A pragmatic two-scale homogenization technique for ferromagnetic cores at high frequencies

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Abstract. The strong interaction between hysteresis and eddy currents in steel laminations subjected to a time-varying magnetic field cannot be resolved without a coupled finite element modelling at the level of a single lamination. Direct use of such a lamination model into the finite element modelling of an electrical machine, for instance, can be achieved by homogenization methods which, however, make it necessary to modify the finite element code. In this paper, we propose a pragmatic homogenization approach with no implementation required, provided the simulation code at hand has enough flexibility in the description of the material laws. The lamination model is used to identify the parameters of an appropriate macroscopic irreversible material law.

1. Introduction

An important aspect to be considered for the accurate calculation of iron losses in ferromagnetic laminated cores is the strong interaction between eddy currents (skin effect) and hysteresis inside individual laminations. This interaction cannot be resolved without a coupled finite element (FE) modelling at the level of a single lamination. The direct use of such a model in the FE modelling of a full electrical device (e.g. an electrical machine, a transformer) can be achieved by homogenization methods like those proposed in e.g. [1–3, 7]. Those methods, however, are rather technical and make it necessary to modify significatively the finite element code.

This paper proposes a pragmatic alternative, i.e. an homogenization strategy with no implementation required, provided the simulation code at hand has enough flexibility in the description of the material laws. The lamination model, able to resolve accurately the fields inside individual laminations is used to identify the parameters of an appropriate macroscopic irreversible material law. Once the material parameters are identified, the behaviour of laminated cores at higher frequencies and in the presence of higher harmonics can be accounted for in the macroscopic FE model.

The paper is organized as follows. Section 2. succinctly describes the 1D single sheet magnetodynamic model. From this model, which accounts for eddy currents and hysteresis, a relationship between the macroscopic flux density and magnetic fields is obtained as explained in section 3.. The implementation of the homogenized model is then described in section 4..

2. 1D single sheet FE model

The quantitative description of the interplay between hysteresis and eddy currents in a single lamination done by solving one-dimensional (1D) eddy current equations, which involves the magnetic field strength $\mathbf{h}$, the magnetic flux density $\mathbf{b}$ and the electric field strength $\mathbf{e}$ in a material with a conductivity σ and a non-linear hysteretic relationship $\mathbf{b}(\mathbf{h})$.

Considering an individual lamination of thickness $2d$ with an upper surface normal vector $\mathbf{n} = (0,0,1)$, the domain of analysis ω is a line parallel to $\mathbf{n}$, across half the thickness, and far from the edges. The boundary condition at the center of the lamination is $\operatorname{curl}\mathbf{h}(0) \times \mathbf{n} = 0$, whereas a given external field $\mathbf{h}(d)$ is applied at the surface of the lamination. A h-field formulation (1) is preferred because the magnetic field is the natural driving quantity of the hysteretic behaviour. The FE equations are

$$\int_{\omega} \left(\dot{\mathbf{b}}(\mathbf{h},\text{history}) \cdot \mathbf{h}' + \sigma^{-1} \operatorname{curl}\mathbf{h} \cdot \operatorname{curl}\mathbf{h}'\right) d\omega = 0 \quad \forall \mathbf{h}' \quad (1)$$

with $\mathbf{h} \equiv (0,h(z),0)$. The hysteresis behaviour is described by the term $\dot{\mathbf{b}}(\mathbf{h},\text{history})$ in (1). As an implementation, we use the *BH hysteresis model*, developed in parallel by Bergqvist [4] and Henrotte [5]. This flexible and accurate model builds on a thermodynamic representation of magnetic hysteresis in terms of an energy density ρ^{Ψ} (consisting of a stored energy term and an empty space term) and a friction-like dissipation potential $\dot{\rho}^{Q}$:

$$\rho^{\Psi} = \rho_{\text{st}}(\mathbf{J}) + \mu_0 \frac{\mathbf{h}^2}{2} \quad , \quad \dot{\rho}^{Q} = \kappa \left|\dot{\mathbf{J}}\right|. \quad (2)$$

Practically, the saturation of the material is represented by the anhysteretic curve, i.e. a curve giving the magnetic susceptibility $\chi := J/h_r$ as a function of $|\mathbf{h}_r|^2$, with $\mathbf{h}_r \equiv \partial_{\mathbf{J}}\rho_{\text{st}}$ the vector field that represents the history of the material. The flux density is then

$$\mathbf{b}(\mathbf{h},\mathbf{h}_r) = \mu_0\mathbf{h} + \chi(|\mathbf{h}_r|^2)\,\mathbf{h}_r, \quad (3)$$

and its time derivative in terms of the unknown $\mathbf{h}$ writes (note the dyadic product $\mathbf{h}_r\mathbf{h}_r$)

$$\dot{\mathbf{b}}(\mathbf{h},\mathbf{h}_r) = \left(\mu_0 + \left(\chi(|\mathbf{h}_r|^2)\mathbb{I} + 2\dot{\chi}(|\mathbf{h}_r|^2)\,\mathbf{h}_r\mathbf{h}_r\right)\partial_{\mathbf{h}}\mathbf{h}_r\right)\dot{\mathbf{h}} \equiv \underline{\mu}^{\partial}\dot{\mathbf{h}}$$

where $\mathbb{I}$ is the identity. More details are found in [6] or in the full paper.

A typical discretization is done with 50 equidistant nodes and 360 time steps per period. Iron losses per unit surface are given by the flux of the Poynting vector $\mathbf{e}(d) \times \mathbf{h}(d)$ across the lamination surface. Simulation results obtained with the single sheet model are shown in Fig. 1.

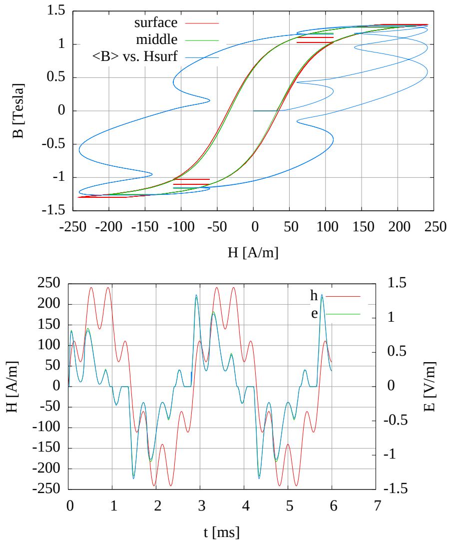

Figure 1. The top picture shows the true $b-h$ cycle (at the surface at in the middle of the sheet), as well as the apparent (measurable) $<b>-h$ cycle in the presence of higher harmonics. The bottom picture shows h and e fields at the lamination surface in the same situation. Iron losses evaluate as the product of these two curves.

3. Homogenization

The relation between the single lamination model and the macroscopic FE model is described by the homogenization relationships :

$$|\mathbf{H}| = h(d) \quad , \quad |\mathbf{B}| = <b> \equiv \frac{1}{d}\int_0^d b(z)\mathrm{d}z. \tag{4}$$

The idea of the pragmatic two-scale model is to identify from simulations with the single lamination model a regular material relationship of the form

$$\mathbf{H}_{\mathrm{hom}} = \nu(B)\mathbf{B} + \lambda(B,H_c)\dot{\mathbf{B}}. \tag{5}$$

where the function $\nu(B)$ is the parametric representation of a general reluctivity curve and the function $\lambda(B,H_c)$ is another parametric function with a cut-off at the value H_c related with the coercivity of the considered material. As a first step, this relationship is assumed isotropic. The parameters of the functions $\nu(B)$ and $\lambda(B,H_c)$ are identified by the least squares method.

4. Homogenized FE model

We shall denote fields at the macroscopic scale with a capital letter. The most widely used formulation for electrical applications is the magnetic vector potential $\mathbf{A}$ 2D formulation. The weak formulation of Ampere's law then reads :

$$\int_\Omega \mathbf{H}_{\mathrm{hom}} \cdot \mathrm{curl}\,\mathbf{A}' d\Omega = \int_\Omega \mathbf{j}_s \cdot \mathbf{A}'\, d\Omega \quad \forall \mathbf{A}' \tag{6}$$

with $\mathbf{j}_s$ the current density and $\mathbf{H}_{\mathrm{hom}}$ given by (5). Figure 2 and 3 represent the magnetic flux density waveform along the lamination sheet over a period in time on the surface and in the middle of the sheet at two different magnetic flux density levels. The classical skin effect is clearly recognized at a flux density level of 1 T. In contrast, it is realized that in case of a saturated

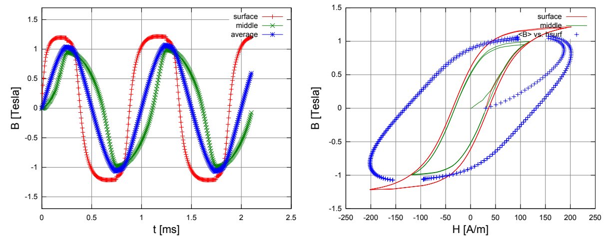

Figure 2. Left: Magnetic flux density along the lamination sheet over a period in time at $B = 1\,\mathrm{T}$ and $f = 1000\,\mathrm{Hz}$. Right: Corresponding hysteresis loops.

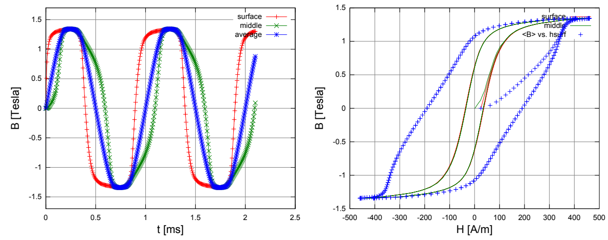

Figure 3. Magnetic flux density along the lamination sheet over a period in time at $B = 1.3\,\mathrm{T}$ and $f = 1000\,\mathrm{Hz}$. Right: Corresponding hysteresis loops.

material, i.e., $B = 1.3\,\mathrm{T}$, the skin effect disappears and a saturation front moves through the material. At a larger frequency, skin effect still dominates. Looking at the instantaneous profiles of B within lamination depth, it is apparent that these profiles are not spatially uniform. Saturation naturally limits the increase of B, but an important phase-shift appears between B waveforms located at different depths within sheet. This gives rise to a phenomena, a kind of saturation front, that strongly affects the loss mechanism. Calculating these instantaneous magnetic flux density profiles, it is evident that, the induced eddy currents strongly influence the local B-H dynamic cycles, compare Fig. 2 and 3. This underlines the necessity to solve the strongly coupled phenomena, hysteresis and induced nonlocal eddy currents, simultaneously. Disregarding this, leads to significant differences in field and loss predictions.

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