# Sensorless control for surface mounted permanent magnet synchronous machines at low speed

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Abstract — This paper proposes a sensorless speed control based on a new extension of the torque producing flux (active flux) observer for the surface mounted permanent magnet synchronous machines (SPMSM) without additional high frequency signal injection. From the estimated torque producing flux, the rotor position and speed can be calculated at low speed due to their independency. Two approaches of the torque producing flux observer are presented and compared. The results show the stability and robustness of the expansion of the torque producing flux observer at low speed for the SPMSM.

## I. NOMENCLATURE

List of the symbols used in this work is as following:

Symbol	Description
$i_{a,b,c}$	Stator currents in 3 phases
$u_{a,b,c}$	Stator voltages in 3 phases
$i_s^{s} = (i_{\alpha}, i_{\beta})^T$	Stator currents in $\alpha\beta$ -coordinates
$\hat{i}_s^s = (i_\alpha, i_\beta)^T$	Estimated stator currents in $\alpha\beta$ - coordinates
$u_s^{s} = (u_a, u_\beta)^T$	Stator voltages in $\alpha\beta$ -coordinates
$i_s^r = (i_d, i_q)^T$	Stator currents in $dq$ -coordinates
$\Psi_s^s$	Stator flux in $\alpha\beta$ -coordinates
$\widehat{\Psi}_{s}^{s}$	Estimated stator flux in $\alpha\beta$ -
	coordinates
$\widehat{\Psi}_{si}^{s}$	Estimated magnet flux in dq-
	coordinates
$\widehat{\Psi}^{s}_{act}$	Estimated active flux in $\alpha\beta$ -
	coordinates
R	Stator resistance
$L_d, L_q$	Inductances (direct axis &
	quadrature axis)
р	Number of pole pairs
$\theta_{mech}$	Rotor position (mechanical)
$\theta_{el} = \theta_{mech} p$	Rotor position (electrical)
$\omega_{mech}$	Rotor speed (mechanical)
$\omega_{el}$	Rotor speed (electrical)
Т	Sampling time

# II. INTRODUCTION

Sensorless control for electrical machines plays an important role in industry applications, in which the number of hardware components and system costs can be significantly reduced. Besides, low installation space requirement and less electromagnetic compatibility problems are also advantages of the sensorless control principles.

There are two categories of sensorless control for the surface mounted permanent magnet synchronous machines, which are used in two different speed ranges, i.e. high speed range and low speed range. The sensorless control at low speed is playing an important role for the research. Many advantageous approaches have already been successfully developed.

In [1-4], the high frequency signal injection method is used in order to diagnose the magnetic saliency, which contains the information about the rotor position and rotor speed. It is one of the most used methods, which are appropriate for low speeds.

In [5], the rotor position is obtained from a predefined ramp function of the rotor speed. The rotor position can be determined through the integral of the rotor speed. This approach for low speed is switched to the one for high speed range, after the rotor ramps up with a constant q-current along this predefined ramp from standstill to a fixed high speed range.

A similar method is used in [6], where a I-f feedforward control is realised at low speed for rotor position and rotor speed estimation. In relation to [5], a reference frequency of stator current is predefined. The stator currents  $i_d = 0$  and  $i_q$ =constant are operated separately. With the aid of the reference frequency, the reference rotor position can be detected.

In addition, the non-linearity of stator inductance can be utilized for the rotor position estimation [7, 8]. At this, the self-inductance and mutual-inductance are considered, which are dependent on the rotor position. The difference between the stator voltages in free-wheeling mode operation and in converter-active-operation is determined. The information about the rotor position can be detected from this difference.

"Back EMF" method is usually used for the rotor position and rotor speed estimation. Matsui's observer is an extension of the "back EMF" observer [9]. Two redundant parameter models are established: an electrical parameter model and a mechanical parameter model, which contain the information about the rotor position and the rotor speed. An optimal experimental approach is required, in order to provide the



Fig. 1. Sensorless field oriented speed control scheme.

extended "back EMF" method for the rotor position and rotor speed estimation at low speed.

In literature, a torque producing flux concept [10,11] provides the speed estimation at low speed without the common approach of signal injection. Such methods are suitable for interior permanent magnet synchronous motor (IPMSM). This paper introduces a new extension for SPMSM, which is based on the torque producing flux method and combines a disturbance feedforward.

The paper is organized as follows: Section III: Sensorless Field Oriented Speed Control, Section IV: Observer model, Section V: Estimation and control results, Section VI: Conclusion.

#### III. SENSORLESS FIELD ORIENTED SPEED CONTROL

As it is already known from field oriented control that due to the linear independence of the stator currents in dqcoordinates, it is possible to control two current components  $i_d$ and  $i_q$  separately. The direct axis current  $i_d$  is set to zero in order to control the torque produced by quadrature axis current  $i_q$ . Only the q-component is responsible for the construction of a torque control so that the current control of q-component can superimpose the speed control. An exact rotor position is required to control the PMSMs, which can be obtained by installing an encoder on the rotor shaft. However, this encoder increases the cost. Therefore, a sensorless control with position observer is designed to provide the information about exact rotor position. Detailed information can be found in the following sections.

A complete block diagram representation for a field oriented control of PMSM using a voltage regulated space vector PWM voltage source inverter is shown in Fig. 1. By the sensorless speed control, the observers estimate the rotor position and speed using the stator currents  $i_s^s$  and voltage  $u_s^s$ in the  $\alpha\beta$  coordinate system, which are calculated from the measured stator currents  $i_{a,b,c}$  and voltages  $u_{a,b,c}$ . The estimated rotor position is used for the Park's transformation and the estimated speed is fed back to the speed control.

## IV. OBSERVER MODEL

# A. Observer model with flux feedback

The principle of the torque producing flux observer with flux feedback is shown in Fig. 2 [10]: the aim of this method is an accurate estimation of the torque producing flux (active flux).

With the help of the voltage model (1) [10], the estimated stator flux  $\hat{\Psi}_s^s$  can be calculated from the measured current  $i_s^s$  and voltage  $u_s^s$ :

$$u_s^s = R_s i_s^s + d\Psi_s^s / dt.$$
 (1)

The current model [10]

$$\widehat{\Psi}_{si}^{r} = \Psi_{si,d} + j\Psi_{si,q} = \begin{pmatrix} \Psi_{F} \\ \mathcal{O} \end{pmatrix} + \begin{pmatrix} L_{d} & \mathcal{O} \\ \mathcal{O} & L_{q} \end{pmatrix} \begin{pmatrix} i_{d} \\ i_{q} \end{pmatrix}$$
(2)

is defined to estimate the magnet flux  $\widehat{\Psi}_{si}^{r}$ , which has to be transformed in  $\alpha\beta$ -coordinates:

$$\widehat{\Psi}_{si}^{s} = T^{-1}(\theta) \cdot \widehat{\Psi}_{si}^{r}.$$
(3)

Details about the transformation are described in the appendix.



Fig. 2. Structure of the torque producing flux observer with flux feedback.

As shown in Fig. 2, the difference between the estimated  $\widehat{\Psi}_{s}^{s}$  and  $\widehat{\Psi}_{si}^{s}$  is fed back to the voltage model through the PI compensator gain. Thereby, the estimation of the stator flux can be corrected and improved:

$$\widehat{\Psi}_{s}^{s} = \int \left( u_{s}^{s} - R_{s} i_{s}^{s} + u_{comp} \right) dt \,. \tag{4}$$

The compensation value  $u_{comp}$  in s-domain is described as:

$$u_{comp} = \left(k_p + k_i/s\right) \cdot \left(\widehat{\Psi}_{si}^s\left(s\right) - \widehat{\Psi}_s^s\left(s\right)\right),\tag{5}$$

where  $k_p$  is the proportional gain and  $k_i$  is integral gain, which can be experimentally ascertained.

In order to get closer insight into the characteristics of the permanent flux in  $\alpha\beta$ -coordinates, the active flux is defined as[10]:

$$\widehat{\Psi}_{act}^{s} = \widehat{\Psi}_{s}^{s} - L_{q}i_{s}^{s} \tag{6}$$

where  $L_q$  is the q-axis stator self inductance and  $\Psi_F$  is the stator flux. The rotor position  $\hat{\theta}_{el}$  can be estimated by

$$\hat{\theta}_{el} = \arccos\left(\hat{\Psi}_{act,\beta} / \sqrt{\hat{\Psi}^2_{act,\alpha} + \hat{\Psi}^2_{act,\beta}}\right) + n \cdot \pi, n = N_0$$
(7)

### A. Observer model with current feedback

The principle of the observer with current feedback is shown in Fig. 3: similar to the flux observer with flux feedback, this observer model consists of a current model (8) and a voltage model (9) [11]:

$$\begin{pmatrix} \hat{i}_{\alpha} \\ \hat{i}_{\beta} \end{pmatrix} = T^{-I} \begin{pmatrix} \hat{\theta} \end{pmatrix} \cdot \begin{pmatrix} I/L_d & 0 \\ 0 & I/L_q \end{pmatrix} \cdot T \begin{pmatrix} \hat{\theta} \end{pmatrix} \cdot \begin{pmatrix} \hat{\Psi}_{\alpha} \\ \hat{\Psi}_{\beta} \end{pmatrix} + \frac{\Psi_F}{L_d} \cdot \begin{pmatrix} \cos(\hat{\theta}) \\ -\sin(\hat{\theta}) \end{pmatrix}; \quad (8)$$

$$\frac{d}{dt} \begin{pmatrix} \hat{\Psi}_{\alpha} \\ \hat{\Psi}_{\beta} \end{pmatrix} = -R_s \cdot \begin{pmatrix} i_{\alpha} \\ i_{\beta} \end{pmatrix} + \begin{pmatrix} u_{\alpha} \\ u_{\beta} \end{pmatrix} + K \cdot \begin{pmatrix} i_{\alpha} - \hat{i}_{\alpha} \\ i_{\beta} - \hat{i}_{\beta} \end{pmatrix}.$$
 (9)

 $\hat{\Psi}_{\alpha}$  and  $\hat{\Psi}_{\beta}$  are estimated stator fluxes in  $\alpha\beta$ -coordinates and used to determine the present stator current. Through the proportional control factor *K*, the difference between the estimated stator current  $\hat{i}_s^s = (\hat{i}_{\alpha}, \hat{i}_{\beta})^T$  and the measured stator current  $i_s^s = (i_{\alpha}, I^{\beta})^T$  is the feed-back signal of the voltage model to be minimized. The active flux  $\hat{\Psi}_{act}^s$  is determined by (6). The rotor speed can be defined with estimated rotor position  $\hat{\theta}_{el}$  (7) as

$$\hat{\omega}_{mech} = \dot{\hat{\theta}}_{mech} = \frac{d\hat{\theta}_{mech}}{dt} = \frac{d}{dt}\frac{\hat{\theta}_{el}}{p}$$
(11)

or with an equation as a function, that depends on the difference between the previous and the current values of the estimated active flux:



Fig. 3. Structure of the torque producing flux observer with current feedback.

$$\omega_{mech} = \left(\theta_{mech,n} - \theta_{mech,n-1}\right) / T, \qquad (12)$$

$$\omega_{mech} = \frac{1}{T} \cdot \frac{1}{p} \cdot \left[ \arctan\left(\frac{\Psi_{act,\beta_n}}{\Psi_{act,\alpha_n}}\right) - \arctan\left(\frac{\Psi_{act,\beta_{n-1}}}{\Psi_{act,\alpha_{n-1}}}\right) \right], \quad (13)$$

where *p* is number of pole pairs and *T* is the sampling time.

### A. Extension of observer model

In order to be able to improve the estimation results, a compensation of the observer is developed. Thereby, the uncertainty of the machine parameters is considered, e.g.: the non-linearity of the stator inductance  $L+\underline{L}$  and the change of resistance with temperature  $R+\underline{R}$ . Furthermore, the measurement accuracy could also affect the estimation results. The above-mentioned variables are defined as the disturbance variable of the observer system. Fig. 4 describes the principle of the disturbance variable compensation. At this, the estimated current  $\hat{i}_s^s = (\hat{i}_{\alpha_s}, \hat{i}_{\beta})^T$  is selected as the input variable of the compensator.

Afterwards, the estimated current is corrected. With the help of experiments, the correlation between the estimated current und the disturbance variable  $\xi$  can be simplified to

$$\xi(x) = k_1 x^2 + k_2 x . \tag{14}$$

The parameter  $k_1$  and  $k_2$  can be ascertained from measurement.



Fig. 4. Compensation of observer model.



Fig. 5. Experimental test platform.

# V. ESTIMATION AND CONTROL RESULTS

The parameters of the SPMSM used in the simulation and experiment are tabulated in Tab. I. In contrast to the interior permanent magnet synchronous motor (IPMSM), the stator inductance along quadrature axis and direct axis ( $L_q$  and  $L_d$ ) of SPMSM has the same values. The above presented methods were implemented for the SPMSM.

The experimental system setup and testing setup with hardware components are declared in Fig. 5. The above depicted sensorless control method is implemented and explicated on a dSPACE platform for the permanent magnet synchronous machines. Thereby, a three-phase asynchronous machine (ASM) is utilised as a load machine, which is controlled by an inverter in order to provide the desired torque. The dSPACE CLP1103 is used to control the drive system (Fig.5).

The stator currents and voltages are mearsured and these information are transmitted to the dSPACE platform. A torque gauge bar is installed on the shaft between the asynchronous machine and the PM machine for the torque measure.

The estimated rotor position is leaded to the PC, on which the Control Desk is installed for the control. The return of the PC is given back to the dSPACE again. The inverter inherits the aproval and the suitable signals, which are the inputs of the PM machine. During the experiment, the temperatur of the machine is monitored in order that the machine overheating can be avoided.

The estimation result of the current by using flux observer with current feedback without compensation at the speed of 30 rpm is shown in Fig. 6, where the measured current is illustrated. It can be seen that the chronological sequence of the estimated current is similar to the measured current. However, its peak value does not accord with the peak value of the measured current  $i_s^s$ .

This deviation cannot be rectified by the adjustment of the proportional control factor K (Fig. 3). The reason for this is that the estimated current and measured current  $i_s^s$  are coupled by the control factor K, the voltage model and the current model.







Fig. 7. Estimated current with compensation.



Fig. 9. Comparison of two methods.

Furthermore, the inaccurate parameters of the PMSM have negative impact on the estimated current and the estimated rotor position, which influence each other. It is intricate to minimize the estimation error only by changing the control factor K.

Fig. 7 illustrates the current estimation result by using compensation (Fig. 4), which does not strongly depend on the motor parameters variation. The negative impacts on the estimation are considered, e.g. the stator resistance change due to the motor temperature rise and influence of inductance variation. The estimation error is considerably minimized.

An incremental encoder was used to measure the rotor position which was considered as reference. The estimated rotor position and the measured rotor position are shown in Fig. 8. By comparison, although having a tiny time delay around 20 ms to measured rotor position.

The results of the developed sensorless speed control are shown in Fig. 9. Both of the approaches are stable at low speed. However, the controller with "flux feedback" results in overshoots and is even instable at the speed of 5 rpm. When compared to "flux feedback", the "current feedback" shows improved stability and performance at low speed.

The observer with "current feedback" provides better results in comparison to the one with "flux feedback". The reason behind is that the flux  $\hat{\Psi}_s^s$  (Fig. 2) is not directly measured by the "flux feedback" and it is calculated from the measured stator currents und voltages.

Because of this additional conversion, the values of flux  $\hat{\Psi}_s^s$  could actually differ from the real value. By "current feedback", the estimated current is compared to the measured current without further transformation (Fig. 3). This leads to less overlay error.

### VI. CONCLUSION

In this paper two torque producing flux (active flux) observer models for sensorless speed control of the surface mounted permanent magnet synchronous machines are presented.

An extension of the observer is developed in order to improve the estimation results. Thereby, the uncertainty of the machine parameters is considered und the estimation error is minimized.

The observer model with current feedback provides better result in comparison to the observer model with flux feedback and shows improved stability and performance at low speed.

#### **APPENDIX**

Transformation from *abc* to  $\alpha\beta$ :

$$\begin{pmatrix} I_{\alpha} \\ I_{\beta} \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} I_{a} \\ I_{b} \\ I_{c} \end{pmatrix} .$$
 (A1)

Transformation from  $\alpha\beta$  to dq:

$$\begin{pmatrix} I_d \\ I_q \end{pmatrix} = \frac{2}{3} \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix}.$$
 (A2)

Transformation from dq to  $\alpha\beta$ :

$$\begin{pmatrix} I_d \\ I_q \end{pmatrix} = \frac{2}{3} \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix}.$$
 (A3)

TABLE	I	
SPECIFICATIONS	OF	PMSM

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Parameters and constraints	Value
Number of pole pairs <i>p</i>	4
Maximum speed $n_{max}$	2000 [rpm]
Rated speed $n_N$	4500 [rpm]
Rated power $P_N$	10.3 [kW]
Rated phase to phase voltage $U_N$	380 [V]
Maximum permitted motor current Imax	75 [A]
Rated motor current $I_N$	21.2 [A]
Maximum torque $T_{max}$	67 [Nm]

Rated torque $T_N$	49.2 [Nm]
Mass moment of inertia J	60·10 <sup>-3</sup> [kg·m2]
Stator resistance $R_s$	0.2 [Ω]
Stator inductance (quadrature axis) $L_q$	0.005 [H]
Stator inductance (direct axis) $L_d$	0.005 [H]
Excitation flux $\psi_F$	0.2735 [Vs]
Time constant (quadrature axis)	0.025 [H/ Ω]
$t_q = L_q/R$	
Time constant (direct axis)	0.025 [H/ Ω]
$t_d = L_d / R$	
Coefficient of friction µ	0

The field oriented control is used in order to support the sensorless control for the permanent magnet synchronous machine. Thereby, the PID controller is implemented in the control system.

The stator currents  $i_q$ ,  $i_d$ ; the rotor speed  $\omega$  and the rotor position  $\theta$  are controlled separately. The differential equation of the ideal PID controller in parallel structure:

$$u(t) = K_p \cdot \left[ e(t) + \frac{1}{T_{reset}} \int_0^t e(\tau) d\tau + T_{rate} \frac{d}{dt} e(t) \right], \qquad (A4)$$

where  $K_p$  is the proportional gain,  $T_{rate}$  is the rate time and the reset time is  $T_{reset}$ . The PID controller can also be described as transfer function:

$$\frac{U(s)}{E(s)} = K_p \cdot \left[ I + \frac{1}{T_{reset} + s} + T_{rate} \cdot s \right].$$
(A5)

The controller parameters of the PID control for the sensorless field oriented control are listed in TABLE II.

Controlled variable	Controller parameters	Value
Stator current	Proportional gain $K_p$	1
(quadrature axis)	Rate time $T_{rate}$	0.25
$i_q$	reset time $T_{reset}$	0
Stator current	Proportional gain K <sub>p</sub>	1
(direct axis)	Rate time <i>T</i> <sub>rate</sub>	0.25
$i_d$	reset time $T_{reset}$	0
Rotor Speed	Proportional gain $K_p$	4.54
ω	Rate time <i>T</i> <sub>rate</sub>	0
	Reset time <i>T</i> <sub>reset</sub>	0.09
Rotor position $\theta$	Proportional gain $K_p$	20
	Rate time $T_{rate}$	0

 TABLE II

 CONTTOLER PARAMETERS OF PID CONTROLLER

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#### BIOGRAPHIES

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