Galerkin Projection Method for Sliding Interfaces in Finite Element Analysis of Electrical Machines

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***Abstract*— This paper proposes the application of the Galerkin projection method to implement the relative motion of stator and rotor in the FE simulations of electrical machines. The non–conforming representation of stator and rotor regions impose no restriction on time or space discretization. The symmetry and sparsity of the system of FE equations are preserved. The method is applied to the 2D simulation of the cogging torque of a synchronous machine and the results are compared with a conforming moving band approach with remeshing of the air gap at each time step.**

I. Introduction

Several approaches to simulate the movement within a Finite Element analysis (FEA) of electrical machines have been developed. Static and transient analysis of the machines require a flexible variation of the rotor position. An obvious and early adopted approach is the moving band (MB) technique [4] whose principle is to re-generate at each time step a single layer of conforming finite elements in a thin annulus–shaped region of the air gap. However, in practical the automatic remeshing of the air gap is only tractable for 2D rotating machines. For linear motion in 2D and motion in 3D models, air gap remeshing would imply invoking a full–fledged automatic mesh generator at each time step, which is impractical. The mortar element method (MEM) was proposed in [8] and applied to a 2D machine problem in [1]. The Lagrange multiplier (LM) method has been extensively investigated in [2]. Both MEM and LM can be extended to 3D problems, but the MEM requires an additional integration mesh [9], and for the LM the conditioning worsens significantly [6].

The non–conforming approach proposed in this paper is based on a mesh–to–mesh Galerkin projection method (GPM) which has been introduced in [5]. While applications described in [7] project a known field from a source to a target mesh, in this paper the GPM is implemented in a standard FE assembly process.

II. Projection Method and Formulation

Let $L^2(\Omega)$ be the space of square integrable functions on $\Omega \subset \mathbb{R}^n$, $n = 1, 2, 3$. The scalar product relative to the L^2–norm of F and G is defined as

$$(F, G)_\Omega = \int_\Omega F(x) \cdot G(x)\, \mathrm{d}x. \tag{1}$$

A field $F^\Omega \in L^2(\Omega)$ can be interpolated in a discrete domain as:

$$F^\Omega = \sum_{i=1}^{n} f_i \alpha_i^\Omega \tag{2}$$

where α_i^Ω is the shape function associated to the node or edge i and f_i is the corresponding coefficient.

The sliding boundaries of the rotor (master) and the stator (slave) domain are denoted Γ_N and Γ_M respectively. Note that the choice of the master boundary is based on the discretization: The boundary with the largest number of unknowns is chosen as the master boundary. Let $p : \Gamma_N \mapsto \Gamma_M$ be a bijective mapping. Consider the two magnetic vector potential fields $F^N \in L^2(\Gamma_N)$ and $G^N \in L^2(\Gamma_M)$. One wishes to have $\gamma\left(F^N \circ p^{-1}\right) = G^M$ on Γ_M and $F^N = \gamma\left(G^M \circ p\right)$ on Γ_N with $\gamma = \pm 1$ according to whether the identification between Γ_N and Γ_M is a symmetry (or an identity) or an antisymmetry. This writes in weak form:

$$\int_{\Gamma_M} \left(\gamma\left(F^N \circ p^{-1}\right) - G^M\right) \alpha_k^M \,\mathrm{d}\Gamma_M \quad = 0, \quad \forall k = 1 \ldots m \tag{3}$$

$$\int_{\Gamma_N} \left(F^N - \gamma\left(G^M \circ p\right)\right) \alpha_k^N \,\mathrm{d}\Gamma_N \qquad = 0. \quad \forall k = 1 \ldots n \tag{4}$$

It can be shown that (3) and (4) loose their symmetry on the discrete level. Thus, the idea is to use only the projection p and avoid its inverse p^{-1} by applying p to (3) which leads to the following formulation:

$$\int_{\Gamma_N} \left(\gamma F^N - \left(G^M \circ p\right)\right)\left(\alpha_k^M \circ p\right) \,\mathrm{d}\Gamma_N \quad = 0, \tag{5}$$

$$\int_{\Gamma_N} \left(F^N - \gamma\left(G^M \circ p\right)\right) \alpha_k^N \,\mathrm{d}\Gamma_N \qquad = 0. \tag{6}$$

If, after discretization, F^N and G^M are expressed by (2), (5) and (6) can be written in matrix form

$$\begin{bmatrix} \mathbf{A} & -\gamma\mathbf{B} \\ -\gamma\mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \mathbf{f} \end{bmatrix} = 0 \tag{7}$$

with the components of $\mathbf{A}$ and $\mathbf{D}$ being:

$$A_{mj} = \int_{\Gamma_N} \left(\alpha_m^M \circ p\right)\left(\alpha_j^M \circ p\right) \,\mathrm{d}\Gamma_N, \tag{8}$$

$$D_{ni} = \int_{\Gamma_N} \alpha_n^N \alpha_i^N \,\mathrm{d}\Gamma_N. \tag{9}$$

And the components of $\mathbf{B}$ and $\mathbf{C}$ expand to:

$$B_{mi} = \int_{\Gamma_N} \alpha_m^N \left(\alpha_i^M \circ p\right) \, \mathrm{d}\Gamma_N, \tag{10}$$

$$C_{nj} = \int_{\Gamma_N} \left(\alpha_n^M \circ p\right) \alpha_j^N \, \mathrm{d}\Gamma_N. \tag{11}$$

Obviously $B_{mi} = C_{jn}$ and the resulting system (7) is symmetric. The GPM can be incorporated into the equation system of any standard Galerkin FE formulation. Furthermore, no restriction regarding the degree or the type of the degrees of freedom are imposed.

III. Application: Cogging Torque

The GPM has been implemented in the *i*MOOSE–package [10]. The cogging torque of a permanent magnet synchronous machine with surface mounted magnets has been studied. The torque is calculated according to Arkkio's method [3]. The numeric field solution is obtained by means of a standard magnetic vector potential FE formulation combined with either the GPM or the MB technique. Despite its non–conformity, the mesh density for GPM, as shown in Fig. 1, is identical to the one for MB. The mapping $p = f(\varphi)$ is the rotation about the center of the rotor by the angle φ. The simulated cogging torques, normalized to the nominal torque T_0, are compared in Fig 2. Additionally, the relative difference between the GPM and the MB is shown. The GPM has as well been applied to meshes with slightly differing numbers of unknowns n in Γ_N and m in Γ_M $(0.75 < n/m < 1.25)$. The results are similar to the ones shown in Fig. 2.

IV. Discussion

Numerical results show a good agreement between the non–conforming GPM and the conforming MB. In general it can be stated, that the torque calculated by the GPM follows a smoother waveform compared to the one of the MB. The higher harmonics in the MB waveform are suspected to stem from the remeshing process. The differences between the GPM and the MB do not follow a certain pattern. Larger differences occur around the maximum as well as around the zero crossing of the torque waveform.

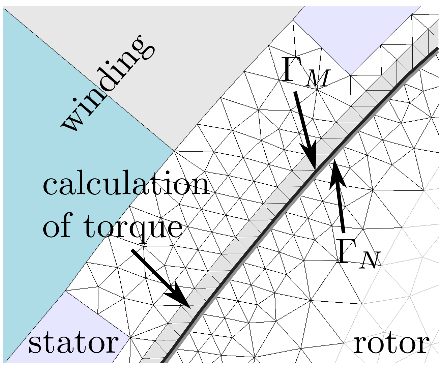

Fig. 1. Nonconforming elements at Γ_N and Γ_M of rotor and stator

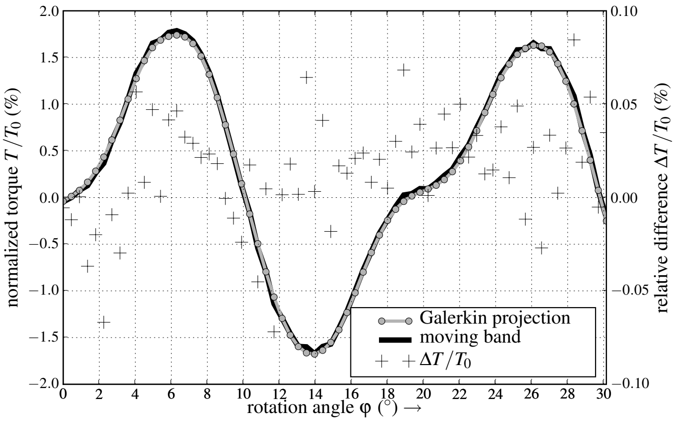

Fig. 2. Cogging torque vs. rotation angle by means of GPM and MB

V. Conclusion

This abstract presents a non–conforming method to model the sliding interfaces in electrical machines for FEA. The method is applied to the simulation of the cogging torque of a permanent magnet machine and the results are compared to a classical MB technique. The first results and the straightforward implementation of the GPM compared to MEM or LM approaches for 3D problems promise a flexible and versatile approach to deal with sliding conditions in electrical machines. The application to 3D problems as well as further investigations regarding energy conservation, error estimations, numerical integration and a comparison to measurements will be presented in the full paper.

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