ds} = R_s i_{ds} + \sigma L_s \frac{di_{ds}}{dt} + (1-\sigma) L_s \frac{di_\mu}{dt} - \omega_e \sigma L_s i_{qs} \tag{20}$$

$$u_{qs} = R_s i_{qs} + \sigma L_s \frac{di_{qs}}{dt} + \omega_e (1-\sigma) L_s i_\mu + \omega_e \sigma L_s i_{ds} \tag{21}$$

The symbols u_{ds}, u_{qs}, i_{ds} and i_{qs} represent the stator voltage and current in d- and q-axis, respectively. R_s, L_s and σ denote the stator ohmic resistance, stator inductance and leakage coefficient. ω_e describes the synchronous frame speed and i_μ the magnetizing current.

The entire model can be considered as linear parameter varying (LPV) system with parameters R_s, L_s and ω_e. However, the parameter ω_e has a large range of variation and will be therefore excluded from the robust MPC in this work by using the decoupling components, so that the system dynamic is not diminished substaintially. The normalized values of R_s



(a) Orthogonal partition.

(b) Approximation control laws.

Fig. 2: Off-line optimization by solving approximated mp-SDP.

and L_s as minimum and maximum are defined as 0.5 and 2. The discretized system can be then described as:

$$\begin{aligned} x_{k+1} &= A_k x_k + B u_k \\ A_k &\in Co\left\{A_1, A_2, A_3, A_4\right\}, \end{aligned} \tag{22}$$

where A_l correpsonds to the arbitrary combination of the defined extreme values of resistance and inductance.

Since the min-max optimization introduced till now deals with the regulation problem, the state space formulation of IM should be reconstructed in such a way, that the state variables converge at the zero point in steady state. The model for reference tracking problem is defined as:

$$\begin{bmatrix} \Delta x_{k+1} \\ x_{k+1} - r_{k+1} \end{bmatrix} = \begin{bmatrix} A_k & 0 \\ A_k & I \end{bmatrix} \begin{bmatrix} \Delta x_k \\ x_k - r_k \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} \Delta u_k, \tag{23}$$

where r represents the reference value and is considered as constant within the prediction horizon, $\Delta x_k = x_k - x_{k-1}$. In Fig. 1 the control structure of the current control loop is given. The kalman filter is used to reduce the impact of measurement disturbance of current on the computation of Δx.

Fig. 2 shows the off-line optimization result. The state-space vector is partitioned orthogonally and illustrated in Fig. 2a. The dark gray region containing origin represents the terminal ellipsoid calculated by the optimization. The gray ones represent the regions, in which the parameter vectors are feasible. Depending on the error tolerances defined for the approximation, the regions can be divided coarse or fine. In this work the state space is partitioned in 416 feasible quadrats. All the regions are structured in a quadratic tree including the relationships to their super- and subregions, whereby the corresponding solution can be searched substaintially efficiently. In Fig. 2b the approximated control laws are presented in the state space. Here is to identify that according to the parameter vectors the control laws build up a continuous piecewise affine function. The relative error of the approximated mp-SDP solutions compared to the original SDP solutions is given in Fig. 3. The errors due to the approximation are marginal.

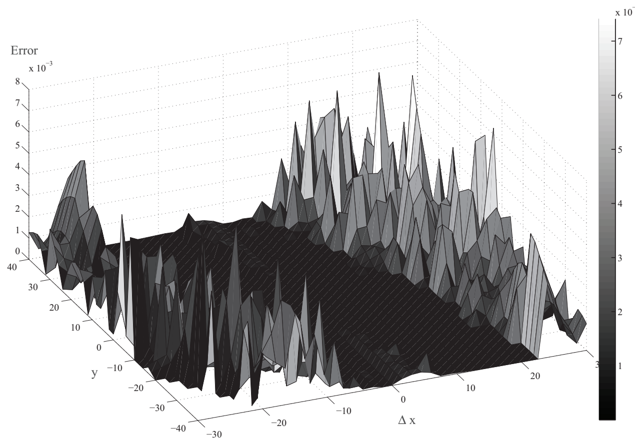


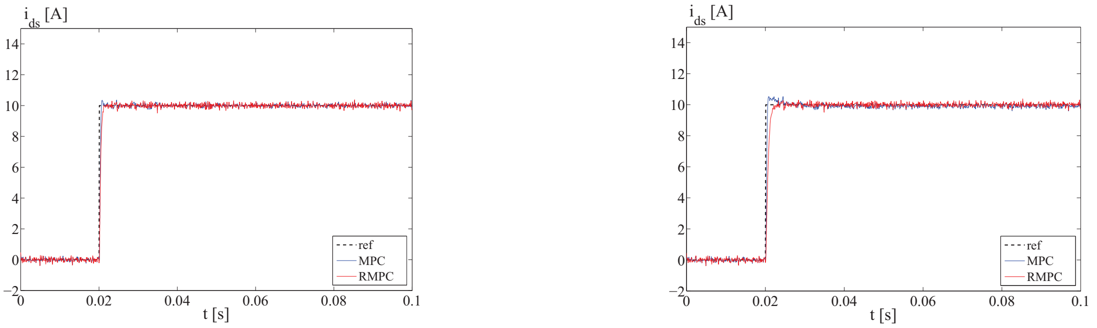
Fig. 3: Relative error of approximated solutions.

IV. Simulative and Experimental Results

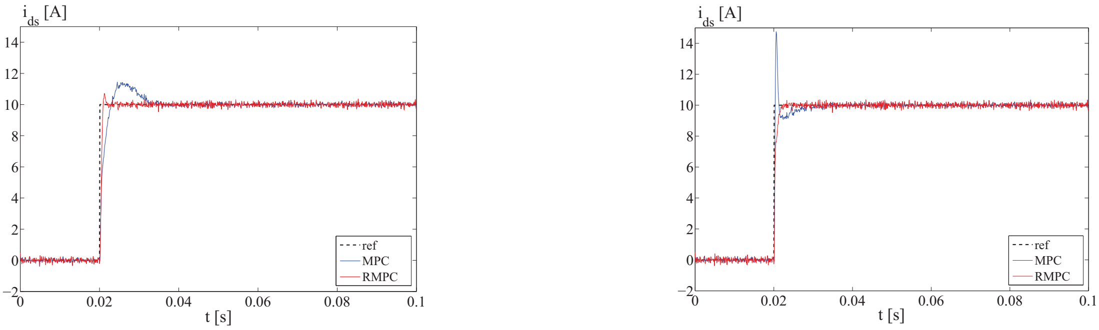
In this section both the simulative and the experimental results of the proposed approach are compared to those of the MPC method, in which the impact of the parameter variation is constructed as process disturbance and compensated to the system input.

To show the performance due to parameter mismatch, four cases are chosen for the simulation. In the first case, the parameters in the system and in the controller conform to each other. This corresponds to the operating points at room temperature with noload magnetizing current. In the second case, the actual resistance is higher than the one in the controller, which occurs at high operating temperature. In the third and the last cases, the mutual inductance set by the controller is half and twice of the actual one respectively, which means a higher and lower magnetising current taking place in the full load and in the field-weakening in each case.

Fig. 4 shows the current responses on the d-axis from both control strategies. r_s and l_m are the ratios of the ohmic resistance and mutual inductance in the controller to the ones

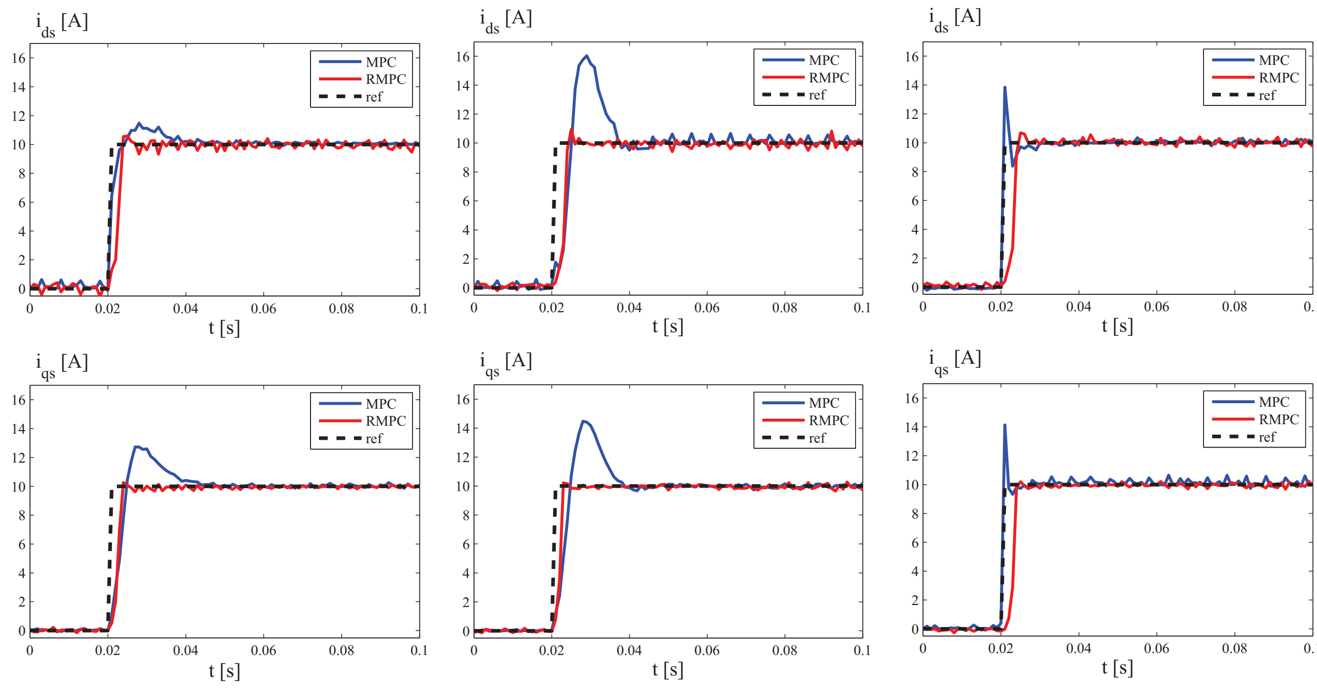


(a) $r_s = 1$ & $l_m = 1$. (b) $r_s = 2$ & $l_m = 1$.



(c) $r_s = 1$ & $l_m = 0.5$. (d) $r_s = 1$ & $l_m = 2$.

Fig. 4: Simulation results.



(a) $l_m = 1$. (b) $l_m = 0.5$. (c) $l_m = 2$.

Fig. 5: Experimental results.

in the motor. The simulation results of these four cases are presented in Fig. 4a - 4d, respectively.

In Fig. 4a it is apparent that for the case without parameter mismatch, both control strategies have achieved a similar good performance, whereat the system controlled by MPC with disturbance model has a little higher dynamic when compared to the one controlled by the proposed approach. In the second case, the system controlled by general MPC has a small overshooting due to resistance mismatch while RMPC shows a similar performance as in the first case. In the third case, the system performances of both controllers are impacted. However, by RMPC the overshooting is smaller and the system dynamic is much higher. In Fig. 4d the performance of the general MPC is further deteriorated. A significant overshooting amounting 50% takes place. In contrast a good system performance is exhibited by RMPC.

The experimental results from a laborary test bench are shown in Fig. 5. A slightly different from the simulation, the mutual inductance in the controller is varied merely on the test bench, because it affects the current control performance much more significantly than the stator resistance according to the simulation results. The results in Fig. 5 are comparable to the ones in the simulation. The overshooting shown in Fig. 5a and 5b by the general MPC is larger than in the simulation, which is caused probably by the unknown dead time in the system.

Concerning the simulative and the experimental results, the general MPC for current control in induction motors is robust against parameter mismatch and variation. However, it is not easy to prove, that the robustness can be gauranteed over entire operating points by this strategy. Furthermore, the system performance degrades in case of parameter variation. To achieve a high robustness of this strategy the system dynamic has to be sacrificed. Instead, the proposed approach incorporates the parameter variation systematically into the formulation of the optimization problem, whereby a general robustness of the system is given. By reducing the conservativeness of the optimization a good system performance can be achieved as well.

V. Conclusions

To ensure the performance and the robustness of the system, the parameter variation in IM is described in the proposed control approach by using polytopic uncertainties and LMIs, which is formulated as SDP problem. The real-time application is realized by solving approximated mp-SDP off-line and applying othorgonal partitioning and quadratic search tree. The simulative and experimental results indicate that by means of the proposed approach both the system dynamic is ensured and the overshooting is significantly suppressed.

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