Uncertainty quantification and sensitivity analysis in electrical machines with stochastically varying machine parameters

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***Abstract*—Fabrication processes of electrical machines are tainted with production tolerances, resulting in undesired deviations in the later machines' behaviour. In order to estimate their influence on the machine design, worst-case estimations are commonly employed. To achieve more accurate probabilistic predictions, this paper proposes the application of the spectral stochastic finite element method (SSFEM). Instead of supplying only behavioural margins as in worst case analysis, the SSFEM allows to connect each output size's value to a probability of its occurrence, enabling the calculation of failure rates and parameter sensitivities. The presented approach is subsequently applied to evaluate magnetization deviations in a permanent-magnet synchronous-machine (PMSM) as well as their influence on the cogging torque.**

***Index Terms*—electrical machines, production tolerances, spectral stochastic finite element method, uncertainty**

I. INTRODUCTION

Production tolerances that influence the behaviour of an electrical machine can be deviations in terms of material characteristics, geometry [1] or excitation. In the presence of production tolerances, the nominal data of a large set of produced machines varies slightly between each fabricated machine instance. In order to create machine designs which prove to be less sensitive to production tolerances (so-called robust designs), a more accurate method for the uncertainty assessment than worst case analysis is required. Employing a non-intrusive spectral stochastic meta-modelling approach [2] allows to solve the described dilemma. It permits a straightforward calculation of central moments (mean values, variances, kurtosis), failure probabilities and sensitivity indices [3], hence enabling the creation of robust designs. A non-intrusive SSFEM is applied to evaluate magnetization deviations in a permanent-magnet synchronous-machine (PMSM) and their influence on the cogging torque.

II. METHODOLOGY

Based on a fixed set of calls to the FE-model $\mathcal{M}(\mathbf{x})$, a continuous polynomial response surface $\widetilde{\mathcal{M}}(\mathbf{x})$ is built:

$$\mathbf{y} = \widetilde{\mathcal{M}}(\mathbf{x}) = \sum_{i=0}^{P} \alpha_i \cdot \psi_i(\mathbf{x}) \tag{1}$$

The orthogonal base polynomials $\psi_i(\mathbf{x})$ are chosen according to the probability-density-function (PDF) $f_{\mathbf{x}}(\mathbf{x})$ of the inputs, which are assumed to be independent. The polynomial coefficients α_i follow from

$$\alpha_i = \int_{\mathbf{x} \subset \mathbb{R}^n} \psi_i(\mathbf{x}) \cdot \mathcal{M}(\mathbf{x}) \cdot f_{\mathbf{x}}(\mathbf{x})\ d\mathbf{x}\ . \tag{2}$$

With (1), the mean and the variances can be easily evaluated.

III. RESULTS

The presented methodology has been applied to calculate the cogging torque's sensitivity to magnet-production tolerances of a PMSM with six surface-mounted magnet-poles. Each pole's remanence flux-density has been allowed to vary uniformly in an interval $1.08T \leq B_r \leq 1.18T$. Employing a second order meta-model with 27 coefficients and 75 FE evaluation-calls yields a mean value of $\mu_{T,\text{cog}} \approx 0.21\,\text{Nm}$ and a standard-deviation of $\sigma_{T,\text{cog}} = 0.005\,\text{Nm}$. A comparison to a 2000 sample Monte-Carlo simulation shows an error of 0.05% in the mean value and less than 2.5% difference in the standard deviation, proving the method's accuracy. The torque's relative probability density function can be estimated with successive calls to the meta-model and is displayed in Fig 1 with preceding application of a Gaussian kernel density estimation. The stochastic analysis of further input variations (e.g. slot-opening width and excentricity) along with a sensitivity assessment will be presented in the full paper.

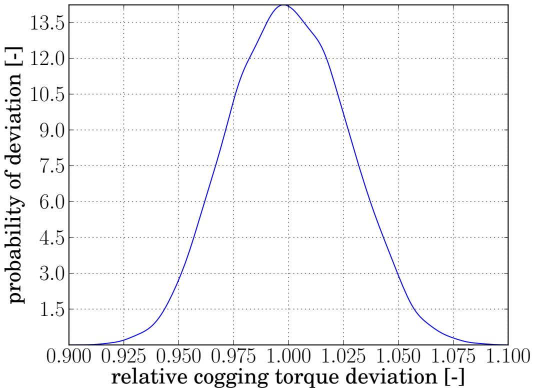

Fig. 1. Probability distribution of relative deviations in the cogging torque's peak value for the presented magnet deviations based on a kernel density estimation executed on the meta-model.

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