# Calculation of the flux distribution of three phase five limb power transformers considering nonlinear material properties

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***Abstract* - This paper describes an approach to calculate the flux distribution inside the core of power transformers for arbitrary working points. The method is based on a linearised lumped parameter representation of the transformer. The lumped parameters are extracted from a 2D or 3D Finite Element model of the transformer. Nonlinear material properties of the transformer core are taken into account. To achieve a numerical stable behaviour the magnetic flux linkage is taken as a state variable of the coupled simulation problem.**

## I. INTRODUCTION

The power flow in the energy transmission grid can be controlled by phase-shifting transformers (PST). Different principles and applications are given in [1]. In PSTs of the two core design especially the stray flux of the exciter transformer, caused by load currents, is predominantly in phase or in phase opposition to the exciting flux of the core, similar to the conditions in regular transformers under capacitive or inductive load. Due to the superposition of exciting and stray flux significant local variations of the core fluxes may occur. Since saturation of the core definitely has to be avoided possible load-dependent local flux variations have to be considered already in the design stage.
This paper describes an approach to investigate the flux distribution inside the transformer core using a linearised lumped parameter model of the transformer under various load conditions. The model is based on a lumped inductance matrix representation, whose values are extracted by means of the FEM whenever the magnetic energy within the transformer changes notably. The approach is applied to a three phase five limb exciter transformer of a PST considering the nonlinear behaviour of the transformer core.

## II. THEORETICAL FRAMEWORK

This section describes the theoretical framework of the electromagnetic calculations. It is described how to determine the operating point of a transformer considering nonlinear material properties and furthermore how the flux distribution inside the transformer core is calculated. The transformer is represented by linearised lumped parameter and therefore by the differential equations

$$\mathbf{u}(t) = \mathbf{R} \cdot \mathbf{i}(t) + \partial_t \mathbf{\Psi}(\mathbf{i}(t)), \qquad (1)$$

with the phase voltages $\mathbf{u}$, the winding resistance matrix $\mathbf{R}$, the phase currents $\mathbf{i}$, and the flux linkage matrix $\mathbf{\Psi}$. The inductance matrix is extracted systematically using a method described in [2,3]. Based on the parameter extraction subsection *A* describes a model for the nonlinear calculation of the working point. Subsection *B* describes the calculation of the magnetic flux distribution inside the transformer core.

### *A. Nonlinear calculation of the working point*

Considering nonlinear material properties the inductance matrix depends on the working point, i.e. the load, and thus on the saturation state of the transformer core. This yields a coupled simulation problem. The differential equation system of the electric circuit equation (1) can be solved by using a lumped parameter model representing the field problem. The field problem is solved by a computationally expensive Finite Element calculation. The extracted lumped parameters from the FEM are updated with an arbitrary update rate of $\Delta T_{FE}$ which is lower than the circuit problem time step. Figure 1 depicts a flowchart of the weak coupled simulation.

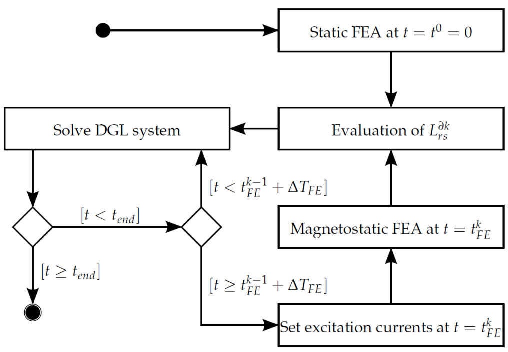



Fig.1. Flowchart of the weak coupled simulation.

Initially, the tangent inductance matrix ( $L_{rs}^{\partial k} = \left.\frac{\partial \Psi_r}{\partial i_s}\right|_{t_{FE}^k}$ ) is extracted by a static Finite Element Analysis. The circuit problem is solved for a time interval up to the update rate of the field problem. At the update point the excitation currents which are calculated by the circuit equations are set and an updated tangent inductance matrix is extracted and the circuit problem is solved for the next time interval. The coupled simulation ends if a defined end time $t_{end}$ is reached.

Taking the phase current as state variable for the coupled problem, the circuit equation system (1) for a transformer in no load operation yields

$$\partial_t \mathbf{i} = \mathbf{L}^{\partial -1}[\mathbf{u} - \mathbf{Ri}]. \qquad (2)$$

For this model there is no feedback between flux linkages calculated by the circuit problem ($\mathbf{\Psi}=\mathbf{L}^{\partial}\mathbf{i}$) and the field problem ( $\boldsymbol{\Psi} = \int_{\Omega} \mathbf{w}_r \cdot \mathbf{a}$ , with the current shape function $\mathbf{w}_r$, the coil domain $\Omega$ and the magnetic vector potential **a**). The linearisation of the tangent inductance matrix leads to an error which propagates in time. Due to the missing feedback the simulation will diverge as it is depicted in Figure 2. The Figure depicts the induced voltage in one phase of the transformer at starting operation. To decrease the transient phenomenon the magnitude of the exciting voltage is increased with an exponential function. At t ≈ 150ms the simulation diverges. The flux linkage and consequentially the induced voltage calculated by the Finite Element model exceed the induced voltage calculated by the circuit problem, because of erroneously impressed currents.

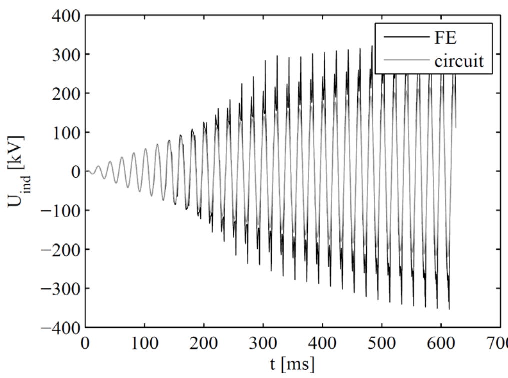



Fig.2. Comparison of the induced voltages of one phase.

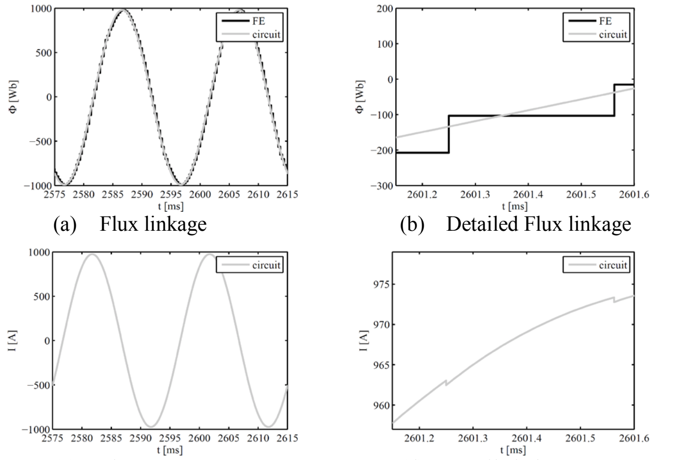
A stable behaviour for the coupled problem can be achieved by taking the flux linkage as state variable. At each extraction step the tangent inductance matrix and the flux respectively the flux linkage $\boldsymbol{\Psi}\big|_{t_{FE}^k}$ is extracted from the magneto static Finite Element analysis at the time step $t_{FE}^k$ . The circuit equation yields

$$\int_{-\infty}^{t} (\mathbf{u} - \mathbf{R}\mathbf{i})dt = \boldsymbol{\Psi}\big|_{t_{FE}^k} + \frac{\partial \boldsymbol{\Psi}}{\partial \mathbf{i}}\bigg|_{t_{FE}^k} \left(\mathbf{i}(t) - \mathbf{i}\big|_{t_{FE}^k}\right), \qquad (3)$$

and consequently

$$\mathbf{i}(t) = \left(\frac{\partial \boldsymbol{\Psi}}{\partial \mathbf{i}}\right)^{-1}\Bigg|_{t_{FE}^k} \left(\int_{-\infty}^{t} (\mathbf{u} - \mathbf{R}\mathbf{i})dt - \boldsymbol{\Psi}\big|_{t_{FE}^k}\right) + \mathbf{i}\big|_{t_{FE}^k} . \qquad (4)$$

The current delta is calculated on basis of the flux linkages of the circuit equation and of the Finite Element model. At each FE extraction point $t_{FE}^k$ the deviation of the flux linkage from the circuit problem and the FE problem is reset. Thus, discontinuities in the current characteristic are possible. Figure 3 depicts the flux linkage and the current of one phase of the transformer. The discontinuities at the FE extraction points can be used as an error indicator to control the step width of the FE extraction points.




(a) Flux linkage (b) Detailed Flux linkage

(c) Phase current (d) Detailed phase current

Fig.3. Flux linkage and current considering flux as state variable.

### *B. Magnetic flux distribution inside the transformer core*

The flux is determined by the circulation of the magnetic vector potential over a closed contour C

$$\varphi = \oint_C \mathbf{a} dC . \qquad (5)$$

Since the vector potential solution is available for each extraction point equation (5) can directly be applied to the field solution of the working point.

## III. CONCLUSIONS

This paper presents an approach for the calculation of the flux distribution of power transformers considering nonlinear material. The calculation is based on a linearised lumped parameter representation of the transformer. By using the magnetic flux linkage as a state variable a numerical stable behaviour is obtained to accurately determine the transformer's working point. The full paper will present extended results of the flux density distribution of a three phase five limb power transformer considering various load cases. The flux density distribution will be compared using linear and non-linear material properties, as well as 2D and 3D Finite Element models of the transformer.

## REFERENCES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