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Stochastic models for the evaluation of magnetisation faults

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Abstract

Purpose – The introduction of stochastic deviations due to production faults into the finite element (FE) simulation of electrical machines requires suitable error-model. These models should describe the occurring deviations from the ideal case. Permanent magnets, which can be used as rotor excitations in synchronous machines (PMSM), are one out of many possible sources for the aforementioned stochastic production variations. Fitting measured magnet variations to simulation models with the aim of describing the occurring production deviations, however, poses a problem due to two reasons: to begin with, only data of measured flux-densities are available. Second, a solution of the inverse problem is required to obtain data about changes inside the magnet. This paper, therefore, presents two solutions to this problem.

Design/methodology/approach – Two error-models, one based on knowledge about the magnetisation process, the other one built upon principal component analysis, are presented. Both models are evaluated by parametrising them, using a set of measured flux-density data from magnets. Afterwards, each model's applicability and reproduction quality is assessed.

Findings – Both models still have some drawbacks. While the first model seems to be too coarse grained for certain variations, the second model lacks applicability for a high reproduction quality.

Originality/value – The comparison of both methods reveals guidelines, which methodology should be applied for predicting which variations. Furthermore, solutions are shown, how to mitigate the problems of the two presented models.

Keywords Finite element method, Model evaluation, Inverse problem, Magnetisation error, Stochastic deviation

Paper type Research paper

1. Introduction

Most electrical machines nowadays can be distinguished by two groups: on the one hand, there are highly specialised machines, produced in small numbers and strongly optimised towards one or several constraints (typical examples are applications in medical care and military) and on the other hand machines produced in bulk production (used in automotive or household appliances). As this set represents the largest market share it is typically constructed using a robust design pattern (Wang *et al.*, 1999), in order to minimize the influence of production variations onto the targeted nominal machine data. The rising cost pressure of mass production, however, enforces the usage of more cost effective materials and processes, resulting in an increasing number of deviating input conditions. In consequence, the robust design of a machine becomes more difficult and needs to be verified.

Tools for the propagation of uncertainties across electromagnetic models recently came into the focus of research and development and are available, as for instance the

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spectral stochastic finite element (FE) method (Ghanem and Spanos, 2003), applying stochastic Galerkin (Rosseel *et al.*, 2010) or non-intrusive projection/regression methods (Sudret, 2007). Nevertheless, for some input variables, e.g. magnets in a PMSM, valid deviation-models for the employment in simulation as well as information about realistic parameters (likelihoods) for these models still are missing.

Until now, most deviation-models are based upon simple assumptions. In the case of magnets this typically means the hypothesis of an over the magnet constant variation in its remanence flux-density or magnet misplacement on the rotor (Gasparin *et al.*, 2009), their occurrence being based upon a Gaussian probability density function (PDF) (Heins *et al.*, 2011). These postulations obviously reduce the result quality of the new sophisticated uncertainty propagation methods, because the error-model's precision directly influences the prediction quality of the entire system. Hence these assumptions need either to be proven or in case of inaccuracy to be enhanced.

In order to improve prediction quality when compared to the aforementioned models, a physically motivated model (Jurisch, 2007), representing the state of the art and being deduced from the magnet production process, is compared to a new, non-physical model which is based on the idea of a superposition of multiple independent magnet fragments. Both models are parametrised using a set of self-conducted radial flux-density measurements along the surface of each magnet.

As a result, the advantages and drawbacks of both techniques are compared using measured magnetisation curves. Furthermore, it is shown how to mitigate the problems of the two presented models. The results finally allow a finer grained and therefore more accurate simulation of PMSMs under the consideration of intrinsic magnet deviations arising from the production process.

2. Models

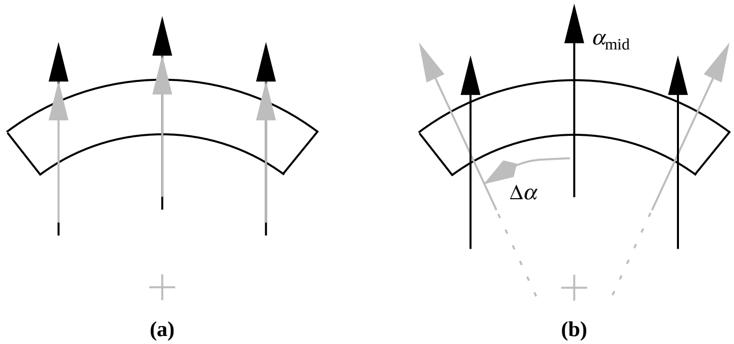
In order to include variation analysis of electrical machines into "conventional" FE analysis, propagation methods and variation models are needed to consider the system's uncertainties. This section presents two possible magnetisation variation models, which themselves are independent of the FE, but have to fulfil the typical boundary conditions of FE before being applied there. The fulfilment of the boundary conditions is however considered as given here, moving considerations concerning the models' application in FE simulation into Section 4.

2.1 Model A – production process based modelling

Model A is based on the knowledge about the production process and comprises a subset of the presented variations in Jurisch (2007). Two possible variations are allowed:

(1) ξ_1. Constant, global changes in the magnet's remanence flux-density (Figure 1(a)). Such variations can arise either from a weaker remanence flux-density created in the magnetisation process, or an error in the main direction of the targeted magnetisation angle, resulting again in a weaker flux-density in the magnet's radial flux-density component.

(2) ξ_2. Spatial deviations of the remanence flux-density depending on the angle $\Delta\alpha$ relative to the magnet's middle α_{mid} (Figure 1(b)). This parameter represents magnetisation disturbances that arise from a magnetic field which is applied to



Notes: (a) Constant remanence flux-density weakening, modelled by parameter ξ_1; (b) angle deviations, growing in dependence of their distance towards the magnets edges, modelled by parameter ξ_2

Figure 1. Considered magnet variations (grey) in contrast to ideal magnetization (black) for model A

the green body of the magnet during its pressing step of the production process and is used to align magnet particles. This field is not necessarily the same field as the one used in the magnetisation process and may deviate in its orientation, especially towards the magnet's edges. The resulting flux-density therefore will be a superposition of this field with the intended magnetisation field.

The effective description of the magnetic excitation therefore is extended by the random variables ξ_1, ξ_2 and the angle difference $\Delta\alpha$ with respect to the magnet's middle α_{mid} and can be written in a Cartesian decomposition as:

$$\vec{B}(\Delta\alpha, \xi_1, \xi_2) = B_r(\xi_1) \cdot \begin{pmatrix} \cos(\alpha_{mid} + \Delta\alpha(\xi_2)) \\ \sin(\alpha_{mid} + \Delta\alpha(\xi_2)) \\ 0 \end{pmatrix} \tag{1}$$

2.2 Model B – segmented magnetisation model

Determining the PDF of the random variables described in Section 1 proves to be difficult due to the necessary solution of the inverse problem and a certain information loss as described in Section 4. This new model hence is motivated by the variation analysis of measurement data (Jolliffe, 2002) and bases upon the following steps:

- At first, a principle component analysis (PCA) of the radial flux-density measurement data for all magnets is performed.
- In parallel, a segmented model of the magnet under test is created, treating the magnet as a sum of one mm wide, directly adjacent located segment parts (Figure 2). The radial flux-density along the measurement circumference is simulated for one single segment, setting the excitation for all other segments to zero, stepwise executing this procedure for all magnet segments.
- The first principal component of the PCA is approximated using a weighted sum of all single segment excitations. This process is comparable to signal reconstruction from a sampled, digital signal as described in Vaseghi (2007)

and afterwards repeated for all significant principal components. Principal components, whose influence on the absolute measurement variance is below a certain limit (e.g. 0.5 percent), are defined to be insignificant and therefore neglected.

- The weighted PCA approximation coefficients are implemented into the magnet-model for each magnet segment.
- Realisations of different magnetisations can finally be created using weighted accumulations of the principal components according to their probability of occurrence.

Using this approach, the magnet is modelled as the superposition of a set of different magnetisation shapes (respectively principal components), each shape depending on a random variable ξ_i, yielding equation (2):

$$\vec{B}(\alpha, \vec{\xi}\ ) = \sum_{i}^{n} M_i(\alpha) \cdot X(\xi_i) \tag{2}$$

3. Measurements

In order to evaluate the presented magnetisation fault models and determine the intrinsic remanence flux-density variations for a given set of magnets, a test-bench as shown in Figure 3 has been constructed. There, the magnets are rotated around the x-axis of the mounting, allowing a measurement of the radial flux-density component in the picture's y-direction with a Hall-sensor. In this way, 52 magnets have been measured pointwise along a constant circumference above each magnet's surface. Twenty six magnets have got their north-pole on the outer side, the other is oriented group vice-versa.

The results for the "north-up" group of magnets are shown in Figure 4. Repetition measurements to determine the reproducibility of the test-bench have been taken. The goal was to prevent manual misplacement errors to be mistaken for magnetisation errors. Differences between repetitive measurements were found to be below 0.5 percent. Remanence flux-density variations between different magnets were determined to exceed 25 percent in the worst case.

4. Results

For both models the challenge consists in the solution of the inverse problem. Hence the goal for each magnet is to find a set of parameters in the FE-models, which reflect the measured flux-density with the least possible error. For both model parametrisations,

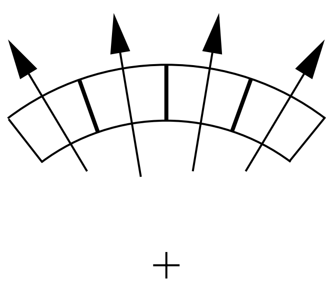


Figure 2.
Considered segmented magnet model, for the purpose of clearness shown with a reduced number of segments

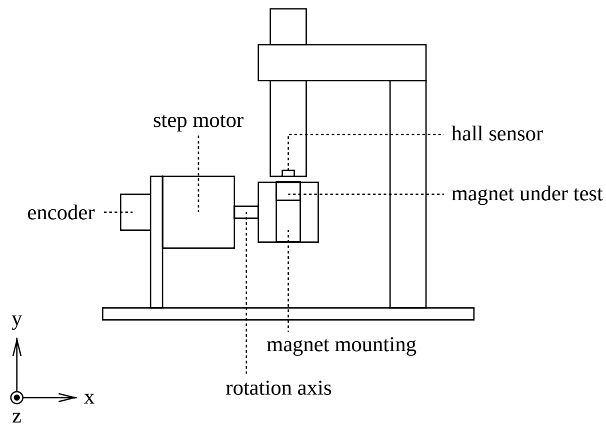


Figure 3.
Sketch of the test-bench used for measuring the magnets' radial flux-density components

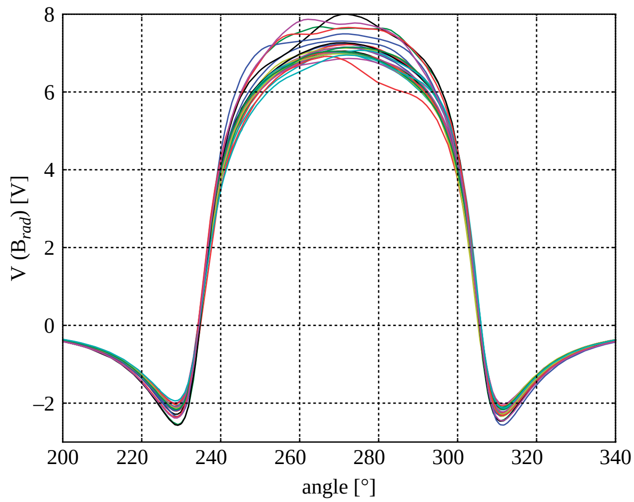


Figure 4.
Plot of Hall-voltages of the radial flux-densities of 26 magnets measured 1.6 mm above each magnet's surface

a 2D-FE model is selected, because its accuracy above the magnet proves to be sufficient precise. This selection furthermore allows a considerable reduction of simulation time when compared to 3D-simulations having the same mesh-density along the magnet's circumference. The circumference beside the magnet is not considered in the following, restricting evaluation to the arc-segment located above the magnet's surface (230°. . .310°), with the magnets middle located at the angle of 270°.

4.1 Model A – parametrisation

In the beginning, a full factorial sampling of the parameter space of ξ_1 and ξ_2 must be simulated once using the selected FE model. Afterwards, the quadratic error of the difference between the simulated radial flux-density $B_{rad,sim}$ and the measured radial flux-density $B_{rad,mes}$ is minimised by optimal parameters selection of ξ_1 and ξ_2 using least-square regression as given in equation (3):

$$\min_{\xi_1,\xi_2}\left|\sum_{\alpha=230^\circ}^{310^\circ}\left[B_{rad,sim}(\alpha,\xi_1,\xi_2)-B_{rad,mes}(\alpha)\right]^2\right| \tag{3}$$

Figure 5 shows the comparison between the optimal parametrised simulation results and their corresponding measurements for two selected magnets. In the left graph (Figure 5(a)), the automatic parameter detection offers a very good approximation in the considered area of interest. The right graph (Figure 5(b)) shows the curve of a damaged magnet. Here the applied algorithm detects different parameters than those presented in the picture, resulting in a more compressed graph with a reduced ξ_1.

In consequence, it can be deducted that the presented approach reproduces the assumed variations in angle and strength of the flux-density very good. Local errors, however, pose a problem for this model, as they cannot be accounted for in this approach and furthermore result in a biased parameter detection.

4.2 Model B – parametrisation

For parameter determination of model B, a principal component analysis (PCA) over all measured radial flux-densities of one group of magnets has to be performed (Figure 6). Afterwards, the six most important principal components – consisting of the intended magnetisation (Figure 6(a)) and its main deviations (Figure 6(b)) – must be reconstructed. The applied segment model consists of 24 segments (Figure 8).

Figure 7 shows the reconstruction of the measured field using the weighted PCA approach for the same two magnets as in Figure 5. For both magnets the PCA approach allows a better field reconstruction. The price paid is an increase in random variables by a factor three and a highly complicated and error prone implementation of the magnetization configuration inside the FE, here requiring 144 parameters per magnet and simulation.

Figure 8 finally depicts the FE-model of type B used to simulate magnet no. 13. The weighted sum of the applied principal components yields a piecewise constant, unidirectional magnetisation per segment. Each segment's color saturation here represents the strength of its magnetisation.

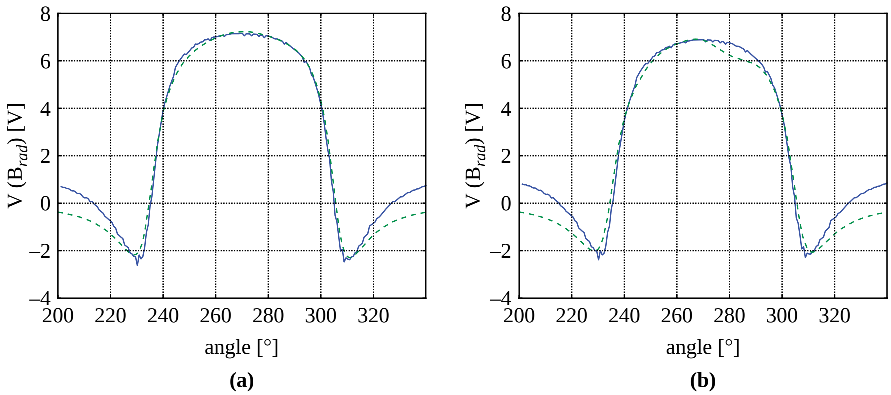
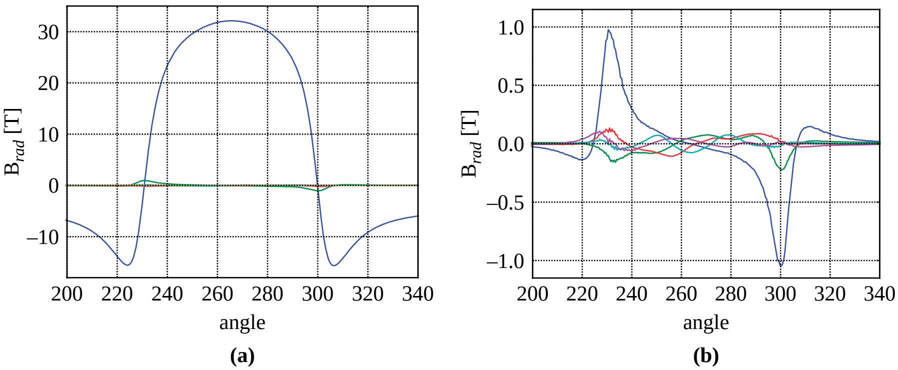


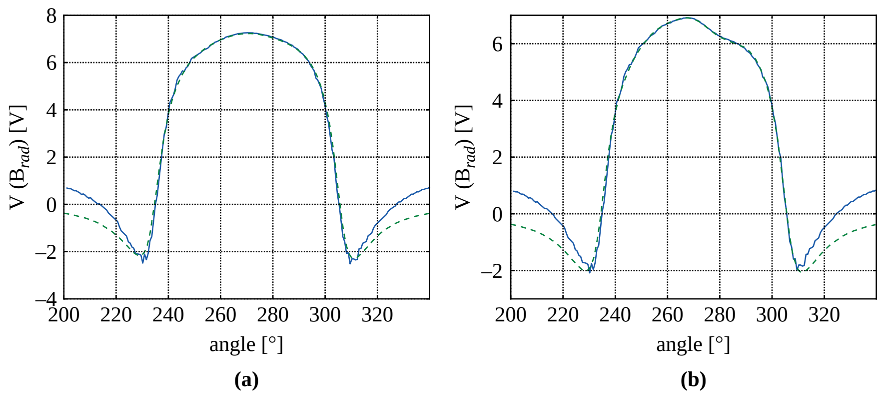
Figure 5.
Parametrised and scaled FE-results (solid) in comparison to measurements (dashed)

Notes: (a) Magnet #1, ξ_1 = 1.047 and ξ_2 = –0.05; (b) magnet #13, ξ_1 = 0.95 and ξ_2 = –0.15



Notes: (a) All principal components; (b) zoom into principal components without main component.

Figure 6.
Results of the principal component analysis for all "north-up" magnets



Notes: (a) Magnet #1, using six principal components; (b) magnet #13, using six principal components

Figure 7.
Parametrised and scaled FE-results (solid) in comparison to measurements (dashed)

5. Conclusions

Two models for the representation of magnetisation deviations in permanent magnets have been presented. Both models have been parametrised using FEM-simulations and a set of self-conducted radial flux-density measurements along the surface of 52 magnet samples. Model A allows considerations of the production process, but is not able to account for local magnet faults, which can be measured. Model B promises more accuracy and enables the simulation of local magnet faults. Its bulkiness in terms of additional random variables and the fact, that the postulated segmentation of the magnets implicates discontinuous jumps in excitation are severe drawbacks which seem to outweigh its benefits. In consequence model A is recommended for use in typical machine design cycles, whilst model B should be only used for final verification.

A promising step might be to fuse both models, in the way that model A might be allowed to be superposed by local magnet faults. This would enable to consider local

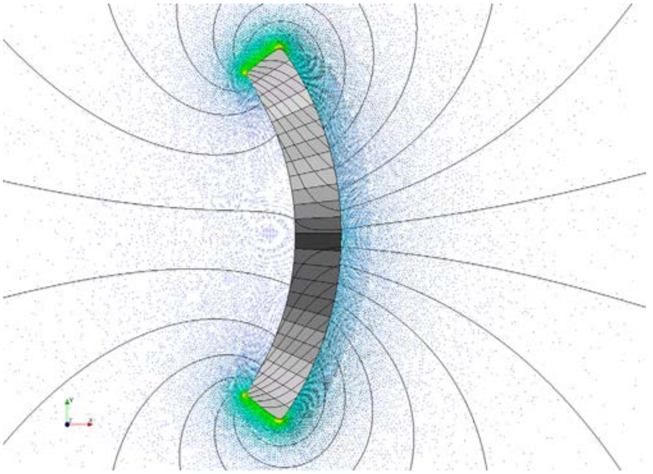


Figure 8.
Piecewise constant magnetisation parameters of the unidirectional/diametral magnetised magnet no. 13 determined for model B

Note: Darker segments correspond to higher values of remenance flux-density

errors without an too expensive increment of random variables. Furthermore, the resulting model would again be physical and therefore simplify to draw conclusions for possible improvements of the production process. The next steps require to apply the described models into a complete machine simulation to determine each model's resolution limit with respect to machine sizes as torque ripple or induced voltages.

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