



Computational cost-effective modelling of non-linear characteristics in permanent magnet synchronous motors

Modelling
of characteristics
in PMSMs

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Abstract

Purpose – In the optimisation of electrical drives, the required degree of detail in the simulation increases constantly. Especially, the industrial demand on multi-objective optimisation craves for highly efficient models. The purpose of this paper is to propose a hybrid model for the computation of the air-gap field of a permanent magnet synchronous motor (PMSM) combining analytic and numeric methods.

Design/methodology/approach – The classic conformal mapping (CM) approach is improved by the numeric approximation of the required ansatz-functions. This approach allows to consider the non-linear permeability of the applied materials and complex geometries. The non-linear permeance-function is described by a one-dimensional wave varying in time and space.

Findings – The permeance-function has to be derived for different load cases at the actual stage.

Research limitations/implications – A physical motivated modelling allowing for an appropriate interpolation between different load cases is planned in further research.

Practical implications – The proposed approach is applied to a surface mounted PMSM. It is validated by means of a non-linear finite element analysis.

Originality/value – The hybrid model offers to consider rotors with buried magnets using the CM approach. It is possible to either use analytic or numeric modelling of rotor ansatz-function, stator current ansatz-function and permeance-function with the proposed approach. Non-linear permeability of iron is modelled by means of a wave representation of the permeance-function. This can significantly reduce the computational cost in the design and optimisation stage of electrical machines.

Keywords Electrical machines, Modelling, Electromagnetism, Mathematical analysis

Paper type Research paper

1. Introduction

Numerical methods such as the finite element analysis (FEA) are usually applied for the field calculation in electrical machines. This method is characterised by a high level of detail in the modelling, such as non-linear permeability of iron and exact modelling of the geometry. However, this approach is computationally expensive. Especially in connection with parameter variations or geometrical optimisation, FEA yields to an unacceptable computational effort. A model, which is accurate in the significant properties, can lead to a reduction of the computational costs. One suitable approach is the conformal mapping (CM) (Zarko *et al.*, 2006). But the standard CM does not allow considering non-linear permeability of iron, loaf shaped permanent magnets or internal mounted permanent magnets. Therefore, a parameterisation of the CM ansatz-function by FE simulations is proposed.



2. Brief introduction to the classical CM approach

The air-gap field computation by CM is generally obtained from the solution of a linear Laplace problem, describing the specific geometry, assuming the magnetic core with infinite permeability. Since this system is linear and time invariant, the impact of the field excitation by magnets and coils as well as the influence of the slotting is modelled individually. In case of a slottless stator, the radial flux-density $B_r(\Theta)$ and the tangential flux-density $B_\varphi(\Theta)$ can be expressed in terms of a Fourier series:

$$\vec{B}(\Theta) = \sum_{n=0}^{\infty} (B_{r,n} \cdot \vec{e}_r + B_{\varphi,n} \cdot \vec{e}_\varphi) e^{jnp\Theta} \quad (1)$$

where n is the frequency order, p the number of pole pairs and Θ the mechanical angle of the field wave. In this representation of the air-gap field, the Fourier coefficients $B_{r,n}$ and $B_{\varphi,n}$ are the solution of a linear Laplace problem with magnets and a slottless stator depending on the magnetization type (Zhu and Howe, 1993; Zhu *et al.*, 2002; Hanselman, 2003). Stator slotting, which significantly influences the magnetic field distribution, is generally modelled by permeance-functions. These complex permeance-functions λ consider the radial and tangential effect of the slotting on the slottless field distribution and can be obtained by Schwarz-Christoffel transformations (Zarko *et al.*, 2006, 2008). Correlating the field distribution with slotting, B , to the field without slotting equation (1), yields the permeance λ :

$${}^s\vec{B}(t) = \vec{\lambda}^* \cdot \vec{B}(t) \quad (2)$$

$$\vec{\lambda}^* = \begin{pmatrix} \lambda_r & \lambda_\varphi \\ -\lambda_\varphi & \lambda_r \end{pmatrix} \quad (3)$$

which describes the individual characteristic of the slotting on the field. The current ansatz-function describes the magnetic field of the stator winding. It can be obtained analytically by three successive CMs (Binns, 1963). The armature field ${}^a\vec{B}(t)$ is given by:

$${}^a\vec{B}(t, I) = \begin{pmatrix} p\vec{B}(\sqrt{2}Ie^{j\omega_s t + 0^\circ}) \\ p\vec{B}(\sqrt{2}Ie^{j\omega_s t + 120^\circ}) \\ p\vec{B}(\sqrt{2}Ie^{j\omega_s t + 240^\circ}) \end{pmatrix} \cdot \begin{pmatrix} e^{j\phi_q + 0^\circ} \\ e^{j\phi_q + 120^\circ} \\ e^{j\phi_q + 240^\circ} \end{pmatrix} \quad (4)$$

where the angle ϕ_q defines the relative phase orientation to the quadrature axis of the machine and ω_s is the stator current angular frequency. The complete air-gap field distribution of the slotted machine can be described consequently by:

$${}^g\vec{B}^{CM}(I) = {}^a\vec{B}^{CM}(I) + {}^s\vec{B}^{CM} \quad (5)$$

3. Idea of the hybrid model

The classic CM approach lacks in modelling of non-linear effects. Still, the CM formalism, the associated time and space-harmonic analysis together with the significant acceleration of simulation time when compared to FEA, are so practical that

it is worth to seek for an approximate representations of equation (2) in case of saturation. Therefore, the idea is to estimate each non-linear phenomenon by means of FEA models. The CM approach and the FEA deduced ansatz-functions can be used in combination in order to reduce the computational effort. It may be sufficient to model non-linear characteristics only in the rotor ansatz-function or the permeance-function depending on the application. To proof the proposed concept, the rotor field, the armature field and the stator permeance-function are calculated independently by FEA.

3.1 Hybrid concept rotor

The solution of the linear Laplace problem in case of a slottless stator can generally be found for all rotor types where saturation can be neglected. This may be the case for surface mounted permanent magnet motors. A linear field computation for internal mounted permanent magnet motors mismatches the air-gap flux-density completely, since the flux of the permanent magnets is closed through the wedges within the rotor. In the non-linear situation, these thin steel bridges are highly saturated building a natural flux barrier between the poles of the machine. Therefore, classical CM is not applicable for all kinds of geometry, where the main physical working principle is based on saturation. In order to approximate equation (1) a FEA of a rotor in an unslotted stator is performed. As an example, a rotor with buried PM is presented. The geometries and the field solution are shown in Figure 1.

The stator is modelled by means of a Neumann boundary condition. The symmetry of a rotor can be utilized for a further reduction of the model complexity. As an example, the radial and tangential air-gap flux-density, which is generated by the rotor are shown in Figure 2(a) and (b). The complete unslotted rotor field is determined by reflection and repetition of the obtained FEA results.

3.2 Current ansatz-function

The current ansatz-function can either be determined by analytic approaches or based on a loaded, linear FEA. In this case, the FEA is performed linearly, because all saturation effects are to be modelled into the non-linear permeance-function. Therefore, a linear FEA under load condition is required. For the determination of the current ansatz-function, equation (3) writes for the FEA case:

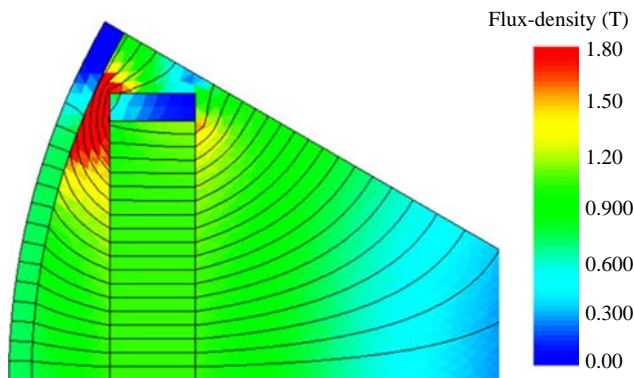
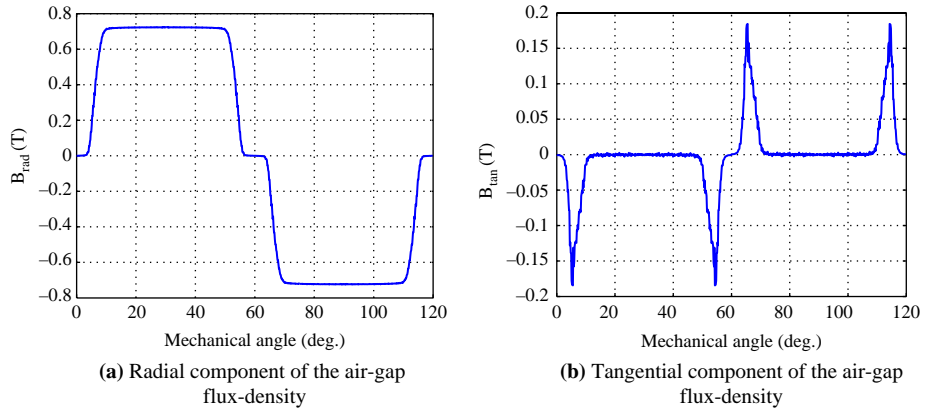


Figure 1.
Half pole pitch
of a PM rotor

Figure 2.
Radial and tangential
air-gap flux-density
of an internal-mounted
permanent magnet
rotor for on-pole pair



$$\vec{B}^{FEA} = \lim \vec{\lambda}^* \cdot \left({}^a \vec{B}^{FEA} + {}^{PM} \vec{B}^{FEA} \right) \quad (6)$$

whereby ${}^a \vec{B}^{FEA}$ is the current ansatz-function, ${}^{PM} \vec{B}^{FEA}$ the rotor ansatz-function (compare to equations (1) and (4)), and $\lim \vec{\lambda}$ is the linear permeance-function, which can be determined by CM. ${}^a \vec{B}^{FEA}$ can be determined by solving equation (6). Figure 3 shows the obtained current ansatz-functions. The deviation between both current ansatz-function can be explained by the approach dependent different modelling of the stator geometry. In case of the CM approach, the teeth are assumed to be blocks and the slot depth is infinite. The FEA approach allows for the consideration of the actual slot shape with appropriate modelling and simulation effort.

3.3 Hybrid concept permeance-function

The CM approach neglects the slot depth and slot shape. It is shown that the sensitivity of the air-gap field to these geometrical properties is very low (Binns, 1963). Therefore, this simplification is valid without significant reduction of accuracy. However, the influence of the saturation of the teeth and teeth-tips is not modelled in the CM approach. Teeth-tip saturation is unavoidable in permanent magnet synchronous

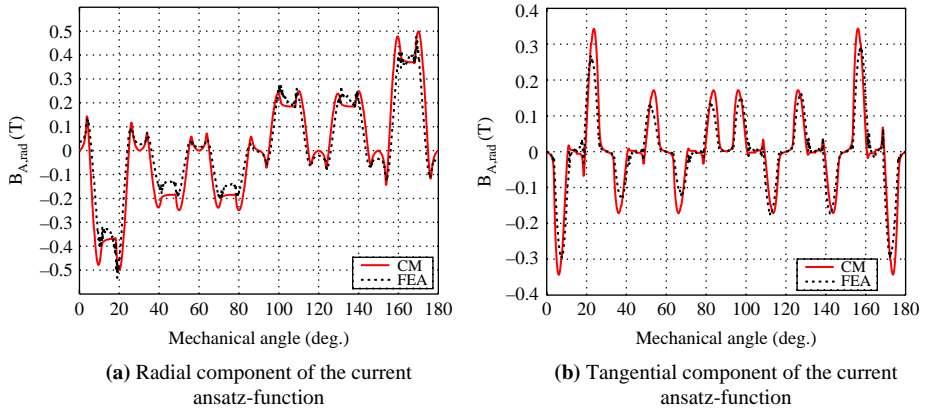


Figure 3.
Comparison of radial and
tangential component of
the current
ansatz-function obtained
by FEA and CM for one
time step

motor (PMSM) with distinct pole shoes, even under no-load condition. A comparison of the torque obtained by CM and FEA approach for different load cases is presented in Hafner *et al.* (2010). It is shown, that the influence of the saturation cannot be neglected. Hence, these effects are taken into account to increase the accuracy of the hybrid model compared to the classic CM method. For the determination of the non-linear permeance-function $\vec{\lambda}$, a FEA model of the motor with non-linear B-H characteristics under load condition is required. $\vec{\lambda}$ can be determined by:

$$\vec{\lambda}|_{r\rho} = \frac{\vec{B}^{FEA}}{a\vec{B} + {}^{PM}\vec{B}} \quad (7)$$

The required ansatz-function to determine the non-linear $\vec{\lambda}$ are the current ansatz-function $a\vec{B}$ and the rotor ansatz-function ${}^{PM}\vec{B}$, which can either determined applying the CM approach or the FEA approach. In order to compute the air-gap field for different load cases, the analytic determined current ansatz-function $a\vec{B}^{CM}$ is useful to apply, because all non-linear effects are modelled into the permeance-function in this case. The resulting non-linear permeance-function can be described by a one-dimensional wave. A comparison of the non-linear permeance-function $\vec{\lambda}$ with the linear one $\vec{\lambda}^{CM}$ shows a good agreement. These two permeance-functions are shown in Figure 4 for one time step. The saturation effect can be pointed in the tips of the radial permeance-functions. The linear permeance-function is symmetric, whereby the leading and trailing edge-effect due to saturation is present for the non-linear permeance-function.

3.4 Application of the proposed approach

The proposed approach is applied to a PMSM. The parameters of the machine are defined in Table I and Figure 5. The machine has concentrated, single-layer winding in the stator and loaf shaped surface PMs in the rotor. The non-linear permeance-function is determined for nominal load. The radial and tangential air-gap field is simulated applying a linear model and the proposed hybrid model. For comparison purpose, the results of these two models are compared relatively with regards to a non-linear FEA. Equal meshes are used for all simulations. In case of nominal load, the hybrid model

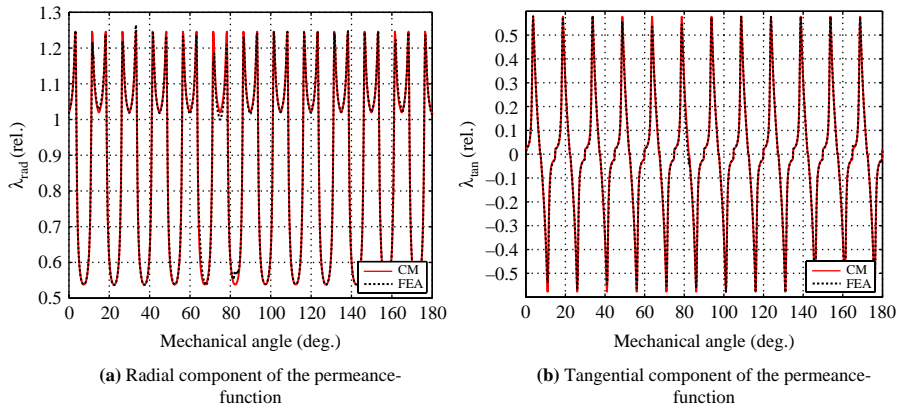


Figure 4. Comparison of radial and tangential component of the permeance-function obtained by FEA and CM for one-time step

and the non-linear FEA agree exactly. This is expected since the permeance-function describes exactly this load case. The linear model differs from the FEA model. The relative deviation of the air-gap flux-densities and the torque of linear and hybrid model with respect to FEA are shown in Figures 6 and 7. The deviation in the flux-density of the linear model is up to 200 per cent. The mean deviation in torque of the linear model to the FEA is about 1 per cent. For 110 per cent nominal load, the hybrid model applying the rated load permeance-function differs to the non-linear FEA. A comparison of the deviation of the linear and hybrid model with respect to non-linear FEA in the air-gap flux-density and torque is shown in Figures 8 and 9 for this load case. The deviation in torque is about 1.5 per cent for linear model and 0.5 per cent for the hybrid model respectively compared to non-linear FEA. Still the hybrid model delivers closer results to the FEA compared to the linear model.

Table I.
Parameters for sizing
and electromagnetic
evaluation

p	11	Number of pole pairs
N_s	24	Number of stator teeth
h_m	3 mm	Permanent magnet height
r_{orr}	59.15 mm	Outer rotor radius (incl. PM)
h_δ	0.85 mm	Air-gap height
h_{sth}	17.5 mm	Stator tooth height
r_{osr}	85 mm	Outer stator radius
h_{stw}	8 mm	Stator tooth width
h_{sow}	8 mm	Slot opening width
l_z	70 mm	Length
B_r	1.25T	Remanence flux-density

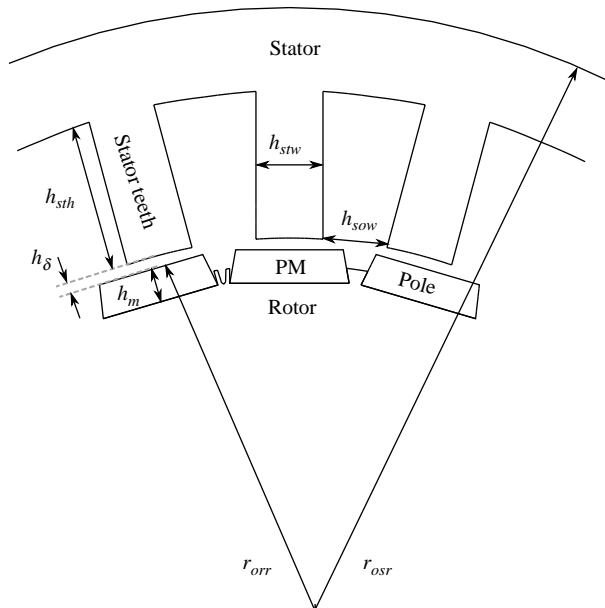


Figure 5.
Definition of the machines
geometric parameters

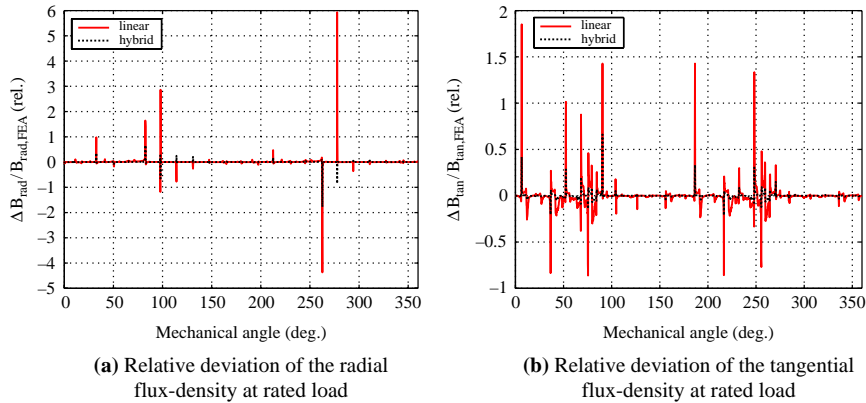


Figure 6. Relative deviation of the flux-density at rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA

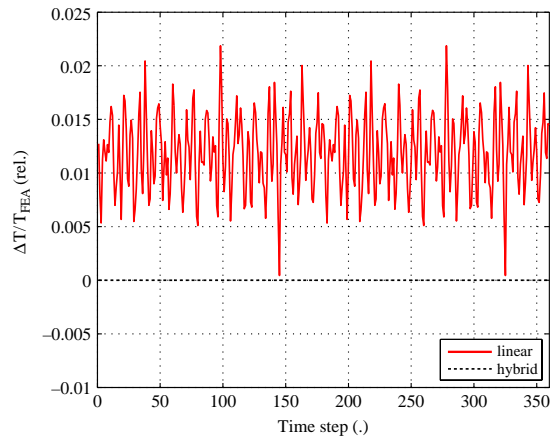


Figure 7. Relative deviation of the torque at rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA

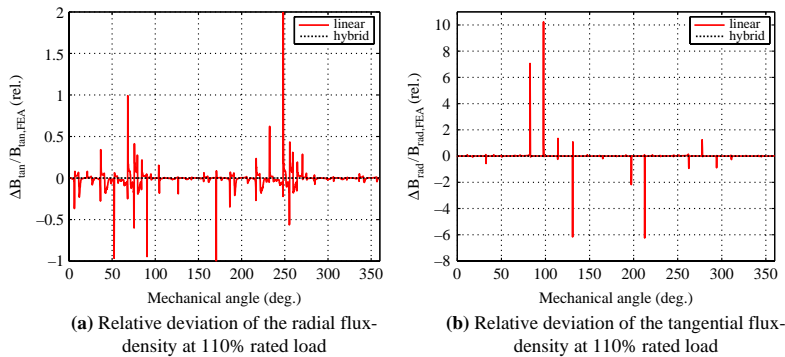
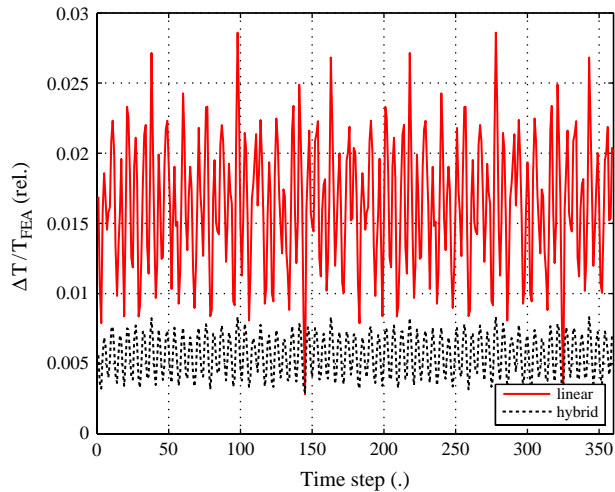


Figure 8. Relative deviation of the flux-density at 110 per cent rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA

Figure 9.
Relative deviation of torque at 110 per cent rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA



4. Conclusion

This paper proposes an approach to determine the required CM ansatz-functions, in order to model a PMSM computational cost-effective. The model is based on the CM approach, where each field component is modelled individually. In order to take non-linear permeability into account, CM ansatz-functions are approximated numerically. The air-gap flux-density of the armature winding and the rotor in an unslotted air-gap and the stator permeance are calculated separately. The saturation effect is modelled into the stator ansatz-function. This ansatz-function depends on the exciting current density and rotor position. The model shows a good agreement to FEA results. Load dependent permeance-function can be defined in order to improve the estimated air-gap for dynamic simulations. It is shown that a linearization of the state of saturation in the proximity of a load case is possible. The determination of the permeance-function is computationally expensive, but once this is calculated the air-gap field of the PMSM can be estimated based on inexpensive multiplications. Further improvement of the proposed approach is ongoing. An application of the proposed approach for control problems is planned, where the processing power is limited in general.

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David Franck received his Diploma in Electrical Engineering in 2008 as Engineer from the Faculty of Electrical Engineering and Information Technology at RWTH Aachen University. Since 2008, he has worked as a Researcher at the Institute of Electrical Machines (IEM) at RWTH Aachen University. He is currently working towards his doctoral degree in the area of noise and vibration of electrical machines. David Franck is the corresponding author and can be contacted at: david.franck@iem.rwth-aachen.de

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Kay Hameyer received his MSc degree in Electrical Engineering from the University of Hannover and his PhD degree from the Berlin University of Technology. After his university studies, he worked with the Robert Bosch GmbH in Stuttgart as a Design Engineer for permanent magnet servo motors and vehicle board net components. Until 2004, Professor Kay Hameyer was a Full Professor for Numerical Field Computations and Electrical Machines with the KU Leuven in Belgium. Since 2004, he has been Full Professor and the Director of the IEM at RWTH Aachen University in Germany. In 2006 he became Vice Dean of the faculty and from 2007 to 2009 he became the Dean of the Faculty of Electrical Engineering and Information Technology of RWTH Aachen University. His research interests are numerical field computation, the design and control of electrical machines, in particular permanent magnet excited machines, induction machines and numerical optimisation strategies. Dr Kay Hameyer's work is concerned with the magnetic levitation for drive systems, magnetically excited audible noise in electrical machines. Dr Kay Hameyer is the author of more than 250 journal publications, more than 500 international conference publications and is the author of four books. Dr Kay Hameyer is a member of VDE, IEEE senior member, fellow of the IET and fellow of WIT.