# Computational cost-effective modelling of non-linear characteristics in permanent magnet synchronous motors


**Abstract**

**Purpose** In the optimization of electrical drives, the required degree of detail in the simulation increases constantly. Especially the industrial demand on multi-objective optimisation craves for high efficient models. For this purpose, a hybrid model for the computation of the air-gap field of a permanent magnet synchronous motor (PMSM) is proposed, combining analytic and numeric methods.

**Design/methodology/approach** The classic conformal mapping approach is improved by the numeric approximation of the required ansatz-functions. This approach allows to consider the non-linear permeability of the applied materials and complicated geometries. The non-linear permeance-function is described by a one-dimensional wave varying in time and space.

**Findings** The permeance function has to be derived for different load cases at the actual stage.

**Research limitations/implications** A physical motivated modelling allowing for an appropriate interpolation between different load cases is planed in further research.

**Practical implications** The proposed approach is applied to a surface mounted permanent magnet synchronous machine (PMSM). It is validated by means of a non-linear Finite-Element-Analysis (FEA).

**Originality/value** The hybrid model offers to consider rotors with buried magnets with the CM approach. It is possible to either use analytic or numeric modelling of rotor ansatz-function, stator current ansatz-function and permeance-function with the proposed approach. Non-linear permeability of iron is modelled by means of a wave representation of the permeance-function. This can significantly reduce the computational-cost in the design- and optimisation stage of electrical machines.


**Keywords** electrical machines, electromagnetic modelling, non-linear material parameters, conformal Mapping, air-gap permeance

**Paper type** Research paper


## 1 Introduction

Numerical methods such as the FEA are usually applied for the field calculation in electrical machines. This method is characterised by a high level of detail in the modelling, such as non-linear permeability of iron and exact modelling of the geometry. However, this approach is computational expensive. Especially in connection with parameter variations or geometrical optimization FEA yields to an unacceptable computational effort. A model, which is accurate in the significant properties, can lead to a reduction of the computational costs. One suitable approach is the conformal mapping (CM)(Zarko, Ban & Lipo 2006). But the standard CM does not allow considering non-linear permeability of iron, loaf shaped permanent magnets or internal mounted permanent magnets. Therefore, a parameterisation of the CM ansatz-function by FE simulations is proposed.

# 2 Brief introduction to the classical CM approach

The air-gap field computation by conformal mapping is generally obtained from the solution of a linear Laplace problem, describing the specific geometry, assuming the magnetic core with infinite permeability. Since this system is linear and time invariant (LTI), the impact of the field excitation by magnets and coils as well as the influence of the slotting is modelled individually. In case of a slottless stator, the radial flux-density $B_r(\Theta)$ and the tangential flux-density $B_\varphi(\Theta)$ can be expressed in terms of a Fourier Series

$$\vec{B}(\Theta) = \sum_{n=0}^{\infty} \left(B_{r,n} \cdot \vec{e}_r + B_{\varphi,n} \cdot \vec{e}_\varphi\right) e^{\jmath np\Theta} \tag{1}$$

where $n$ is the frequency order, $p$ the number of pole pairs and $\Theta$ the mechanical angle of the field wave. In this representation of the air-gap field, the Fourier coefficients $B_{r,n}$ and $B_{\varphi,n}$ are the solution of a linear Laplace problem with magnets and a slottless stator depending on the magnetization type (Zhu & Howe 1993), (Zhu, Howe & Chan 2002), (Hanselman 2003). Stator slotting, which significantly influences the magnetic field distribution, is generally modelled by permeance-functions. These complex permeance-functions $\vec{\lambda}$ consider the radial and tangential effect of the slotting on the slottless field distribution and can be obtained by Schwarz-Christoffel transformations (Zarko et al. 2006), (Zarko, Ban & Lipo 2008). Correlating the field distribution with slotting, ${}^sB$, to the field without slotting (1), yields the permeance $\vec{\lambda}$,

$$ {}^s\vec{B}(t) = \vec{\lambda}^* \cdot \vec{B}(t) \tag{2}$$

$$\vec{\lambda}^* = \begin{pmatrix} \lambda_r & \lambda_\varphi \\ -\lambda_\varphi & \lambda_r \end{pmatrix} \tag{3}$$

which describes the individual characteristic of the slotting on the field. The current ansatz-function describes the magnetic field of the stator winding. It can be obtained analytically by three successive conformal mappings (Binns 1963). The armature field ${}^a\vec{B}(t)$ is given by

$$ {}^a\vec{B}(t, I) = \begin{pmatrix} {}^p\vec{B}\left(\sqrt{2}Ie^{\jmath\omega_s t+0^\circ}\right) \\ {}^p\vec{B}\left(\sqrt{2}Ie^{\jmath\omega_s t+120^\circ}\right) \\ {}^p\vec{B}\left(\sqrt{2}Ie^{\jmath\omega_s t+240^\circ}\right) \end{pmatrix} \cdot \begin{pmatrix} e^{\jmath\phi_q+0^\circ} \\ e^{\jmath\phi_q+120^\circ} \\ e^{\jmath\phi_q+240^\circ} \end{pmatrix} \tag{4}$$

where the angle $\phi_q$ defines the relative phase orientation to the quadrature axis of the machine and $\omega_s$ is the stator current angular frequency. The complete air-gap field distribution of the slotted machine can be described consequently by

$$ {}^g\vec{B}^{CM}(I) = {}^a\vec{B}^{CM}(I) + {}^s\vec{B}^{CM} \tag{5}$$

# 3 Idea of the hybrid model

The classic CM approach lacks in modelling of non-linear effects. Still, the CM formalism, the associated time and space-harmonic analysis together with the significant acceleration of simulation time when compared to FEA, are so practical that it is worth to seek for an approximate representations of (2) in case of saturation. Therefore, the idea is to estimate each non-linear phenomenon by means of FEA models. The CM approach and the FEA deducted ansatz-functions can be used in combination in order to reduce the computational effort. It may be sufficient to model non-linear characteristics only in the rotor ansatz-function or the permeance-function depending on the application. To proof the proposed concept the rotor field, the armature field and the stator permeance-function are calculated independently by FEA.

## 3.1 Hybrid concept rotor

The solution of the linear Laplace problem in case of a slottless stator can generally be found for all rotor types where saturation can be neglected. This may be the case for surface mounted permanent magnet motors. A linear field computation for internal mounted permanent magnet motors mismatches the air-gap flux-density completely, since the flux of the permanent magnets is closed through the wedges within the rotor. In the non-linear situation, these thin steel bridges are highly saturated building a natural flux barrier between the poles of the machine. Therefore, classical CM is not applicable for all kinds of geometry, where the main physical working principle is based on saturation. In order to approximate (1) a FEA of a rotor in an unslotted stator is performed. As an example a rotor with buried PM is presented. The geometries and the field solution are shown in Fig. 1.

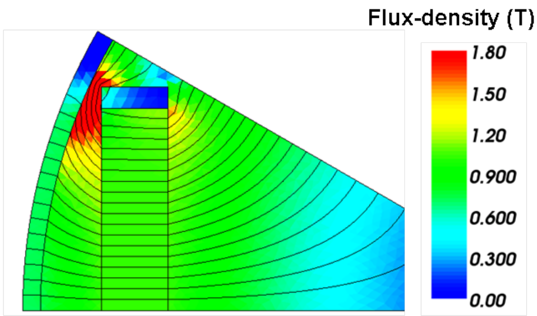
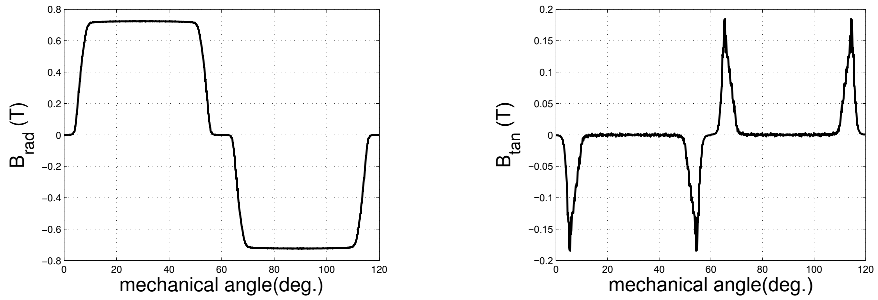



Figure 1: Half pole-pitch of a PM rotor.

The stator is modelled by means of a Neumann boundary condition. The symmetry of a rotor can be utilized for a further reduction of the model complexity. As an example the radial and tangential air-gap flux-density, which is generated by the rotor are presented in Fig. 2(a) and 2(b). The complete unslotted rotor field is determined by reflection and repetition of the obtained FEA results.




(a) Radial component of the air-gap flux-density.

(b) Tangential component of the air-gap flux-density.

Figure 2: Radial and tangential air-gap flux-density of an internal mounted permanent magnet rotor for on pole pair.

## 3.2 Current ansatz-function

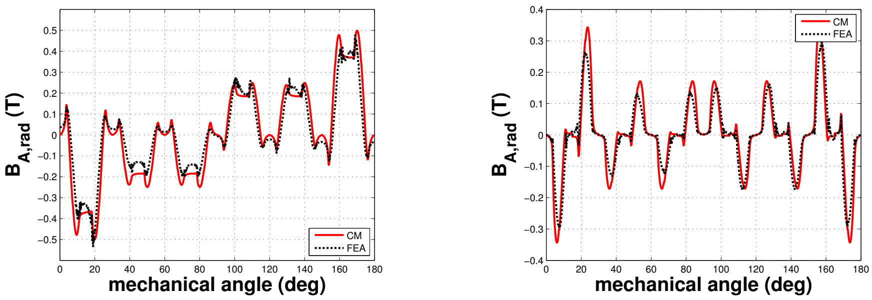
The current ansatz-function can either be determined by analytic approaches or based on a loaded, linear FEA. In this case, the FEA is performed linearly, because all saturation effects are to be modelled into the non-linear permeance-function. Therefore, a linear FEA under load condition is required. For the determination of the current ansatz-function equation (5) writes for the FEA case

$$\vec{B}^{FEA} = \left( {}^{a}\vec{B}^{FEA} + {}^{PM}\vec{B}^{FEA} \right) \cdot {}^{lin}\vec{\lambda}^{*} \tag{6}$$

whereby ${}^{a}\vec{B}^{FEA}$ is the current ansatz-function, ${}^{PM}\vec{B}^{FEA}$ the rotor ansatz-function (comp. to (1) and (4)), and ${}^{lin}\vec{\lambda}$ is the linear permeance-function, which can be determined by CM. ${}^{a}\vec{B}^{FEA}$ can be determined by solving (6). Fig. 3 shows the obtained current ansatz-functions. The deviation between both current ansatz-function can be explained by the approach dependent different modelling of the stator geometry. In case of the CM approach the teeth are assumed to be blocks and the slotdepth is infinite. The FEA approach allows for the consideration of the actual slot shape with appropriate modelling and simulation effort.

## 3.3 Hybrid concept permeance-function

The CM approach neglects the slot depth and slot shape. It is shown that the sensitivity of the air-gap field to these geometrical properties is very low (Binns 1963). Therefore this simplification is valid without significant reduction of accuracy. However, the influence of the saturation of the teeth and teeth-tips is not modelled in the CM approach. Teeth-tip saturation is unavoidable in PMSM with distinct pole shoes, even under no-load condition. A comparison of the torque obtained by CM and FEA approach for different load cases is presented in (Hafner, Franck & Hameyer 2010). It is shown, that the influence of the saturation can not be neglected. Hence, these effects are taken into account to increase the accuracy of the hybrid model compared to the classic CM method. For the determination of the non-linear permeance-function $\vec{\lambda}$ a FEA




(a) Radial component of the current ansatz-function.

(b) Tangential component of the current ansatz-function.

Figure 3: Comparison of radial and tangential component of the current ansatz-function obtained by FEA and CM for one time step.

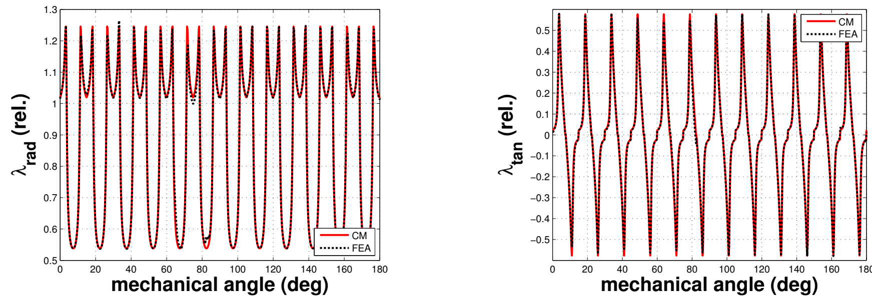
model of the motor with non-linear B-H characteristics under load condition is required. $\vec{\lambda}$ can be determined by

$$\vec{\lambda}^{*} = \frac{\vec{B}^{FEA}}{{}^{a}\vec{B} + {}^{PM}\vec{B}} \tag{7}$$

The required ansatz-function to determine the non-linear $\vec{\lambda}$ are the current ansatz-function ${}^{a}\vec{B}$ and the rotor ansatz-function ${}^{PM}\vec{B}$, which can either determined applying the CM approach or the FEA approach. In order to compute the air-gap field for different load cases, the analytic determined current ansatz-function ${}^{a}\vec{B}^{CM}$ is useful to apply, because all non-linear effects are modelled into the permeance-function in this case. The resulting non-linear permeance-function can be described by a one-dimensional wave. A comparison of the non-linear permeance-function $\vec{\lambda}$ with the linear one $\vec{\lambda}^{CM}$ shows a good agreement. These two permeance-functions are shown in Fig. 4 for one time step. The saturation effect can be pointed in the tips of the radial permeance-functions. The linear permeance-function is symmetric, whereby the leading- and trailing edge-effect due to saturation is present for the non-linear permeance-function.

## 3.4 Application of the proposed approach

The proposed approach is applied to a PMSM. The parameters of the machine are defined in Tab. 1 and Fig. 5. The machine has concentrated, single-layer winding in the stator and loaf shaped surface-PMs in the rotor. The non-linear permeance-function is determined for nominal load. The radial and tangential air-gap field is simulated applying a linear model and the proposed hybrid model. For comparison purpose, the results of these two models are compared relatively with regards to a non-linear FEA. As described in (Hafner et al. 2010) the difference between linear FEA and CM is negligible, therefore the investigated linear model is a FEA model in order not to evaluate discretisation errors. Equal meshes are used for all simulations. The proposed approach is applied to a PMSM. The parameters of the machine are defined in Tab. 1 and Fig. 5. The machine has concentrated, single-layer winding in the stator and loaf shaped




(a) Radial component of the permeance-function.

(b) Tangential component of the permeance-function.

Figure 4: Comparison of radial and tangential component of the permeance-function obtained by FEA and CM for one time step.

surface-PMs in the rotor. The non-linear permeance-function is determined for nominal load. The radial and tangential air-gap field is simulated applying a linear model and the proposed hybrid model. For comparison purpose, the results of these two models are compared relatively with regards to a non-linear FEA. As described in (Hafner et al. 2010) the difference between linear FEA and CM is negligible, therefore the investigated linear model is a FEA model in order not to evaluate discretisation errors. Equal meshes are used for all simulations. In case of nominal load the hybrid model and the non-linear FEA agree exactly. This is expected since the permeance-function describes exactly this load case. The linear model differs from the FEA model. The relative deviation of the air-gap flux-densities and the torque of linear and hybrid model with respect to FEA are presented in Fig. 6 and 7. The deviation in the flux-density of the linear model is up to 200%. The mean deviation in torque of the linear model to the FEA is about 1%. For 110% nominal load the hybrid model applying the rated load permeance-function differs to the non-linear FEA. A comparison of the deviation of the linear and hybrid model with respect to non-linear FEA in the air-gap flux-density and torque is presented in Fig. 8 and 9 for this load case. The deviation in torque is about 1.5% for linear model and and 0.5% for the hybrid model respectivly compared to non-linear FEA. Still the hybrid model delivers closer results to the FEA compared to the linear model.

# 4 Conclusion

This paper proposes an approach to determine the required CM ansatz-functions, in order to model a PMSM computational cost-effective. The model is based on the CM approach, where each field component is modelled individually. In order to take non-linear permeability into account CM ansatz-functions are approximated numerically. The air-gap flux-density of the armature winding and the rotor in an unslotted air-gap and the stator permeance are calculated separately. The saturation effect is modelled into the stator ansatz-function. This ansatz-function depends on the exciting current density and rotor position. The model shows a good agreement to FEA results. Load dependent permeance-

Table 1: Parameters for Sizing and Electromagnetic Evaluation

| | | |
|---|---|---|
| $p$ | 11 | Number of Pole Pairs |
| $N_s$ | 24 | Number of Stator Teeth |
| $h_m$ | 3 mm | Permanent Magnet Height |
| $r_{orr}$ | 59.15 mm | Outer Rotor Radius (incl. PM) |
| $h_\delta$ | 0.85 mm | Air Gap Height |
| $h_{sth}$ | 17.5 mm | Stator Tooth Height |
| $r_{osr}$ | 85 mm | Outer Stator Radius |
| $h_{stw}$ | 8 mm | Stator Tooth Width |
| $h_{sow}$ | 8 mm | Slot Opening Width |
| $l_z$ | 70 mm | Length |
| $B_r$ | 1.25 T | Remanence Flux-Density |

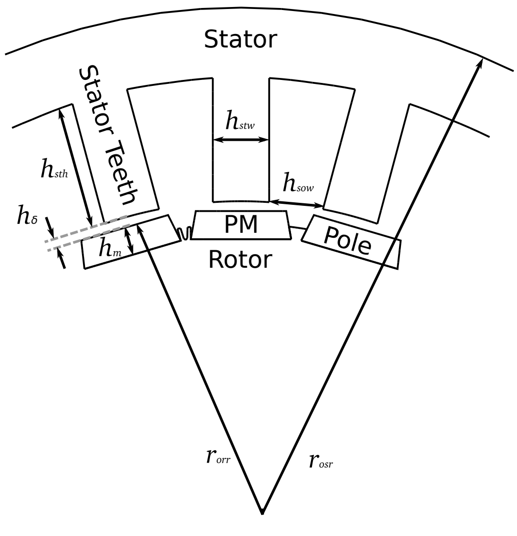
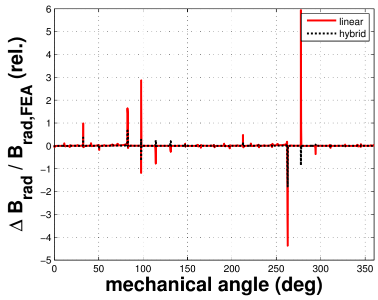
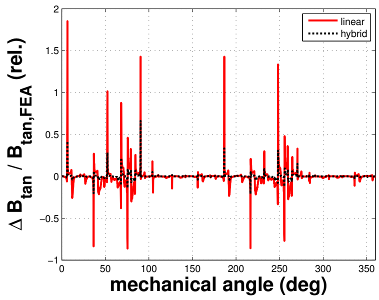



Figure 5: Definition of the machines geometric parameters.




(a) Relative deviation of the radial flux-density at rated load.




(b) Relative deviation of the tangential flux-density at rated load.

Figure 6: Relative deviation of the flux-density at rated load, linear simulation compared to the hybrid simulation with repect to non-linear FEA.

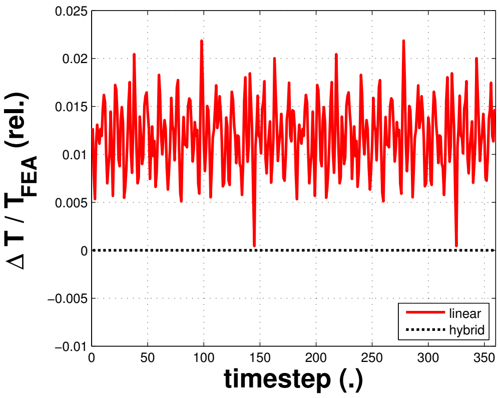
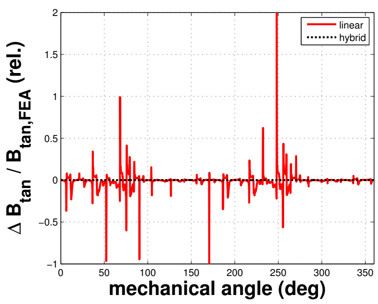
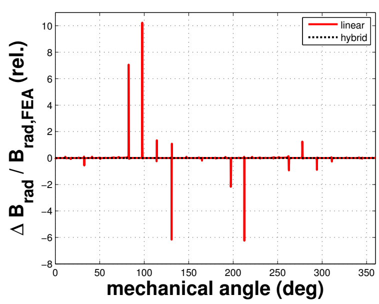



Figure 7: Relative deviation of the torque at rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA.




(a) Relative deviation of the radial flux-density at 110% rated load.




(b) Relative deviation of the tangential flux-density at 110% rated load.

Figure 8: Relative deviation of the flux-density at 110% rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA.

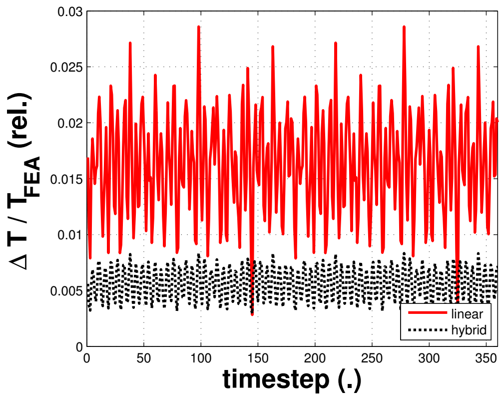



Figure 9: Relative deviation of torque at 110% rated load, linear simulation compared to the hybrid simulation with respect to non-linear FEA.

function can be defined in order to improve the estimated air-gap for dynamic simulations. It is shown, that a linearization of the state of saturation in the proximitivity of a load case is possible. The determination of the permeance-function is computationally expensive, but once this it is calculated the air-gap field of the PMSM can be estimated based on inexpensive multiplications. Further improvement of the proposed approach is ongoing. An application of the proposed approach for control problems is planed, where the processing power is limited in general.

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