OPTIMAL CONTROL OF ELECTROMAGNETIC ACTUATOR CONSIDERING ENERGY LOSS MINIMISATION IN ELECTRIC CIRCUIT

Slawomir Stepien*, Grzegorz Szymanski* and Kay Hameyer**

*Chair of Computer Engineering, Poznan University of Technology, Poznan, Poland
e-mail: Slawomir.Stepien@put.poznan.pl
**Institute of Electrical Machines, IEM RWTH – Aachen, Aachen, Germany
e-mail: Kay.Hameyer@iem.rwth-aachen.de

Keywords: optimal control, LQP problem, distributed parameters model, electromagnetic field.

Abstract

A finite element - based simulation of the single – phase electromagnetic actuator control including 3D electromagnetic field, circuit and movement model coupling to optimal control is presented. The subject of this paper is to study an optimal control technique that minimises quadratic energy function in the electric circuit. As a new contribution an algorithm based on Linear Quadratic Problem coupled with electromagnetic actuator model in a closed – loop control system is proposed.

1 Introduction

Distributed parameters models of electromagnetic devices are used to design and optimisation their construction [5]. These models can be also used to investigate an optimal excitation problem [3,6,8,9].
This paper shows an optimal control impact on dynamic behaviour of the simple electromagnetic actuator. The problem consists of minimising energy losses in the electric circuit of the device shown in Fig. 1.

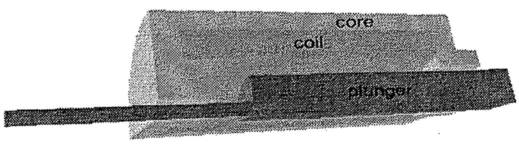

Fig.1 Quarter part of electromagnetic actuator

The problem is solved numerically using 3D model. Also the way to solve the coupled problem numerically with discussion of complexity and difficulties is presented.

2 Actuator model

The electromagnetic actuator is fed by voltage source. The equation that describes electric circuit of the device should be considered as [2,9]:

$$\frac{d}{dt}\oint_l \mathbf{A}(t)\mathbf{dl} = -Ri(t) + u(t) \tag{1}$$

where $u(t)$ represents coil voltage, R – coil resistance and $i(t)$ – coil current. The magnetic linkage flux is represented by the magnetic vector potential **A** as a variable of the 3D electromagnetic field model.

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{A}\right) + \sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V\right) - \sigma(\mathbf{v} \times (\nabla \times \mathbf{A})) = \mathbf{j} \tag{2}$$

$$\nabla \cdot \left(\sigma(\mathbf{v} \times (\nabla \times \mathbf{A})) - \sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma\nabla V\right) = 0 \tag{3}$$

The magnetic vector potential **A** and the electric scalar potential V are electromagnetic field variables [9], μ is a permeability, σ represents conductivity, **v** represents velocity of movable element and **j** current density of the thin coil.
The electric circuit equation (1) is suitable coupled with field equations (2) – (3) creating strong coupled system of equations [2]. The magnetic vector potential **A** and the electric scalar potential V are obtained using the finite element approach. By the way, an unknown coils current is calculated in each iterative step. The total force is obtained from the local forces calculation using Maxwell stress tensor method [1]. The force is a vector of three components $\mathbf{F} = [F_r \quad F_\varphi \quad F_z]^T$.
The displacement of the plunger is assumed as the one degree of freedom motion problem, which is performed along axis z. The displacement s_z and velocity v_z are chosen as a state vector of the mechanical motion.

$$\frac{d}{dt}\begin{bmatrix} s_z \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} s_z \\ v_z \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F_z \tag{4}$$

where m is plunger mass, k is a spring stiffness and b is a damping coefficient. The discrete form of (4) is obtained using the Euler's recurrence method and motion of the plunger in the electromagnetic field is solved considering the movable sub-grid method in the global mesh.

3 Controller design

The model of the device is used as a plant in the control system. The problem consists of finding a voltage course applied on the coil that minimises energy in the electric circuit. The problem is solved using closed loop system with feedback controller [4,7,8,9]. The current in the circuit can be suitable controlled by the voltage that includes in the set of admissible controls D_u:

$$u(t) \in D_u \quad (5)$$

The voltage excitation should minimise the control quality function called optimality criterion:

$$J(u) = \frac{1}{2}\int_0^T (Qi^2 + Pu^2)dt\,, \quad (6)$$

where $Qi^2 + Pu^2$ is the function of power loss, $Q,P > 0$ are weighting factors and T is a control time. Based on linear-quadratic programming [4,7] the voltage excitation function is determined by:

$$\min_{u \in D_u} J(u) \quad (7)$$

The optimal voltage control $u^*(t)$ is found as the current function $i(t)$:

$$u^*(t) = -P^{-1}K(t)\cdot i(t) \quad (8)$$

with the feedback gain $K(t)$. The gain is obtained from differential Riccati equation:

$$\frac{dK(t)}{dt} - \frac{K^2(t)}{P\eta(t)} - RK(t)\left(1 + \frac{1}{\eta(t)}\right) + Q = 0 \quad (9)$$

To get the gain $K(t)$ we can to solve the above equation numerically. For this operation we need also exact estimation of parameter $\eta(t)$.

Based on above we can propose the following control scheme shown in Fig. 2.

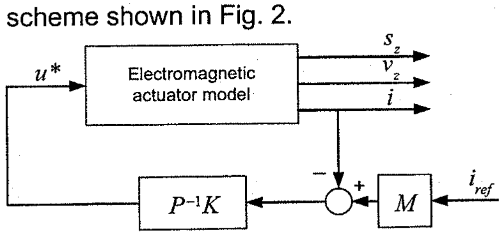

Fig.2 Closed loop control system

To control coil current we should also take the reference current i_{ref} into account in the control law (8). Its modified form is following:

$$u^* = -P^{-1}K(Mi_{ref} - i)\,. \quad (10)$$

Where the coefficient M is calculated from the term:

$$M(t) = -\frac{PR}{K(t)} + 1 \quad (11)$$

4 Control examples

The model of presented system has been used to investigate admissible voltage controls that minimising energy losses in electric circuit for $P = 1/R$ and $Q = R$. The integral function (6) is assumed as $J(u) = \frac{1}{2}\int_0^T (Ri^2 + u^2/R)dt$ and the reference current $i_{ref} = 5A$. In presented device the coil resistance is assumed as R=2,05Ω.

To demonstrate the control possibilities, three kind of simulations are compared: firstly is examined case when starting gain in Riccati equation is taken as K(0)=0, next K(0)=-0,5 and finally K(0)=-1,0. For all cases voltage excitations and coil currents are examined.

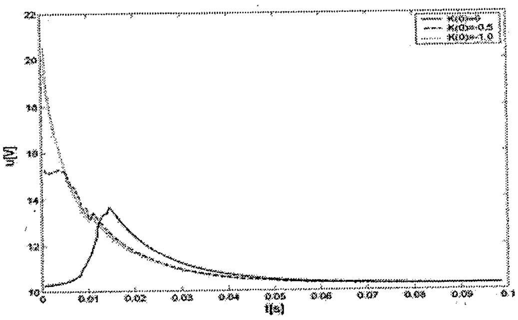

Fig.3 Voltage control signals

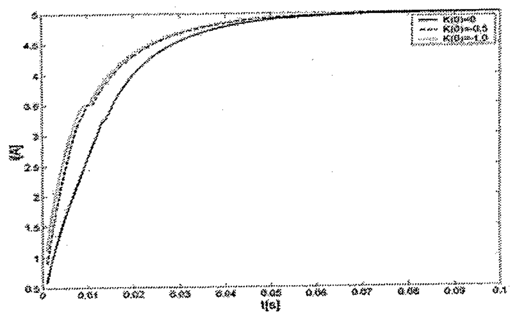

Fig.4 Coil currents

5 Conclusions

This paper demonstrates that 3D field – circuit - movement models of electromagnetic devices can be equipped with optimal controllers to perform energy loss minimization in the electric circuit. The optimal control law can be suitable applied to very complex models and the control process can be solved numerically.

References

[1] W. Demski, G. Szymański, "Comparison of the force computation using vector and scalar potential for 3D", *IEEE Trans. Mag.*, vol. 33, no. 2, pp. 1231-1234, (1997)

[2] K. Hameyer, "Field - circuit coupled models in electromagnetic simulation", *Journal of Computational and Applied Mathematics*, Vol. 168, pp. 125-133, (2004)

[3] G. Jang, J. Chang, D. Hong and K. Kim, "Finite element analysis of an electromechanical field of a BLDC motor considering speed control and mechanical flexibility", *IEEE Trans. Mag.*, Vol. 38, no. 2, pp. 945-948, (2002)

[4] T. Kailath, "Linear Systems", *Prentice-Hall*, (1980)

[5] D. Kim, K. Ship and J. Sykulski, "Applying Continuum Design Sensitivity Analysis combined with standard EM software to Shape Optimisation in Magnetostatic Problems", *IEEE Trans. Mag.*, vol.40, no. 2, pp. 1156-1159, (2004)

[6] P. Kuo-Peng, J. Roel, N. Sadowski, N. Batistela, J. Bastos, "Coupling static converter with control loop and non-linear electromagnetic devices", *IEEE Trans. Mag.*, vol. 37, no. 5, pp. 3514-3517, (2001)

[7] H. Kwakernaak, R. Silvan, "Linear optimal control systems", *Wiley – Interscience*, (1972).

[8] I. R. Melcher, "Continuum Electromechanics", *MIT Press*, Cambridge, MA, USA, (1982)

[9] S. Stepien, A. Patecki, "Modeling and Position Control of Voltage Forced Electromechanical Actuator", *COMPEL: The Int. Journal for Comp. and Mathematics in Electrical and Electronic Engineering*, Vol. 25, No. 2, pp. 412-426, (2006)