Non-conforming Sliding Interfaces for Motion in 3D Finite Element Analysis of Electrical Machines by Magnetic Scalar Potential Formulation Without Cuts

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Abstract— **This paper discusses non-conforming sliding interfaces for motion in combination with a magnetic scalar potential formulation. Lagrange multiplier are used to implement the relative motion of stator and rotor. The utilization of the specific Lagrange multiplier approach implies the application of a magnetic scalar potential formulation in 3D Finite Element (FE) modelling of electrical machines.**

I. Introduction

NOWADAYS several approaches for handling relative motion of stator and rotor in FE analysis of electrical machines are available [1],[2]. Static, transient and especially field coupling simulations of electrical machines require the flexible displacement of the rotor by an arbitrary angle in rotating machines or a distance in translational electric machines. In 2D FE modelling the Moving-Band method can be employed where an annulus-shaped band in the airgap between rotor and stator is remeshed in every timestep. In 3D the Lockstep method [1] is usually applied which is based on a regular discretization of the rotor and the stator surfaces, so that the motion can be described by associating the nodes of the rotor surface by a discrete shift with those of the stator surface. The major disadvantage of this method is the lack of arbitrary displacement because step size is fixed by the discretization. In consequence, a smooth movement leads to a significant increase in the number of elements resulting in an increase of computing time which is highly undesireable. Lagrange multiplier approaches seek to overcome the disadvantages being applicable to 2D as well as 3D problems [3].

II. Motion by Lagrange multiplier

Lagrange multiplier approaches allow the independent discretization of the stationary and moving region of the FE mesh which results in a non-conforming interface between stator and rotor region. As a consequence an arbitrary displacement is possible without restrictions in time or space discretization.

In general, the application of Lagrange multiplier methods yield a saddle point problem, which can not be solved by the usually used krylov subspace algorithms. In order to obtain a symmetric positive definite (SPD) problem instead of the standard basis functions for the Lagrange multiplier, the method utilizes special biorthogonal basis functions as described in [3] and applied to electromagnetic field computation in [2]. Thereby, the resulting SPD system can be solved by krylov subspace algorithms. Previous studies have indicated that it is not feasible to construct such biorthogonal basis function for the magnetic vector potential formulation in a canonical way but for magnetic scalar potential $T-\omega$ formulation [2]. Hence, the implemented $T-\omega$ formulation is described in the following section.

III. Topological Structure

In classical $T-\omega$ formulation the field T is computed in the conducting region only whereas ω is computed in the whole domain [4]. This approach requires the definition of cuts in the non-conducting domain for multiple connected regions because otherwise ω may become multi-valued. In order to avoid these cuts the approach in this paper consists in decomposing the magnetic field h appropriately, which theoretical background has been presented in [5].

Let Ω be a connected mesh, $C \subset \Omega$ the domain of all conductors and ∂C its boundary. Furthermore, let $W^p(\Omega)$ be the set of differential forms of degree p. While div $b = 0$ can be satisfied by introduction of the magnetic vector potential a with $b =$ curl a, the situation is more complicated for curl $h = 0$. The topological structure of the functional space $W^1(\Omega - C)$ has to be considered. Let B^1 be the set of all gradients and Z^1 the set of all curl-free fields on $\Omega - C$. Following De Rham's theorem the quotient vector space $H^1 = Z^1/B^1$ is of finite dimension and equals the number of holes in $\Omega - C$, respectively the number of independent loops N_Σ formed by the conductors.

The magnetic field in $\Omega - C$ can then be expressed by

$$h = \sum_{k=0}^{N_\Sigma} I_k T^k + \text{grad } \omega \tag{1}$$

where I_k is the current in conductor loop k, the fields T^k form a basis for H^1 and ω is a continous scalar potential.

IV. Conclusions

The presented approach allows the 3D simulation of electrical machines including motion by a magnetic scalar potential formulation without cuts. The full paper will contain a detailed description of the $T-\omega$ formulation and the algorithms to construct the T fields. Furthermore, the implemented formulation in the institute's in-house FE-package iMOOSE [www.iem.rwth-aachen.de] is applied to a 3D electrical machine field problem.

References

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