Quasistatic Electromagnetic Field Computation by Conformal Mapping in Permanent Magnet Synchronous Machines

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***Abstract*— In the electromagnetic design of modern servo drives or electrical drives for positioning, torque pulsation, tangential forces and ripple torque are highly undesirable. Since these quantities are directly linked with the occurring harmonic air gap flux density waves, this paper presents a time-saving method to compute the radial and tangential field components in load and no-load condition by conformal mapping in frequency domain. The proposed method is applied to a surface permanent magnet synchronous machine, and compared to numeric results obtained by nonlinear FEA.**

I. Introduction

Nowadays the design of electric machines is mainly accomplished virtually to shorten the time-to-market. Since finite element analysis (FEA) is time consuming and requires a high computational effort, in early stages of the sizing process analytic models are applied that are focused on an estimation of the average or fundamental air gap flux density. Consequently, the effect of air gap field harmonics on the main machine characteristics, like EMF, cogging-torque and load-torque, as well as the impact of geometry variations on those quantities is neglected. Therefore, in this paper, an analytic conformal mapping method in frequency domain for permanent magnet synchronous machines (PMSM) is applied to consider the occurring air gap field harmonics for torque and EMF computation. Even if that approach attributes ideal steel characteristics, a comparison to standard nonlinear FEA yields a good approximation of the air gap flux density spectrum and all derived quantities.

II. Computational Framework

A. Rotor Field Distribution in the Slotless Air Gap

The analytical technique for computing the 2D magnetic no-load field distribution in the air gap of radial-magnetized permanent magnet rotors has been published by [1]. The derived parametric field solution is applicable for internal and external rotors. In further researches the approach has been generalized to radial and parallel magnetizations, [2]. Recently, [3] published a further extension to radial sine and sinusoidal direction magnetizations. In case of a slotless stator, the radial flux density $B_r(\Theta)$ and the tangential flux density $B_\varphi(\Theta)$ are given by

$$B_r(\Theta) + \jmath B_\varphi(\Theta) = \sum_{n=0}^{\infty} (B_{r,n} + \jmath B_{\varphi,n})\, e^{\jmath n p \Theta} \qquad (1)$$

where n is the frequency order, p the number of pole pairs, Θ the mechanical angle. The Fourier-Coefficients $B_{r,n}$ and $B_{\varphi,n}$ depend on the magnetization type of the permanent magnets.

B. Rotor Field Distribution in the Slotted Air Gap

Stator slotting significantly influences the magnetic field distribution:

1) Different radial magnetic permeances of teeth and slot affects the local distribution of the flux in the air gap, so that a flux concentration beneath the stator teeth can be observed.
2) In case of slotting, a characteristic flux density super elevation can be observed over the PM vertices in the air gap.
3) Slotting cross-effects radial and tangenial flux distribution

A common method for modeling these effects on the magnetic field distribution are formally known as "‘permeance functions"’ in literature. In a former research, [4] models the impact of effect 1) on the radial flux density component only. The recent publication [5] derives by four complex conformal mappings, including a Schwarz-Christoffel transformation, a permeance function which takes the effects 1) - 3) into account. Assuming a infinite permeable stator core, this ansatz has shown to yield identical flux density results in comparison to FE simulation applying Neumann boundary conditions.

For that approach, each point of an equidistant sampled arc in the air gap with the length of one slot pitch, a nonlinear iteratively solved, function is evaluated to compute the corresponding complex permeance number

$$\underline{\lambda} = \lambda_r + \jmath \lambda_\varphi \qquad (2)$$

where λ_r and λ_φ represent the permeance variations in radial and tangential direction. The sampling points of the slot pitch can be expressed by a Fourier-Series and extented to the whole air gap, yielding,

$$\underline{\lambda}(\Theta) = \sum_{0}^{N_\lambda} \lambda_{r,n} \cos(n N_s \Theta) + \jmath \sum_{1}^{N_\lambda} \lambda_{\varphi,n} \sin(n N_s \Theta) \qquad (3)$$

where N_s is the number of stator teeth and N_λ the sampling rate depending maximal occurring frequency. Combining (1) and (2), the flux density ${}^s B$ in the slotted air gap can finally be given by:

$$ {}^s\underline{B}(\Theta) = (B_r + \jmath B_\varphi)\, \underline{\lambda}^* \qquad (4)$$

$$ {}^s\underline{B}(\Theta) = B_r\lambda_r + B_\varphi\lambda_\varphi + \jmath B_\varphi \lambda_r - \jmath B_r \lambda_\varphi \qquad (5)$$

C. Armature winding field

The magnetic field distribution of a current flown single slot, assuming a infinite slot depth and a infinite permeability in a slotless stator, can be obtained by three conformal mappings [6]. Since a electric coil occupies two slots with opposite directed currents, a flux density distribution ${}^{c}\underline{B}$ for the whole air gap per coil can be assembled in functions of the coil current I_c and the number coil turns N by

$$ {}^{c}\underline{B}\left(\Theta, N, I_c\right) = N I_c \sum_{1}^{N_a} \left({}^{c}B_{r,n} + \jmath\, {}^{c}B_{\varphi,n}\right) e^{\jmath n \Theta} \tag{6} $$

where N_a denotes the maximal frequency order and the radial and tangential quantities ${}^{c}B_{r,n}$ and ${}^{c}B_{\varphi,n}$ the Fourier-Coefficients of the coil computed for one ampere. According to the winding schema of the PMSM a flux distribution ${}^{p}B$ for each phase can be found by adding shifted terms of (6). In case of a symmetric current load (UVW) I, the allover armature field ${}^{a}\underline{B}$ is given by

$$ {}^{a}B\left(\Theta, I\right) = \left({}^{p}\underline{B}_U, {}^{p}\underline{B}_V, {}^{p}\underline{B}_W\right) \cdot \left(e^{\jmath\phi_q}, e^{\jmath\phi_q + 120^\circ}, e^{\jmath\phi_q + 240^\circ}\right)^T \tag{7} $$

where the angle ϕ_q defines the relative phase orientation to the q-axis of the machine.

D. Field Distribution in the Slotted Air Gap

The magnetic air gap field in case of stator slotting can be assumed as a superposition of the field component fields due to permanent magnet and stator excitation.

Introducing a time-discretization Δ_{FE} and a corresponding time-stepping n_{FE}, the rotor flux distribution for a certain time step t,

$$ t = n_{\mathrm{FE}} \cdot \Delta_{\mathrm{FE}} \tag{8} $$

can be determined as follows:

$$ {}^{s}B^{t}\left(f\right) = {}^{s}\underline{B}\left(\Theta\right) e^{\jmath\omega t} \tag{9} $$

The frequency dependency is implicitly taken into account by:

$$ \Delta_{\mathrm{FE}} = \frac{n}{N_{el} \cdot p} \tag{10} $$

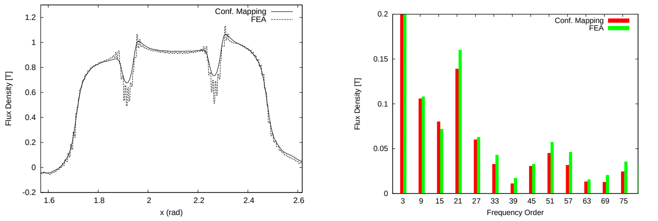
where n is the rotor speed and N_{el} the number of computation steps per electric period. The corresponding time-depended armature field is frequency-shifted by the stator frequency f_1, yielding

$$ {}^{a}B^{t}\left(f, I\right) = {}^{a}\underline{B}\left(\Theta, I\right) e^{\jmath 2\pi f_1} e^{\jmath\omega t} \tag{11} $$

For a given time step t (9) and (11) represent the allover load and no-load flux density distribution in the slotted air gap.

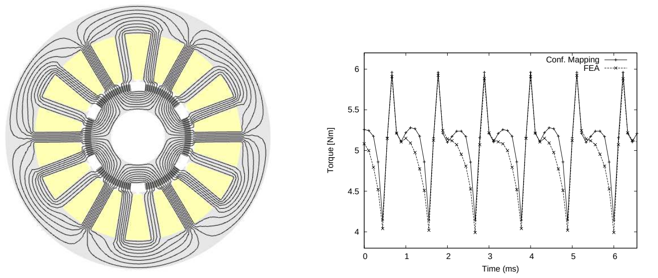
III. Application

To demonstrate the proposed method, a PMSM, designed by in-house sizing software is investigated. Its cross-section together with the field distribution in rated operation is given in fig. 2(a). The FEA flux density is sampled in the air gap for each time step individually. The spectrum for a certain time instance obtained numerically and by (9), (11) is shown in fig. 1(b); the corresponding local flux distribution for a pole pitch is given in 1(a). The resulting torque according to Maxwell stress tensor for both magnetic fields are given in fig. 2(b).



(a) Local field distribution of PMSM pole pitch. (b) Flux density amplitude for different order.

Fig. 1. Flux density distribution in time and frequency domain by FEA and conf. mapping in rated operation.



(a) Six pole PMSM with field lines in rated operation. (b) Torque characteristic over electric period.

Fig. 2. PMSM cross section and torque characteristic over an electric period by FEA and conf. mapping in rated operation.

IV. Conclusion

Electromagnetic field computations are ubiquitous in the design of electrical machines. Even if established finite-element methods yield very accurate result, their high computational effort inhibits an application in early design stages or multi-object optimizations of electrical drives. In that case, it is worth to seek for approximative and time-saving representations. In this paper, the air gap field of a PMSM under load and no-load is computed by conformal mapping in frequency domain. A first demonstration on a PMSM shows, that the obtained occurring harmonic air gap flux waves under load condition are in good agreement to nonlinear FEA results. A detailed comparison in different load conditions between the proposed method and standard simulation will be presented in the full paper submission.

References

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