Extension of a D-Q Model of a Permanent Magnet Excited Synchronous Machine by Including Saturation, Cross-Coupling and Slotting Effects

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***Abstract*—For the description of a permanent magnet excited synchronous machine the idealized equations of common synchronous machines can be used omitting the damping windings and replacing the current excitation by a permanent flux. For a more accurate modeling, nonideal behavior like saturation, cross-coupling magnetization and slotting effects can be included. This is done by extending the common d-q equations by additional elements. This also includes the time dependent variations of the parameters causing additional induced voltages especially at high currents and speed. The elements of the extended equations are extracted from a finite element analysis of the device under test. The derivation of the equations and a simulation of a permanent magnet excited synchronous machine with the applied theory are presented.**

***Index Terms*—Permanent magnet machines, Simulation, Modeling**

LIST OF SYMBOLS

v	Motor voltage.
i	Motor current.
γ	Electrical angel.
ω	Rotational speed.
L	Inductance.
ψ	Flux linkage.
T	Torque.
J	Transformation matrix.
R	Ohmic resistance.
W_m	Magnetic co-energy.
∂	Differential operator.

INDICES

*	Elements of idealized model.
N	Nominal value.
d	Relation to direct axis.
q	Relation to quadrature axis.
dd,qq	Self inductance in d-q reference frame.
dq,qd	Mutual inductance in d-q reference frame or relation to the d-q reference frame if use with a matrix or a vector.
f	Relation to excitation flux.
i	Relation to motor current.
32	Transformation to 2-axis frame.
α	Transformation to rotating system.
t,γ	Derivative with respect to .. .
$i_\mathrm{d}, i_\mathrm{q}$	

I. INTRODUCTION

For the description of a permanent magnet excited synchronous machine (PMSM) the idealized equations of common synchronous machines can be used omitting the damping windings and replacing the current excitation by a permanent flux. Even an anisotropic construction can be included in the description by introducing the d-q inductances. But there are some constraints using this idealized d-q model. In [1] it is shown that there are limitations by applying the d-q theory to machines with concentrated windings. Only if the parasitic effects are small compared to the fundamental wave, this kind of modeling gives accurate results. In order to deal with saturation and cross-saturation an extension of the d-q model by including cross-coupling elements and current dependent inductances is suggested in [2]. For example this extended description becomes important for the simulation of encoderless control strategies [3]. Furthermore, machine faults or manufacturing tolerances that have influence on the machine bahaviour can be simulated. This can be used for the training of condition monitoring systems. A further aspect causing flux harmonics in the machine are the position dependent inductances and excitations (slotting effects). Since these influences are based on local effects they are difficult to describe with a pure analytical approach. In [4] it is suggested to use the finite element analysis (FEA) routinely for the calculation of the reactances. It is possible to extract the inducivities, the back-emf, and the torque of the machine and build up a model in the n-phase stator system. Another possibility is the transformation of the extracted lumped parameters to the d-q reference frame. This has the advantage of keeping the familiar model structure, which also is used for the design of the machine control. To include the aforementioned nonideal effects of the machine into a d-q model, the common description has to be extended.

This paper shows the approach of extending the d-q description of a PMSM to include saturation, cross-coupling and slotting effects. After the mathematical derivation of the extended machine equations the necessary parameters are extracted from an FEA and transformed to the required format. Here, an intention is to keep the model flux-based as far as possible. The characteristics of the designed model are shown by simulation results. A comparison to a more simple

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approach including nonideal effects to a d-q model is given afterwards before the paper is concluded.

II. Idealized model of a PMSM

The common description of a synchronous machine is the idealized d-q-model taking the fundamental wave into account only. In case of a permanent magnet excitation the damping windings are omitted since the machine is operated with a converter and the excitation flux is represented by a constant term. The voltage equations can then be written as:

$$\begin{aligned} v_\mathrm{d}^* &= Ri_\mathrm{d}^* + L_\mathrm{d}^*\partial_\mathrm{t} i_\mathrm{d} - \omega L_\mathrm{q}^* i_\mathrm{q} \\ v_\mathrm{q}^* &= Ri_\mathrm{q}^* + L_\mathrm{q}^*\partial_\mathrm{t} i_\mathrm{q} + \omega L_\mathrm{d}^* i_\mathrm{d} + \omega\psi_\mathrm{f}^* \quad . \end{aligned} \tag{1}$$

The torque of the machine is expressed by

$$T^* = \frac{3}{2}p(\psi_\mathrm{d}^* i_\mathrm{q} - \psi_\mathrm{q}^* i_\mathrm{d}) = \frac{3}{2}p[\psi_\mathrm{f}^* - (L_\mathrm{q}^* - L_\mathrm{d}^*)i_\mathrm{d}]i_\mathrm{q} \quad . \tag{2}$$

where p is the number of polepairs. All nonideal effects of the machine influencing the airgap flux are neglected since only ideal sine waves are assumed in the common description.

III. Extended d-q model

To take account for the nonideal behavior of the machine a bottom-up analysis is performed. Therefore, the voltage equation

$$u(t) = Ri(t) + \partial_\mathrm{t}\psi(t) \tag{3}$$

is solved and transformed to the d-q reference frame. In a first step the overall flux is divided up into the current (index i) and the excitation (index f) based fluxes

$$\psi(t) = \psi_\mathrm{i}(t) + \psi_\mathrm{f}(t) = \psi_\mathrm{i} e^{j(\gamma_\mathrm{i}(t)+\gamma_\mathrm{i,0})} + \psi_\mathrm{f} e^{j(\gamma_\mathrm{f}(t)+\gamma_\mathrm{f,0})} \tag{4}$$

with γ as a general position angle (electrical). In the following the time dependency is omitted. By applying the differential operator to (4) one gets:

$$\begin{aligned} \partial_\mathrm{t}\psi =& (\partial_\mathrm{t}\psi_\mathrm{i})e^{j(\omega_\mathrm{i,0}t+\gamma_\mathrm{i})} + \psi_\mathrm{i}(\partial_\mathrm{t} e^{j(\omega_\mathrm{i}t+\gamma_\mathrm{i,0})}) \\ &+ (\partial_\mathrm{t}\psi_\mathrm{f})e^{j(\omega_\mathrm{f}t+\gamma_\mathrm{f,0})} + \psi_\mathrm{f}(\partial_\mathrm{t} e^{j(\omega_\mathrm{f}t+\gamma_\mathrm{f,0})}) \\ =& (\partial_\mathrm{t}\psi_\mathrm{i})e^{j(\omega_\mathrm{i}t+\gamma_\mathrm{i,0})} + j\omega_\mathrm{i}\psi_\mathrm{i} e^{j(\omega_\mathrm{i}t+\gamma_\mathrm{i,0})} \\ &+ (\partial_\mathrm{t}\psi_\mathrm{f})e^{j(\omega_\mathrm{f}t+\gamma_\mathrm{f,0})} + j\omega_\mathrm{f}\psi_\mathrm{f} e^{j(\omega_\mathrm{f}t+\gamma_\mathrm{f,0})} \quad . \end{aligned} \tag{5}$$

Since the aim of the paper is the creation of a d-q model and the investigated device is a synchronous machine it is reasonable to align (5) to the rotor reference frame (multiplication by $J_\alpha = e^{-j(\omega_\mathrm{f}t+\gamma_\mathrm{f,0})}$ and $\omega_i = \omega_f = \omega$) and project it to the d-q coordinate system (Fig. 1):

$$\begin{aligned} J_\alpha\partial_\mathrm{t}\psi =& \partial_\mathrm{t}\psi_\mathrm{d,i} - \omega\psi_\mathrm{q,i} + \partial_\mathrm{t}\psi_\mathrm{d,f} - \omega\psi_\mathrm{q,f} \\ &+ j(\partial_\mathrm{t}\psi_\mathrm{q,i} + \omega\psi_\mathrm{d,i} + \partial_\mathrm{t}\psi_\mathrm{q,f} + \omega\psi_\mathrm{d,f}) \quad . \end{aligned} \tag{6}$$

As from now the common tensor notation is used instead of the complex one. In the next step the d-q inductances are introduced including the cross-coupling elements by

$$\boldsymbol{\psi}_\mathrm{dq,i} = \begin{pmatrix} \hat{\psi}_\mathrm{d,i} \\ \hat{\psi}_\mathrm{q,i} \end{pmatrix} = \boldsymbol{L}_\mathrm{dq}\boldsymbol{i}_\mathrm{dq} = \begin{pmatrix} L_\mathrm{dd} & L_\mathrm{dq} \\ L_\mathrm{qd} & L_\mathrm{qq} \end{pmatrix} \begin{pmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{pmatrix} \quad . \tag{7}$$

Here, a system without zero-sequence system is assumed. If there is a zero-sequence system the corresponding elements have to be added to (6). With (3), (6), and (7) the electrical system of the PMSM can be written as

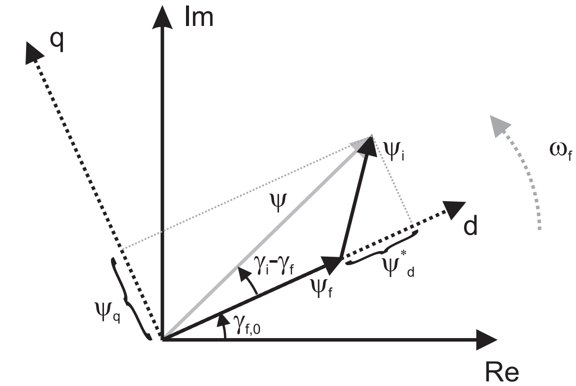

Fig. 1. Transformation to the rotating d-q reference frame ($\gamma_\mathrm{f,0} = \gamma_0$).

$$\begin{aligned} v_\mathrm{d} =& Ri_\mathrm{d} + \partial_\mathrm{t}(L_\mathrm{dd}i_\mathrm{d} + L_\mathrm{dq}i_\mathrm{q}) \\ &-\omega(L_\mathrm{qd}i_\mathrm{d} + L_\mathrm{qq}i_\mathrm{q}) + \partial_\mathrm{t}\psi_\mathrm{d,f} - \omega\psi_\mathrm{q,f} \\ v_\mathrm{q} =& Ri_\mathrm{q} + \partial_\mathrm{t}(L_\mathrm{qd}i_\mathrm{d} + L_\mathrm{qq}i_\mathrm{q}) \\ &+\omega(L_\mathrm{dd}i_\mathrm{d} + L_\mathrm{dq}i_\mathrm{q}) + \partial_\mathrm{t}\psi_\mathrm{q,f} + \omega\psi_\mathrm{d,f} \quad . \end{aligned} \tag{8}$$

The time dependency of the magnitudes can now be calculated by applying the total derivative to (8). The time depending state variables are defined to

$$\boldsymbol{x} = (\gamma \quad i_\mathrm{q} \quad i_\mathrm{d}) \quad \text{with} \quad \gamma = \omega_\mathrm{f}t + \gamma_0 \quad . \tag{9}$$

Exemplary, here the derivative of the direct inductance depending on $\mathbf{x}$ is shown:

$$\begin{aligned} \partial_\mathrm{t}(L_\mathrm{dd}(\gamma, i_\mathrm{q}, i_\mathrm{d})i_\mathrm{d}) =& L_\mathrm{dd}\partial_\mathrm{t} i_\mathrm{d} \\ &+ i_\mathrm{d}(\partial_\mathrm{t}\gamma\partial_\gamma L_\mathrm{dd} + \partial_\mathrm{t} i_\mathrm{d}\partial_{i_\mathrm{d}} L_\mathrm{dd} + \partial_\mathrm{t} i_\mathrm{q}\partial_{i_\mathrm{q}} L_\mathrm{dd}). \end{aligned} \tag{10}$$

Compared to (1) the extended d-q voltage equations contain much more terms if all elements of (8) are expanded like (10) to represent the nonideal effects of the machine.

IV. Determination of the model parameters

The required magnitudes for the model parametrization, i.e. the inductances and the excitation fluxes, are extracted from a finite element analysis (FEA) of the machine.

A. Inductances

The full 3x3 inductance matrix is extracted like shown in [5]. For the usage in the extended d-q description this matrix has to be transformed to the 4 element d-q matrix (7). This is done by solving the flux equation (11).

$$\begin{aligned} \boldsymbol{\psi}_\mathrm{dq,i} =& \boldsymbol{L}_\mathrm{dq}\boldsymbol{i}_\mathrm{dq} = \boldsymbol{L}_\mathrm{dq}\boldsymbol{J}_\alpha\boldsymbol{J}_{32}\boldsymbol{i} \\ =& \boldsymbol{J}_\alpha\boldsymbol{J}_{32}\boldsymbol{L}\boldsymbol{i} = \boldsymbol{J}_\alpha\boldsymbol{J}_{32}\boldsymbol{\psi}_\mathrm{i} \\ \Rightarrow \boldsymbol{L}_\mathrm{dq} =& \boldsymbol{J}_\alpha\boldsymbol{J}_{32}\boldsymbol{L}(\boldsymbol{J}_\alpha\boldsymbol{J}_{32})^{-1} \quad . \end{aligned} \tag{11}$$

In Fig. 2 a parameter field of L_dd is shown with variation of the rotor angle and the current i_q, whereas i_d is constant equal zero. The oscillation in γ direction is based on the slotting effect. The saturation is reflected by the decrease of

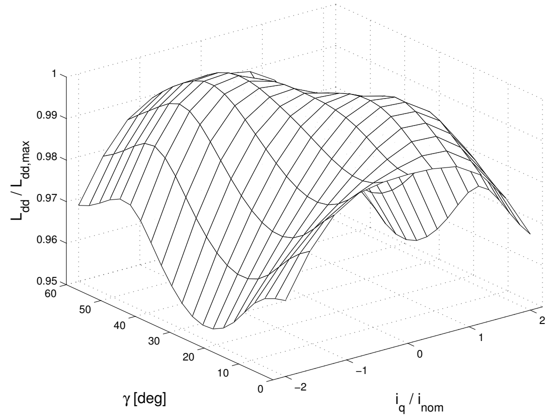

Fig. 2. Saturation and slotting effects observable in the inductances L_{dd} of a PMSM with concentrated windings (related values).

the inductance with increase of the load current (absolute value). The parameter field dependent on i_{d} is shown in Fig. 3. The magnitude of the slotting influence is almost the same as in Fig. 2 but the overall magnitude varies significantly with the magnetizing current. The inductance decreases strongly with positive current. This has its reason in the magnetizing direction, which in this case is directed with the excitation flux and leads to a higher saturation level compared to currentless state. The contrary effect is observable at negative current. Up to the negative nominal current the inductance increases since the magnetization affects against the excitation and therefore decreases the saturation level. Beyond the peak the inductance decreases again because of other local saturation effects.

B. *Excitation flux*

The excitation flux is evaluated by freezing the saturation level of the machines material in ervery element of the finite element model at the actual point of operation. Then, a linear FEA problem based on this material properties is solved with all phase currents equal to zero. The phase fluxes can simply be read after this solving process and determined to

$$\boldsymbol{\psi}_{\mathrm{f}} = \boldsymbol{J}_{\alpha} \boldsymbol{J}_{32} \boldsymbol{\psi}_{\mathrm{dq,f}} \quad . \tag{12}$$

The parameter field of the excitation $\psi_{\mathrm{d,f}}$ is shown in Fig. 4. Here the same influences can be observed as in the in inductance in Fig. 2.

C. *Torque*

The torque of the machine can be described with (2) by adding a further term [6]:

$$T = \underbrace{\frac{3}{2} p(\psi_d i_q - \psi_q i_d)}_{T^*} + \partial_\gamma W'_m \tag{13}$$

that can be calculated by

$$\partial_\gamma W'_m = \frac{3}{2} p(\partial_\gamma(\psi_d id + \psi_q iq)) - \partial_\gamma W_m \quad . \tag{14}$$

In this paper the additional term is determined by calculating the difference between the torque derived form the FEA and T^* which means that the overall torque is exactly the same as the extracted FEA torque. This does not suit the desired modeling methodology since all other magnitudes are derived from extracted or calculated fluxes. Besides that this term can give a validation of the modeling accuracy when compared to the FEA torque. In a future work that will be taken into account.

V. DYNAMIC MODEL

When all parameters are extracted they have to be included in a simulation environment. The used environment is Matlab/Simulink™. The parameters can be stored in lookup tables within a simulink model. The derivatives are calculated in advance. This is done by differentiating a spline interpolation of the parameter fields (e.g. $L_{\mathrm{dd}}(\gamma, i_{\mathrm{q}}, i_{\mathrm{d}})$). Afterwards the derivatives can also be included in lookup tables. This approach has the advantage of avoiding finite differences for the differentiation that can lead to numerical instability.

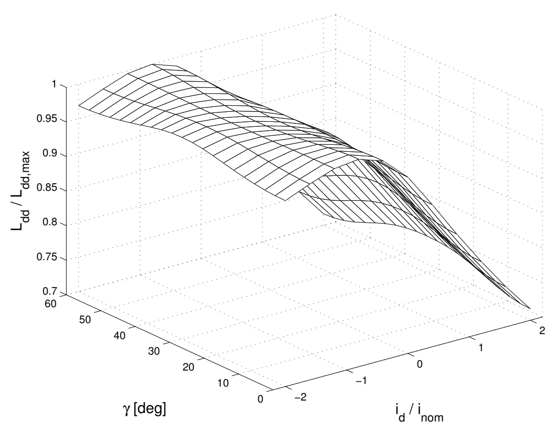

Fig. 3. Saturation and slotting effects observable in the inductances L_{dd} of a PMSM with concentrated windings (related values).

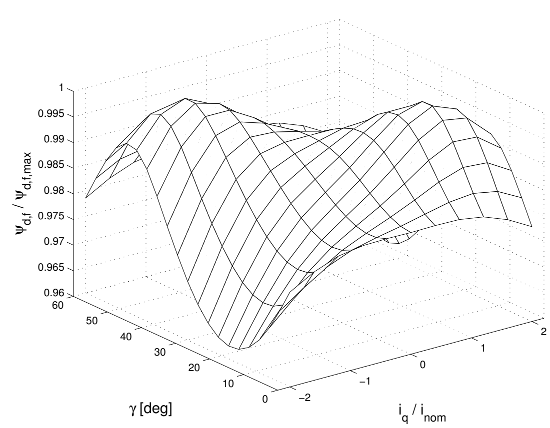

Fig. 4. Saturation and slotting effects observable in the excitation flux $\psi_{\mathrm{d,f}}$ of a PMSM with concentrated windings (related values).

VI. Simulation results

As an example to proof the concept a simulation of a PMSM servo with the given data in table I is performed. It has concentrated windings and the number of slots per pole and phase is $q = \frac{1}{2}$.

Stator resistance	R	$0.9\,\Omega$
Inductance (direct axis)	L_{dd}	0.4 mH
Inductance (quadrature axis)	L_{qq}	0.4 mH
Mass of inertia (overall drive train)	J	$2.69\,\mathrm{kgcm}^2$
Number of pole pairs	p	3
Number of slots	p	9
Nominal speed	T_N	3000 rpm
Nominal torque	T_N	8.4 Nm
Nominal current	I_N	9.3 A

TABLE I
Data of the simulated machine.

The machine is controlled by a PI-cascade (speed and current) with an adjusted load angel equal to zero. Two scenarios are investigated: a startup operation for the dynamical aspects of the model and a load case with constant nominal current at nominal speed for a harmonic analysis. The results are compared to a simulation with an idealized model as described in section II that exactly the same machine parameters as the extended model at currentless condition and position equal zero. The control is also the same for both models.

A. Startup operation

Fig. 5 shows the direct current at the startup operation. The simulation of the idealized model shows a direct current of exactly 0 A, which is clear since the control has a feedforward part that eliminates the influence of i_{q} to the direct axis. In contrast, the extended model has a strong deviation form zero in the direct current based on several influences like the fluctuation of L_{qq} (Fig. 6). This behavior, i.e. saturation, was expected considering the parameter field in Fig. 2. The slotting effect (oscillation in the signals) is also observable. The amplitude of the oscillation is changing with the load current. During the acceleration (up to 6 ms) with two times the rated current the amplitude of the L_{dd} oscillation is three times higher compared to the no-load state.

The developed torque during startup is given in Fig. 7. In contrast to the idealized model a torque pulsation can be observed (especially at maximum torque) as well as the cogging torque in the no-load steady-state after acceleration.

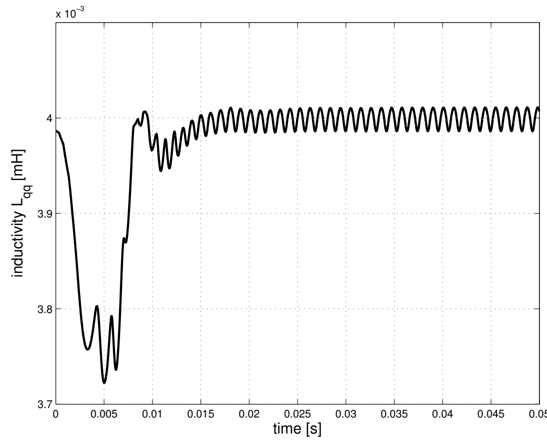

Fig. 6. Inductivity L_{qq} at startup operation.

B. Load case

At nominal load and speed an frequency analysis of the L_{qq} is performed (Fig. 8). Besides the fundamental wave the 6th, 12th and 18th frequency order can be seen, which are exactly the expected orders for a machine with six poles and 9 slots. The amplitude of the major harmonic is rather small with 6‰ of the fundamental wave.

C. Startup simulation with a simplified model

The result of section VI-B raises the question which grade of detail in the modeling is reasonable. A manifest approach

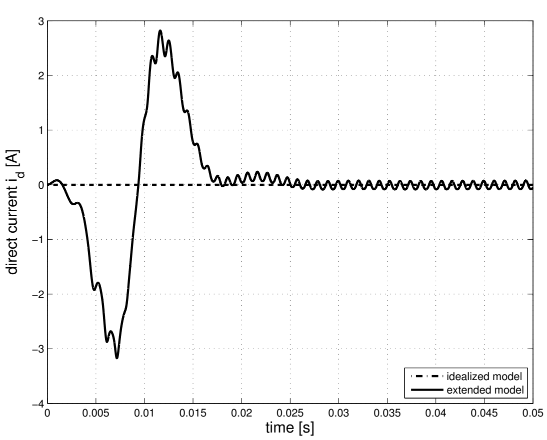

Fig. 5. Direct current at startup operation.

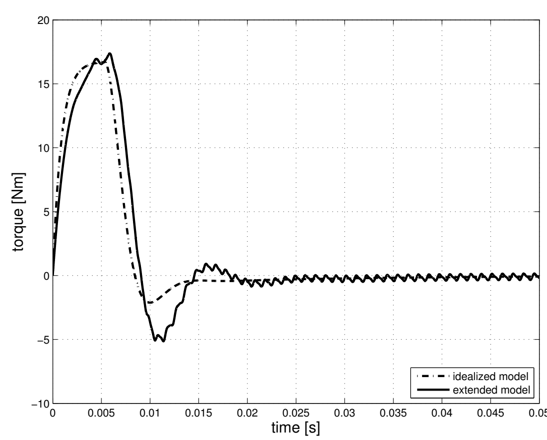

Fig. 7. Torque at startup operation.

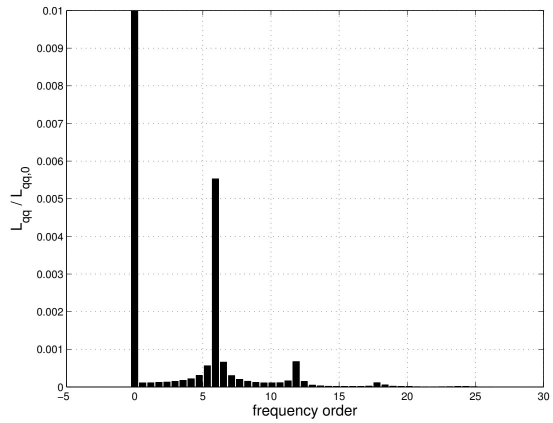

Fig. 8. Frequency analysis of L_{qq} at rated load.

could be replacing the ideal elements of (1) with the extracted parameters

$$\begin{aligned} L_{\mathrm{d}}^* &= L_{\mathrm{dd}}(\gamma, i_{\mathrm{q}}, i_{\mathrm{d}}), \quad L_{\mathrm{q}}^* = L_{\mathrm{qq}}(\gamma, i_{\mathrm{q}}, i_{\mathrm{d}}) \\ \psi_{\mathrm{f}}^* &= \psi_{\mathrm{d,f}}, \quad T^* = T \end{aligned} \tag{15}$$

from the FEA without extending the equations as shown in III. This results in the lack of the derivative terms in the description but could still yield accurate results. For a comparison of the reduced model to the full extended model the startup simulation is taken again. In Fig. 9 and Fig. 10 the d- and the q-current of both simulation models are shown.

Although the shape of the reduced models current is more similar to the the full model than this of the idealized simulation the deviation is rather high anyway. Especially the oscillation in the direct current shows a deviation in the amplitude of the factor seven. This clearly shows that even the use of lumped parameters extracted from an FEA only makes sense if they are combined with a precise description of the machine.

VII. Conclusion

In this paper a method for modeling a permanent magnet excited synchronous machine that includes nonideal effects as saturation, cross-coupling, and slotting effects is shown. These are included by use of lumped parameters (3x3 inductance matrix and the excitation flux) extracted from a previous finite element analysis. For this purpose the parameters are transformed to d-q components since the model is strictly orientated at the d-q reference frame. Whereas the common description of the synchronous machine uses idealized equations the presented model takes all terms into account that arise by solving the fundamental differential voltage equation of the machine.

After the mathematical derivation and the parametrization of the model some simulations are performed. Compared to the idealized machine representation the model extensions reflect the characteristics that are expected for a real machine. Furthermore, a reduced model that only replaces the ideal parameters of the idealized model by those extracted from the FEA without extending the model description is investigated. This model shows only a poor correlation of the results against the full extended model and should not be used.

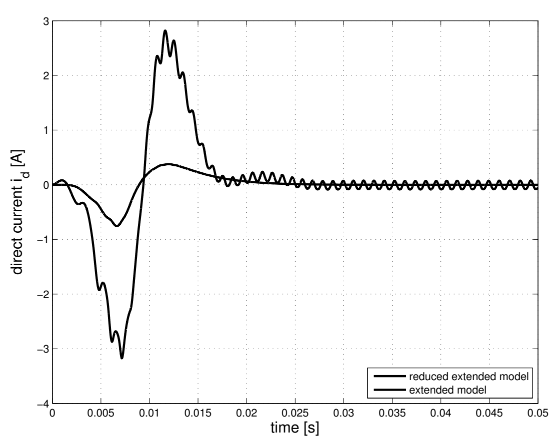

Fig. 9. Direct current at startup operation.

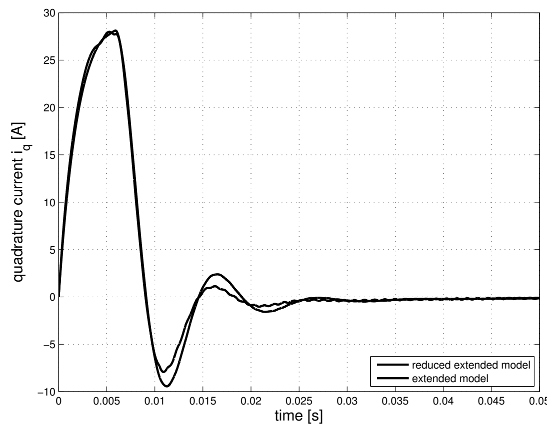

Fig. 10. Quadrature current at startup operation.

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