Uncertainty Quantification and Sensitivity Analysis in Electrical Machines With Stochastically Varying Machine Parameters

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Electrical machines that are produced in mass production suffer from stochastic deviations introduced during the production process. These variations can cause undesired and unanticipated side-effects. Until now, only worst case analysis and Monte Carlo simulation have been used to predict such stochastic effects and to reduce their influence on the machine behavior. However, these methods have proven to be either inaccurate or very slow. This paper presents the application of a polynomial chaos metamodeling at the example of stochastically varying stator deformations in a permanent-magnet synchronous machine. The applied methodology allows a faster or more accurate uncertainty propagation with the benefit of a zero-cost calculation of sensitivity indices, eventually enabling an easier creation of stochastic insensitive, hence robust designs.

Index Terms— **Electrical machines, production tolerances, spectral stochastic finite element method, uncertainty quantification.**

I. Introduction

Electrical machines are subjected to stochastic variations introduced by the production process [1]. Subsequently, each produced machine instance may deviate slightly with respect to its ideal and initial design. As a result, parasitic effects—such as for instance undesired harmonic components in the machine's torque—can occur and will negatively influence its overall performance (e.g., by radiating unanticipated and undesired noise). One possibility to elude these problems is the creation of robust machine designs [2]. However, finding a robust design has proven to be difficult until now, because the standard finite element method (FEM) neither provides any intrinsic possibilities for sensitivity analysis of its (postprocessed) results nor any suitable way to propagate the occurring stochastic deviations onto the considered output sizes. Thus, the available tools to generate robust designs have been limited mostly to worst case analysis, Taguchi-design of experiments (DoE) and crude Monte Carlo (MC) simulation until now. Although worst case analysis proves to be imprecise, MC simulations (or DoE) in combination with the FEM result in high computational costs or, when reducing the required sample count, result in inaccurate predictions again.

To provide better tools for the creation of robust designs, this paper presents the application of a polynomial chaos (PC) metamodeling [3] technique for the simulation of an electrical machine. The choice of a PC approach is motivated by the easy calculation of sensitivity indices within the PC-framework, without being bound to the necessity to model all input deviations as Gaussian (as it is the case with, e.g., kriging). The required PC methodology is briefly recalled in Section II.

Afterward, the technique is applied to consider stochastic geometry deformation modes that are imposed on an electrical machine's stator by being pressed into a housing. Section III details the chosen modeling, and Section IV analyzes the deformation modes influence on the cogging torque of the considered permanent-magnet synchronous machine by calculating the Sobol sensitivity indices directly from the PC metamodel.

This way, a faster or more accurate tool for uncertainty propagation is provided along with the straightforward possibility to calculate sensitivity indices, hence enabling an easier and faster way to calculate and create robust designs.

II. Methodology

In the following, an electrical machine with stochastically deviating dimensions is considered. We assume that the occurring deviations can be expressed as function of a random vector of variables $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. The probability density function (PDF) of $\mathbf{x}$ is supposed to be known and stochastically independent.[1] Because of the randomness contained in $\mathbf{x}$, all studied machine output quantities $\mathcal{M}$ also become random variables in dependence of $x_1, x_2, \ldots, x_n$ and cannot be calculated with a single finite element (FE) calculation anymore. Hence, the overall goal is to find a suitable way to quantify the randomness contained in the machine's output quantities $\mathcal{M}(\mathbf{x})$.

A. Polynomial Chaos Expansion

One way to characterize $\mathcal{M}$ is to determine an explicit expression or approximation $\widetilde{\mathcal{M}}(\mathbf{x}) \approx \mathcal{M}(\mathbf{x})$, a so-called metamodel. The polynomial chaos theory [6] enables the calculation of such an explicit representation

$$\mathcal{M}(\mathbf{x}) \approx \widetilde{\mathcal{M}}(\mathbf{x}) = \sum_{i=0}^{P} \alpha_i \psi_i(\mathbf{x}) \tag{1}$$

[1]For stochastic dependent problems, the introduction of *copulas* can be used to relax this restriction (see [4] along with [5]).

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by approximating the analyzed output quantity $\mathcal{M}(\mathbf{x})$ as a function of a polynomial basis $\psi_i(\mathbf{x})$, $i \in \mathbb{N}$ and its scalar polynomial coefficients α_i. For a given PDF, [6] proposes rules for the construction of the polynomials $\psi_i(\mathbf{x})$. The remaining task is the determination of the coefficients α_i.

B. Coefficient Determination

Two different approaches which allow the calculation of the α_i are *projection* and *regression* [4]. Initially, both methods require Q realization pairs $(\mathbf{x}_k, \mathcal{M}(\mathbf{x}_k))$ of the input random vector $\mathbf{x}$ and its corresponding output $\mathcal{M}(\mathbf{x})$ to find α_i which fulfill

$$\{\alpha_i\} = \arg(\min\{E[\mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x})]^2\}). \tag{2}$$

Afterward, the projection method takes advantage of the polynomial basis orthogonality by calculating the coefficients as

$$\alpha_i = E[\mathcal{M}(\mathbf{x})\psi_i(\mathbf{x})] \tag{3}$$

where $E[\ ]$ is the expectation. The implicit integral in the right-hand side of (3) then can be estimated by

$$E[\mathcal{M}(\mathbf{x})\psi_i(\mathbf{x})] \approx \sum_{k=1}^{Q} \mathcal{M}(\mathbf{x}_k)\psi_i(\mathbf{x}_k)\omega_k \tag{4}$$

where ω_k is the associated integration weight to the point $\mathbf{x}_k$. The values of ω_k depend on the chosen numerical integration scheme, for example, in MC-integration $\omega_k = 1/Q$.

In the regression method, the integral $E[\mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x})]^2$ in the right-hand side of (2) is approximated by

$$E[\mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x})]^2 \approx \frac{1}{Q}\sum_{k=1}^{Q}[\mathcal{M}(\mathbf{x}_k) - \widetilde{\mathcal{M}}(\mathbf{x}_k)]^2. \tag{5}$$

From (1), (2), and (5), it can deduced that

$$\{\alpha_i\} = (\Psi \cdot \Psi^T)^{-1} \cdot \Psi \cdot \mathcal{M} \tag{6}$$

where

$$\mathfrak{M} = \begin{pmatrix} \mathcal{M}(\mathbf{x}_1) \\ \vdots \\ \mathcal{M}(\mathbf{x}_N) \end{pmatrix} \tag{7}$$

and

$$\Psi = \begin{pmatrix} \psi_0(\mathbf{x}_1) & \cdots & \psi_0(\mathbf{x}_N) \\ \vdots & \ddots & \vdots \\ \psi_P(\mathbf{x}_1) & \cdots & \psi_P(\mathbf{x}_N) \end{pmatrix}. \tag{8}$$

In a continuous environment, both approaches create identical results for an infinite polynomial base. With the introduction of a discretization, $(\Psi \cdot \Psi^T)^{-1}$ differs from the identity matrix, yielding more accurate results with use of the regression method. Hence, the regression method is typically preferred in application, although the inversion of $(\Psi \cdot \Psi^T)$ is not guaranteed to be numerically stable [4].

For both methods–projection and regression–the choice of the Q realizations $\mathbf{x}_k$ influence the efficiency of methods. Several works have shown that low-discrepancy sequences (for example, Sobol sequences) are suitable and outperform randomly drawn samples. Nevertheless, the number of realizations Q has to be chosen adequately to obtain good approximations that yield a $\widetilde{\mathcal{M}}(x)$ close to $\mathcal{M}(x)$. If the number of input random variables n is high, then the number Q can become excessively large. In case of the regression method, the number Q must be, for example, at least equal to the number of contributing polynomial chaos terms $P + 1$ that is then calculated as

$$P + 1 = \frac{(n + p_{\max})!}{n! \cdot p_{\max}!} \tag{9}$$

where $p_{\max}$ is the maximum degree of the polynomial chaos [6]. In the end, the time required to calculate all realizations $\mathcal{M}(\mathbf{x}_k)$, $k \in Q$ can become a challenge. To overcome this difficulty, one seeks only polynomial chaos terms whose impact on $\mathcal{M}(\mathbf{x})$ is significant [7], as has been done in this paper. Thus, the number of polynomial chaos terms and hence the number of realizations Q can be reduced drastically.

C. Postprocessing

With the polynomial coefficients, stochastic moments as the model's mean value μ and its variance σ^2 can be calculated. The definition of the expectancy value provides

$$\mu \approx E[\widetilde{\mathcal{M}}(\mathbf{x})] = \int_{-\infty}^{\infty} \widetilde{\mathcal{M}}(\mathbf{x}) \cdot f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} \tag{10}$$

and after inserting the metamodel's definition

$$\widetilde{\mathcal{M}}(\mathbf{x}) = \sum_i \alpha_i \cdot \psi_i(\mathbf{x}) \tag{11}$$

from (11) into (10), one can exploit the polynomials' orthonormality to deduce that

$$\mu \approx \sum_i \alpha_i \cdot \delta_{i0} = \alpha_0. \tag{12}$$

The derivation of the variance σ^2 occurs analogously and results in

$$\sigma^2 \approx \sum_i \alpha_i^2. \tag{13}$$

Eventually, a sensitivity analysis can be performed to evaluate the impact of each input's random variable x_i on the output's variation $\sigma^2_{\mathcal{M}}$. To do so, Sobol sensitivity indices [8] are a suitable choice. The calculation of Sobol sensitivity indices yields values in the interval $[0, 1]$ with S_i close to 0 representing a weak influence and S_i close to 1 indicating a high impact of the input i on the variation of $\mathcal{M}$. Once that the approximation $\widetilde{\mathcal{M}}$ in the polynomial chaos expansion form is available, the Sobol indices can be deduced straightforwardly [9], [10]. Applying the sensitivity indices, design paradigms as robust design and tolerance allocation can finally be implemented.

III. Application

The examination's goal is to analyze the effect of randomly occurring static pressures that deform a PMSMs stator. We here investigate the pressures' influence on the machine's cogging torque in particular. Possible sources for such pressure can be, for example, stator welding seams or fixation points of the stator in its housing. Here it is assumed, that the pressure points occur symmetrically distributed over the stator's circumference, and thus cause stator deformation modes as shown in Fig. 1.

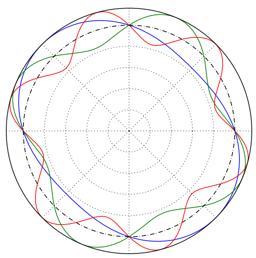

Fig. 1. Stator deformation modes number 2, 4, 6.

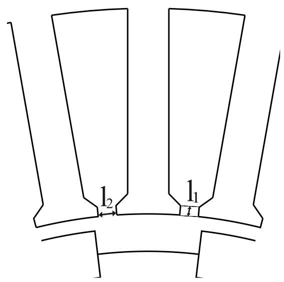

Fig. 2. Stator with varying tooth height (l_1) and slot-opening width (l_2).

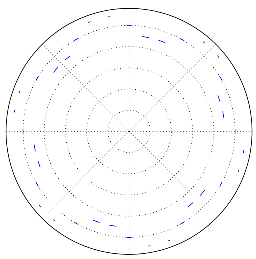

Fig. 3. Variation of the stator's slot-opening width (parameter l_2). Dashes: position and deformation of the slot opening width introduced by the occurrence of mode number 6.

Two effects of this deformation are studied at first separately, and then in combination.

1) The applied pressure on the stator eventually results in variations of the air gap. To avoid the need for a complete mesh remodeling, these changes have been simplified and are represented as variations of the stator tooth height (parameter l_1 shown in Fig. 2), effectively creating a similar air gap variation.
2) The stator's deformation causes variations in the slot opening width between all stator teeth. This effect is modeled with parameter l_2 (compare Figs. 2 and 3).

Applying conformal mapping theory in an approach comparable with [11], the influence of the first 20 modes has been tested for both parameters. Significant changes in the cogging torque can only be observed for the deformation modes $D = \{1, 2, 3, 6, 12\}$. Hence, these modes have been used to model the parameter input variations as

$$l_1(n) = l_1^0 + a_0 + \sum_{k \in D} a_k \cdot \sin\left(k\frac{2\pi n}{36} + a_k^*\right) \quad (14)$$

$$l_2(n) = l_2^0 + b_0 + \sum_{k \in D} b_k \cdot \sin\left(k\frac{2\pi n}{36} + b_k^*\right) \quad (15)$$

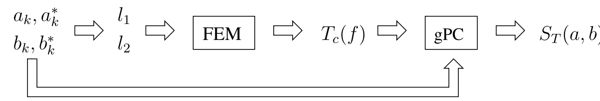

Fig. 4. Simulation flow: tooth height (l_1) and slot opening width (l_2) are calculated from the input random variables a_k, a_k^*, b_k, b_k^* and are used in the FEM. The FEM results together with the corresponding input random variables are employed to calculate the PC metamodel, which enables a straightforward calculation of the Sobol sensitivity indices $S_T(a, b)$.

where n is the stator tooth number, k is the mode number, l_i^0 is the nominal parameter value, a_k and b_k are the independent uniform random variables for the mode k defined in the interval $[-0.02; 0.02]$, and a_k^* and b_k^* are the independent uniform random variables for the mode k defined in the interval $[0; 2\pi]$.

IV. Results

To build the PC metamodels of the presented variations, an A-Φ formulation FE-model is employed with a mesh of 103 126 nodes. Introducing a parallelization with 20 calculation nodes, 400 evaluation points are calculated in 4 days. Within these simulations, the analyzed geometry variations are considered using the transformation method presented in [12]. First, the influence of the parameters l_1 and l_2 on cogging torque is studied separately, then combined simulations with both parameters are executed. Within the metamodels, the cogging torque is directly analyzed in its frequency domain as multiples of one pole pitch of the motor with $\tau = \pi/3$. With the help of the Sobol sensitivity indices, it can then be determined which cogging torque harmonic is originating from or influenced by which input random variable. Fig. 4 shows the the work-flow of the analysis.

The simulations allow the following conclusions for the given geometry.

A. Separate Variations of l_1 and l_2

For both parameter variations, the sixth harmonic of the pole pitch (here $2p = 6$) has a dominating influence on the mean value. This behavior conforms to expectations owing to the 36 stator teeth of the machine. Although the effective air gap distortion modeled by the tooth height variations mainly influences the variance of the harmonics first and second, the slot opening width yields a dominating variance again for the sixth output harmonic. The accordance with expectations confirms the method's correctness. The sensitivity analysis allows to give the following influence mapping of the relevant output harmonics:

1) harmonic 1 is influenced mainly by a_6 and b_6;
2) harmonic 2 is influenced mainly by a_{12} and b_{12};
3) harmonic 6 and 12 are influenced mainly by a_0 and b_0.

B. Interaction of Both Parameters

As the influence of the input modes $D_{\text{neglect}} = \{1, 2, 3\}$ is very small in the separate parameter variations, the corresponding input random variables are discarded for the joint influence analysis. The joint simulation results are given in Table I. It can be observed as follows.

1) The influence of all phase shifts (a_k^*, b_k^*) in the input modes is very weak.

TABLE I
SIMULATION RESULTS VARYING l_1 & l_2: MEAN, VARIANCE AND SENSITIVITY INDICES CALCULATED FROM THE METAMODEL

Harm. of τ_p	Mean	Variance	Sobol	a_0	a_6	a_6^*	a_{12}	a_{12}^*	b_0	b_6	b_6^*	b_{12}	b_{12}^*
1	0.0464	0.816e-3	total	0.057	**0.869**	0.047	0.048	0.037	0.058	0.050	0.048	0.052	0.055
			1^{st} ord.	0.000	**0.816**	0.001	0.000	0.000	0.000	0.002	0.001	0.003	0.003
2	0.0373	0.539e-3	total	0.028	0.035	0.024	**0.918**	0.031	0.029	0.032	0.035	0.023	0.043
			1^{st} ord.	0.002	0.001	0.000	**0.887**	0.001	0.001	0.002	0.000	0.000	0.001
6	0.1822	0.024e-3	total	**0.543**	0.000	0.000	0.000	0.000	**0.456**	0.000	0.000	0.000	0.000
			1^{st} ord.	**0.543**	0.000	0.000	0.000	0.000	**0.456**	0.000	0.000	0.000	0.000
12	0.0186	0.293e-6	total	**0.105**	0.001	0.001	0.001	0.001	**0.895**	0.001	0.001	0.001	0.001
			1^{st} ord.	**0.103**	0.000	0.000	0.000	0.000	**0.893**	0.000	0.000	0.000	0.000

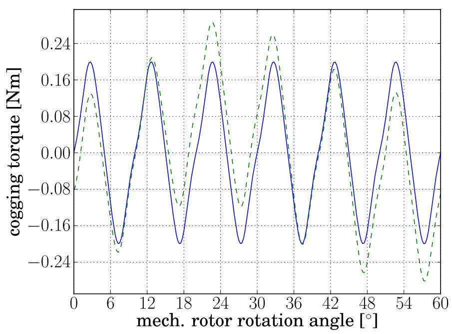

Fig. 5. Comparison of ideal cogging torque compared with a realization that shows a significant low-frequency effect (dashed line).

2) Total and first-order Sobol indices of each random variable are close to equal. It can be deduced that there are nearly no interactions within the model for the simultaneous occurrence of l_1 and l_2.
3) The variance of the harmonics 1 and 2 is dominant compared with the variance of the harmonics 6 and 12. One can reason that the occurring low-frequency effect is significant. This result is also shown in Fig. 5.

V. CONCLUSION

This paper presents the application of a nonintrusive polynomial chaos metamodeling technique for uncertainty quantification in electrical machines. In particular, the influence of randomly occurring stator deformation modes on the cogging torque of a permanent-magnet synchronous machine is modeled, propagated and assessed with the help of Sobol sensitivity indices. The utilization of the generalized polynomial chaos-based uncertainty propagation reduces the required computational effort. Furthermore, it allows to derive a direct correlation between the modeled input random variables and the harmonic components of the machine's cogging torque. This step eventually simplifies the creation of robust machine designs by providing a cause–effect mapping.

The presented methodology is universal and can be applied to arbitrary tolerances in electrical machines, given that the stochastic parameters can be expressed within the FE-model. Future work will investigate the impact of tolerances in the soft magnetic material properties with respect to loss calculations in electrical machines.

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