NON-LINEAR STOCHASTIC VARIATIONS IN A MAGNET EVALUATED WITH MONTE-CARLO SIMULATION AND A POLYNOMAIL CHAOS META-MODEL

Peter Offermann and Kay Hameyer

RWTH Aachen University – Institute of Electrical Machines
Schinkelstr. 4, 52062 Aachen, Germany, e-mail: Peter.Offermann@IEM.RWTH-Aachen.de

***Abstract*—Due to the production process and magnetization, arc segment magnets with radial magnetization for surface-mounted permanent-magnet synchonous machines (PMSM) can exhibit a deviation from the intended ideal, radial directed remanence induction. In such cases, the resulting air gap field may show spatial variations in angle and absolute value of the flux-density. Typical solutions to consider these stochastic variations within the finite element method are worst case estimations and Monte-Carlo simulation [1]. In this paper, a spectral stochastic finite element approach is compared to Monte-Carlo simulation for the calculation of the flux-density above one sole magnet surface. Input distributions are assumed to be Gaussian distributed. The used approach allows representing the flux-density's variations in terms of the magnet's stochastic input variations, which is not possible with pure Monte-Carlo simulation. Furthermore, the resulting polynomial-chaos meta-model can be used to accelerate the calculation of error probabilities for a given limit state function by a factor of ten.**

I. Introduction

The air gap field's shape of a permanent-magnet excited rotor strongly can influence phase and absolute value of certain torque harmonics in PMSMs. For some designs, ideal radial magnetization is desired as it creates a spatial constant field in the air gap. Due to the magnetization process of arc segmet magnets, the desired ideal magnetization is not attainable for all magnets.

Typical solutions to handle the magnet's stochastic variations, introduced by the production respectively magnetization process, have been robust machine design with worst case estimation and Monte-Carlo simulation as applied in [1]. The drawback of these approaches is that they only allow the calculation of error probabilities by counting result samples which fullfill of fail in the sense of the applied limit state function. Expressing the output size's stochastic deviations in terms of the input size's stochastic variations is not possible. Moreover, Monte-Carlo simulation often requires a large number of simulations for an acceptable error accuracy.

In this paper, a polynomial chaos meta-model is compared to a typical Monte-Carlo simulation to calculate the influence of realistic production deviations onto the magnet's created flux-density. In order to separate field changes caused by stochastic variations in the magnet from field changes caused by interaction with the machine's stator yoke, the field of one sole magnet is simulated and evaluated directly above its surface.

300 Simulation have been used for the Monte-Carlo sampling as well as 30 Sampling points for the construction of the polynomial meta-model, which then itself has been Monte-Carlo sampled with the same input samples as the original model. The resulting meta-model closely represents the original model as proven in the assessment of the printed cumulative distribution function (CDF), allowing to express the field's stochastics in terms of the input variation stochastics while requiring approximately only one 10th the sampling size.

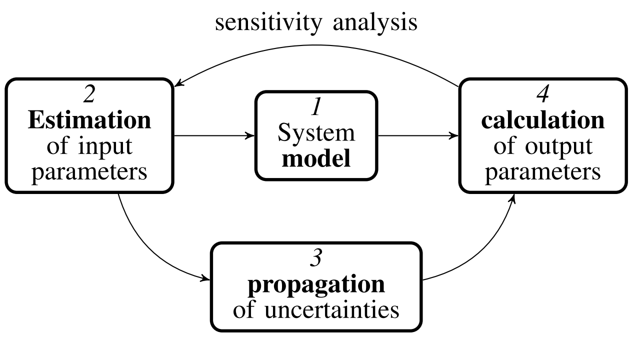


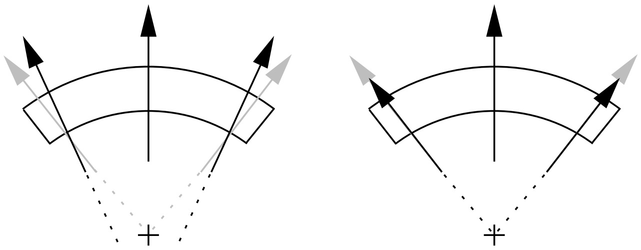
Fig. 1: Chosen approach for uncertainty propagation as proposed in [4].

II. Methodology

Figure 1 shows the chosen approach for the propagation of the magnet's uncertainties: In step 1, the magnet has been modelled using the finite element method (see Fig. 3 for results). The created model allows two possible error configurations:

1) Magnetization errors tending from radial magnetization towards an unidirectional magnetization as illustrated in figure 2(a). The implementation allows for an arbitraty error between both extrems, $\alpha = 0$ representing complete unidirectional magnetization, $\alpha = 1$ representing ideal radial magnetization.
2) Spatial changing remanence induction magnitude, shaped decreasingly from the magnet middle to the magnet border as depicted in figure 2(b), $\beta = 0$ representing a sinusoidal shaped remanence induction, $\beta = 1$ representing an ideal uniform value for the remanence induction over the entire magnet surface.

In step 2, the input distributions for both error cases were estimated. The brochure [3] gives error boundaries for the minimal garantueed magnet's remanence induction in comparison to the typcial achieved remanence induction. The differences between minimal and typical induction vary there from 2% to 5%. Both error cases therefore have been chosen to be normal distributed in a way, that the maximum error of the radial component (located at a spread of 3σ) has been allowed to be 1.5%. Gaussian distributions appear as



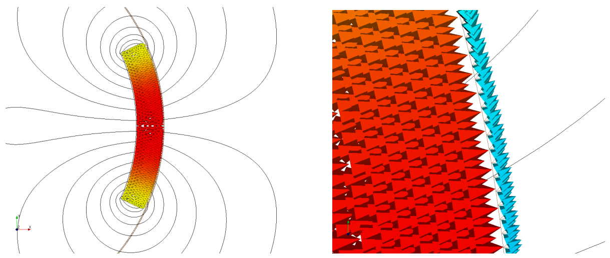
(a) Error type A: deviation of radial magnetization towards undirectional magnetizationd.

(b) Error type B: deviation of local magnetization strength, weakening towards the magnet edge.

Fig. 2: Considered variations (black) in magnet in contrast to ideal magnetization (grey).

a suitable first choice since they are often used in this context and because a good production process should reproduce similar results, with larger deviations being more unlikely than smaller variations. For the propagation of uncertainties in step 3, Monto-Carlo simulation based on the model has been executed. Afterwards a polynomial chaos meta-model (see [2]) has been built from a subset of the Monte-Carlo samples. This meta-model then again was used with Monte-Carlo sampling for the creation of comparable results. As output parameter of interest (step 4), the flux-density above the magnet has been evaluated at 50 1°-steps above the magnet surface.

III. Results



(a) Field plot showing a magnet of error type B.

(b) Zoom into field plot, showing the used evaluation line outside the magnet.

Fig. 3: Field plots of the studied magnet geometry.

Figure 3 shows the resulting field for a magnet of error type B. Since only one sole magnet is considered, the magnet's flux shortens into the same magnet again, leading to higher field densities at the magnet's edge compared to the magnet middle. This can be observed in figure's 3(a) flux lines as well as in the evaluation of the radial field component along the evaluation line (Fig. 3(b)). Figure 4 shows this evaluation for the ideal magnet as well as for a worst case constellation (3σ deviation in both parameters). Figure 5 finally shows the flux-density's cumulative distribution function at the magnet's middle, calculated directly from the model and from the polynomial-chaos meta-model. The graphs of the model and meta-model however overlap so smooth, that only for narrow zooms two curves are visible.

IV. Conclusion

The non-linear stochastic variations of two likely magnet errors have been presented along with their influence onto the flux-density above the magnet's surface. A polynomial-chaos meta-model has been created from simulations for

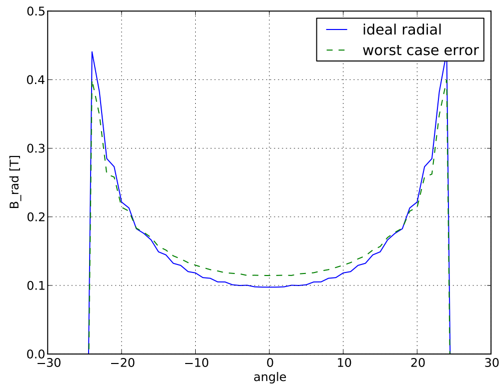


Fig. 4: Comparison of the radial flux-density along the chosen evaluation line for ideal flux density and worst case flux density (dashed).

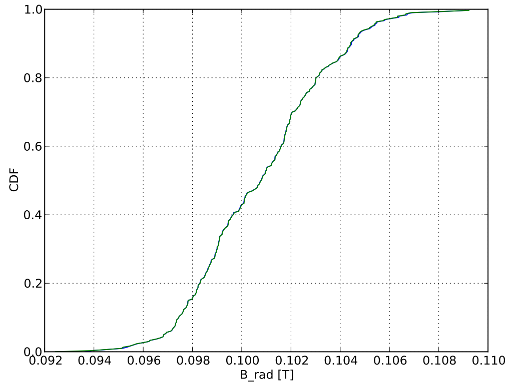


Fig. 5: Cumulative distribution function (CDF) of the flux-density at the evaluation point at the magnet's middle (angle 0°).

the resulting flux density along the evaluation line and has been and compared to the simulation results themselves. The meta-model closely fits the simulation data. Therefore, the considered approach will be used in a next step to estimate the influences of several magnet variations on a rotor onto machine properties as induced voltage, cogging torque, etc.. Furthermore the assumed error distribution functions have to be verified in future work.

V. Acknowledgement

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