Topology Optimization for Compliance Reduction of Magnetomechanical Systems

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This paper presents an optimal approach to designing electrical machines to reduce mechanical deformation caused by magnetic nodal forces in magnetomechanical systems, while maintaining the force calculated using Coulomb's virtual work (CVW) method. It derives a design sensitivity equation by employing the adjoint variable method (AVM) to avoid undertaking numerous sensitivity evaluations for the coupled analysis. IT verifies the sensitivity analysis by using the finite-difference method (FDM). The paper examines a simple core used in a magnetic levitation system for optimal design, demonstrating the strength of the new topology optimization approach.

Index Terms—**Compliance, design sensitivity equation, magnetic nodal force, magnetomechanics, topology optimization.**

I. Introduction

THE significance of machine design considering mechanical deformation increases due to vibration and noise problems in industrial applications. In general, acoustic noise is composed of structure-borne noise and air-borne noise in electrical machines. The former is caused by mechanical vibrations in the machines and the latter is primarily generated by the flow correlation of air between the stator body and the revolving rotor [1], [2]. A source of audible noise is the structure-borne noise emitted from the deformation caused by the magnetic radial force in machines, which is called vibro-acoustics. Therefore, reducing deformation makes the machine quieter. In this context, many researchers have simulated mechanical deformations in coupled magnetomechanical analyses and compared them with experiments to judge accuracy. However, no paper related to a topology optimization algorithm in the coupled fields has been published yet.

Topology optimization is a good approach to design some structures in the beginning steps. Topology optimization is an algorithm that attempts to rearrange the material distribution satisfying the objectives of a problem under given constraints. Thus, the optimized pattern is obtained for an initial conceptual design that differs to the conventional shape optimization. Several optimal examples have been discussed [3]–[6] in electromagnetics and successfully implemented in industrial applications [5]. The topology algorithm is now sufficiently mature and can be extended to multiphysical systems [6]. Some papers regarding the homogenization design method (HDM) [4] have presented structure optimization with a simple design sensitivity analysis (DSA) derived purely in mechanics, with no links to magnetic fields [7]. This indicates that even though the objective function was evaluated using a coupled analysis, the sensitivity was calculated employing only a single mechanical field. Thus, the conventional structure design sensitivity was used for the optimization algorithm, and the optimization problems correspond to the structural ones excited by a constant magnetic force [8]. In this paper, however, a coupled DSA for magnetomechanics is derived and verified using examples.

In the optimization, sensitivity is always used for numerical search methods to provide the optimizer with the best direction in the next iteration. This sensitivity is very important to obtain the final optimum efficiently. The adjoint variable method (AVM) is a unique alternative to the finite-difference method (FDM), which is an approximate approach, for evaluating the sensitivity. The AVM is promising in topology optimization because it requires only an adjoint analysis in a single field regardless of the number of design variables [9]. Note that a number of adjoint analyses are raised in the multicoupled analysis, which is discussed in great detail in this paper.

In magnetomechanical problems, the magnetic force is the most important component describing a coupled term between both fields. Most research relevant to electromagnetics has focused on magnetic force density and employs the Maxwell stress tensor (MST) method since the MST is a simple technique based on magnetic flux density computed in one analysis. If the purpose of a magnetic analysis is only to obtain the force, the force density can be calculated in the air gap. The total force acting on a movable body is identical to the sum of the force density in the air gap. On the other hand, in a coupled magnetomechanical system, the force should be evaluated on the boundary nodes of the body, adjacent to the air elements. Accordingly, nodal forces are applied to the mechanical analysis yielding partially different structural deformations. However, once the force density is calculated in the magnetic analysis, a constant magnetic force is loaded on all nodes for the mechanical analysis. The authors note that the force density on the body is not a good approach for a coupled analysis.

In this paper, the topology optimization of the magnetomechanical system is presented employing a coupled adjoint design sensitivity equation, which is derived using the discrete method. The magnetic nodal force is calculated based on Coulomb's virtual work (CVW) method. In order to reduce

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structural deformation, three different problem sets are defined and optimized.

II. Calculation of Magnetic Nodal Force

The finite-element method regarding both magnetostatics and elastic mechanics is based on the minimization of stored energy [10]. The total energy E equals the sum of the magnetic energy, E_{mg}, and the elastic energy, E_{mc}:

$$E = E_{mg} + E_{mc} = \frac{1}{2}(A^T SA + u^T Ku) \tag{1}$$

where A, S, u and K are the magnetic vector potential, the magnetic stiffness matrix, the displacement, and the mechanical stiffness matrix, respectively.

The partial derivative of the total energy with respect to the unknowns $[A, u]^T$ yields

$$\frac{\partial E}{\partial A} = J = SA + \frac{1}{2}u^T \frac{\partial K}{\partial A}u \tag{2}$$

$$\frac{\partial E}{\partial u} = F_{mc} = \frac{1}{2}A^T \frac{\partial S}{\partial u}A + Ku \tag{3}$$

where J is the magnetic source vector and F_{mc} is the mechanical load vector.

By applying the principle of Coulomb's virtual work (CVW) at constant flux linkage, the magnetic force acting on the movable body is

$$F_{mg} = \frac{\partial E_{mg}}{\partial s} = \frac{1}{2}\left(\frac{\partial A^T}{\partial s}SA + A^T \frac{\partial S}{\partial s}A + A^T S\frac{\partial A}{\partial s}\right) \tag{4}$$

where s denotes the direction of the force to be calculated.

In this paper, the coupling term expressed in the second term on the right-hand side of (2) is ignored for simply coupled magnetomechanics due to the assumption that there is no effect on the magnetic field from the mechanical deformation. Differentiating both sides of (2) yields

$$\frac{\partial S}{\partial s}A + S\frac{\partial A}{\partial s} = \frac{\partial J}{\partial s} = 0. \tag{5}$$

By substituting (5) into (4), the force can be expressed as (6) due to the symmetric matrix S

$$F_{mg} = -\frac{1}{2}A^T \frac{\partial S}{\partial s}A. \tag{6}$$

The matrix form for the finite element coupled system can subsequently be expressed by

$$\begin{bmatrix} S & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ u \end{bmatrix} = \begin{bmatrix} J \\ F_{mc} + F_{mg} \end{bmatrix}. \tag{7}$$

In another way as presented in [11] in which the differentiation of S can be performed by direct differentiation with regard to s, the force is obtained using the directional derivative. The perturbed nodal coordinate vector $\overline{p}$ is defined as

$$\overline{p}(x, y) = p(x, y) + s \cdot v(x, y), v \in V \tag{8}$$

where p is the original nodal coordinate vector and V is a vector space of the virtual displacement field (VDF) which has a unit magnitude on the nodes of the movable body.

The contribution of each node to the force is found by differentiating (6) with respect to the direction in which the force is sought. Since S is a function of the coordinate, the nodal force is obtained by taking directional derivatives

$$F_{mg} = \left.\frac{\partial E_{mg}(p + sv)}{\partial s}\right|_{s=0} = \frac{\partial E_{mg}}{\partial \overline{p}}\frac{\partial \overline{p}}{\partial s}. \tag{9}$$

From the mathematic property $\partial E_{mg}/\partial \overline{p} \equiv \partial E_{mg}/\partial p$, (9) becomes

$$F_{mg} = -\frac{1}{2}\frac{\partial (A^T SA)}{\partial p}v. \tag{10}$$

For the x-direction and the y-direction at node 1, the magnetic nodal force in an element coordinate system can be rewritten as

$$F^{(e)}_{mg.x1} = -\frac{1}{2}A^{(e)T}\frac{1}{\mu^{(e)}}\left(\frac{\partial \hat{S}^{(e)}}{\partial p_{x1}}v_{x1}\right)A^{(e)} \tag{11}$$

$$F^{(e)}_{mg.y1} = -\frac{1}{2}A^{(e)T}\frac{1}{\mu^{(e)}}\left(\frac{\partial \hat{S}^{(e)}}{\partial p_{y1}}v_{y1}\right)A^{(e)} \tag{12}$$

where superscript (e) denotes the element number and the magnetic stiffness matrix $S = (1/\mu) \cdot \hat{S}$, where the hat $(\wedge)$ indicates the matrix without the material property.

Suppose that a four-node quadrilateral element is used, the nodal forces of (11) and (12) in the element coordinate belong to the force vector expressed by

$$F^{(e)}_{mg} = [F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3}, F_{x4}, F_{y4}]^T. \tag{13}$$

The total nodal force on a particular node is found by summing the forces obtained from all surrounding elements. Once the finite-element (FE) model is built, a coefficient representing the element matrix assemblage enclosed by parentheses in (11) and (12) remains constant for the predefined VDF in the magnetic analysis. The coefficient matrix calculated in advance is used repeatedly in each iteration of the optimization process.

III. Design Sensitivity Equation

For magnetomechanical systems, the performance index can be defined as

$$\psi = \psi(u, b)d\Omega \tag{14}$$

where b is a vector of the design variables, expressed by

$$b = [E, \mu]^T \tag{15}$$

where E and μ are the elasticity coefficient and the permeability, respectively. Taking derivatives of (14) with respect to the design variable yields [3]

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \frac{\partial \psi}{\partial u}\frac{du}{db}. \tag{16}$$

Differentiating both sides of (7) yields

$$\frac{dA}{db} = S^{-1} \cdot \left[\frac{\partial J}{\partial b} - \frac{\partial}{\partial b}(S\widetilde{A})\right] \tag{17}$$

$$\frac{du}{db} = K^{-1} \cdot \left[\frac{\partial F_{mc}}{\partial b} + \frac{\partial F_{mg}}{\partial A}\frac{dA}{db} - \frac{\partial}{\partial b}(K\widetilde{u})\right]. \tag{18}$$

Substituting (18) into (16) results in

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \lambda_u^T \cdot \left[\frac{\partial F_{mc}}{\partial b} + \frac{\partial F_{mg}}{\partial A}\frac{dA}{db} - \frac{\partial}{\partial b}(K\widetilde{u})\right]. \tag{19}$$

λ_u is a vector of an adjoint variable for the mechanical systems, and the tilde ($\sim$) indicates a variable that is to be maintained constant for partial differentiation. The adjoint equation corresponding to (19) is written as

$$K\lambda_u = \left[\frac{\partial \psi}{\partial u}\right]^T = F_{eq}. \tag{20}$$

Using (20), the mechanical adjoint variable can be obtained from the original mechanical analysis with the loading vector, which is replaced by an adjoint load, F_{eq}.

The derivative of A with respect to the design variable b in (19) is explicitly impossible such that (17) is substituted for dA/db in (19)

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \lambda_u^T \cdot \left[\frac{\partial F_{mc}}{\partial b} - \frac{\partial}{\partial b}(K\widetilde{u})\right] + \lambda_A^T \left[\frac{\partial J}{\partial b} - \frac{\partial}{\partial b}(S\widetilde{A})\right] \tag{21}$$

where λ_A is a vector of an additional adjoint variable for the magnetic systems, which is computed by an extraordinary adjoint equation

$$[S] \cdot \lambda_A = \left[\frac{\partial F_{mg}(A)}{\partial A}\right]^T \lambda_u = J_{eq}. \tag{22}$$

The adjoint load in the adjoint equation (22) must be treated cautiously for a precise sensitivity evaluation. Since the quadrilateral elements are used in the FE model, it is noted that $[\partial F_{mg}/\partial A]$ regarding the (x, y) coordinates becomes an (8 $\times$ 4) matrix for each element. Equation (22) shows that the magnetic adjoint variable is obtained from the magnetic analysis with an equivalent load vector J_{eq}. The adjoint load is ultimately calculated from

$$J_{eq} = -\left[\frac{\partial (S\widetilde{A})}{\partial p} v\right] \lambda_u. \tag{23}$$

IV. Topology Optimization

A general definition of topology optimization is to seek an optimal material distribution to maximize or minimize an objective function subject to given constraints.

In this research, the objective of the optimization process is to minimize the mechanical compliance caused by the magnetic nodal forces, which is equivalent to minimizing the deformation caused by the magnetic reluctance forces. The radial force causes vibrations that can emit structure-borne noise. Thus, the final effects can be found extensively in noise reduction, which is not discussed in this paper. The quantity of the objective function is evaluated from a magnetomechanical analysis where the force calculated in the magnetic field should be applied as load vectors to the mechanical system. Since the pattern of material distribution in the design domain changes during the optimization process, the changed force is computed for each iteration.

In most papers, general optimization problems contend with volume constraints to reduce the product price. However, a constraint regarding volume is insufficient to solve the optimization problem that aims to minimize compliance when the induced magnetic force is considered. As an important factor in the actuator, the attractive force, i.e., the magnetic force of x-direction in a numerical example, is chosen as an extra constraint.

The in-house code (IHC) developed for these optimization problems controls all processes and computes the magnetic nodal forces that are automatically loaded in the mechanical computations. Accordingly, it is convenient that the adjoint loads for the topology design sensitivity are obtained without extra calculations. The IHC uses an analyzer (ANSYS) to estimate the objective function and volume used as one of constraints, and iteratively evaluates the design sensitivity. The sequential linear programming (SLP) algorithm is used to compute design changes, and materials such as permeability, and Young's modulus are updated consecutively. The material interpolation functions describing the relationship between the material and design variables are defined as

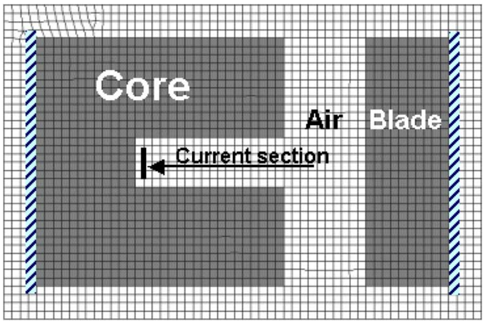
$$\mu = \mu_0 + (\mu_0\mu_r - \mu_0)b^P \tag{24}$$

$$E = b^P E_{\text{initial}} \tag{25}$$

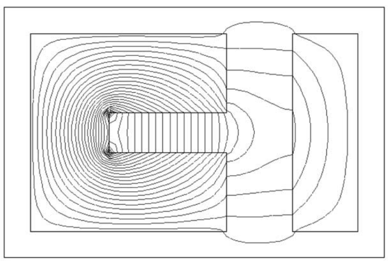
where μ_0 and μ_r denote the permeability of air and the relative permeability of the core, respectively. An artificial design variable b indicates the density distribution between the solid and the void. That is, 1 implies that material exists and 0 indicates a void. The intermediate design variables are removed using the penalty parameter p in the power law, b^P.

V. Numerical Example

For the application of a magnetic levitation system, a C-core structure is used to demonstrate the proposed approach to solve the coupled physics. Fig. 1(a) shows the setup for a numerical example which consists of a simple core and a blade. The currents applied to the current section generate the magnetic flux seen in Fig. 1(b) that causes the magnetic reluctance forces on the actuator. The forces acting on the body are loaded on the nodes and cause consecutive deformations in the mechanical analysis. One sidewall of each structure is fixed to ensure that the structures are deformed due to the stiffness property. In the model, the optimization problem finds an optimal material distribution in the core to minimize the deformation. In order to display the force quantity, a plot program was developed using MATLAB. Each quantity, such as Fx and Fy, and the total forces in the original model, i.e., the model prior to optimization, are shown in Fig. 2. When the total force is applied in the model, the

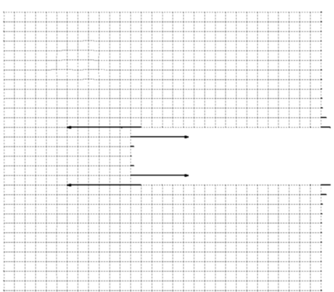


(a)



(b)

Fig. 1. Design (a) domain and (b) plot of magnetic flux lines.



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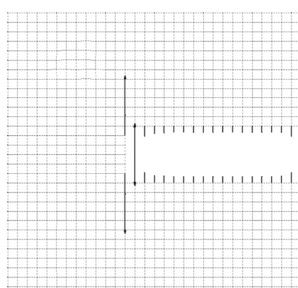

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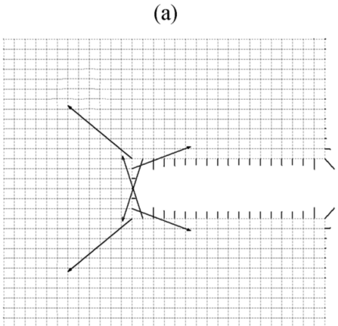

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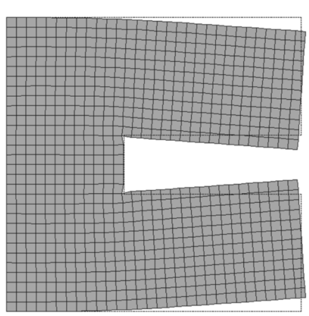

(d)

Fig. 2. Plot of magnetic reluctance force and deformation of the core. (a) Fx (x-direction force); (b) Fy (y-direction force); (c) total force $(\mathrm{Fx}+\mathrm{Fy})$; (d) deformation under total force.

resulting deformed shape of the core is obtained and illustrated in Fig. 2(d).

The same model with a coarse mesh is used to verify the DSA derived using the AVM. Five selected elements as numbered in Fig. 3 are compared with the central FDM. Table I shows that the AVM is in good agreement with the FDM.

The minimization of the mechanical deformation results from the interaction between the improvement in the structural stiffness and the decrease of magnetic reluctance forces in the example. One of the best designs for practical application is to

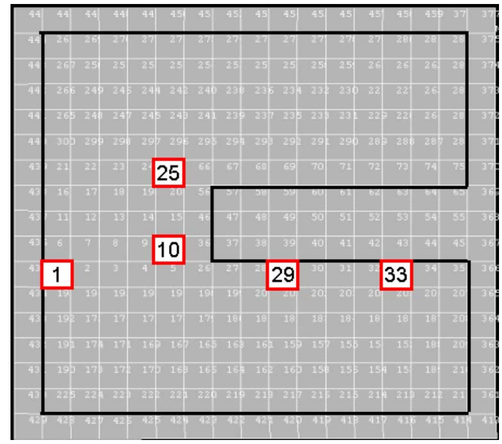

Fig. 3. Core model for sensitivity verification.

TABLE I
SENSITIVITY COMPARISON BETWEEN FDM AND AVM

Element ID	FDM*	AVM	AVM/FDM [%]
1	-8.646×10^{-2}	-8.663×10^{-2}	100.197
10	-2.969×10^{-1}	-2.979×10^{-1}	100.330
25	-2.070×10^{-1}	-2.074×10^{-1}	100.193
29	-1.225×10^{-1}	-1.225×10^{-1}	100.033
33	-1.193×10^{-2}	-1.185×10^{-2}	99.329

* $\Delta\psi = (\psi(b+\delta b)-\psi(b-\delta b))/2\delta b$, central FDM with 1% perturbation

minimize structural deformation while maintaining magnetic force as much as possible. In order to investigate these effects, three optimization cases are examined.

A. *Problem A*

The objective function is to minimize the mechanical compliance, i.e., $E_{mc} = u^T K u$, excited by the attractive force only, i.e., the x-direction force (Fx) in the model. The volume is the only constraint used to limit the optimal design. Hence, the topology optimization problem takes the form

$$\begin{aligned}&\text{Min. } \Psi = \text{Mechanical Compliance caused by Fx}\\&\text{subject to } g = \frac{\int\int_{\Omega 1} bA_r t d\Omega}{0.7\,V_0} - 1 \le 0\end{aligned} \tag{26}$$

bound to $0 < b_{\min} \le b \le 1$ for all $b \in \Omega$.

A_r is the area, t is the thickness, V_0 is the initial volume, and b is the density variable. $b_{\min}$ is the lower bound of the densities introduced to prevent singularity of the equilibrium problem.

B. *Problem B*

The objective function is identical to *Problem A*. The difference is that the force constraint is also considered to maintain the attractive force in comparison with the initial force. Thus, the optimization setup subject to two constraints becomes

$$\begin{aligned}&\text{Min. } \Psi = \text{Mechanical Compliance caused by Fx}\\&\text{subject to } g_1 = Fx_{\text{initial}} - Fx \le 0\\&\qquad g_2 = \frac{\iint_{\Omega 1} bA_r t d\Omega}{0.7\,V_0} - 1 \le 0\end{aligned} \tag{27}$$

bound to $0 < b_{\min} \le b \le 1$ for all $b \in \Omega$.

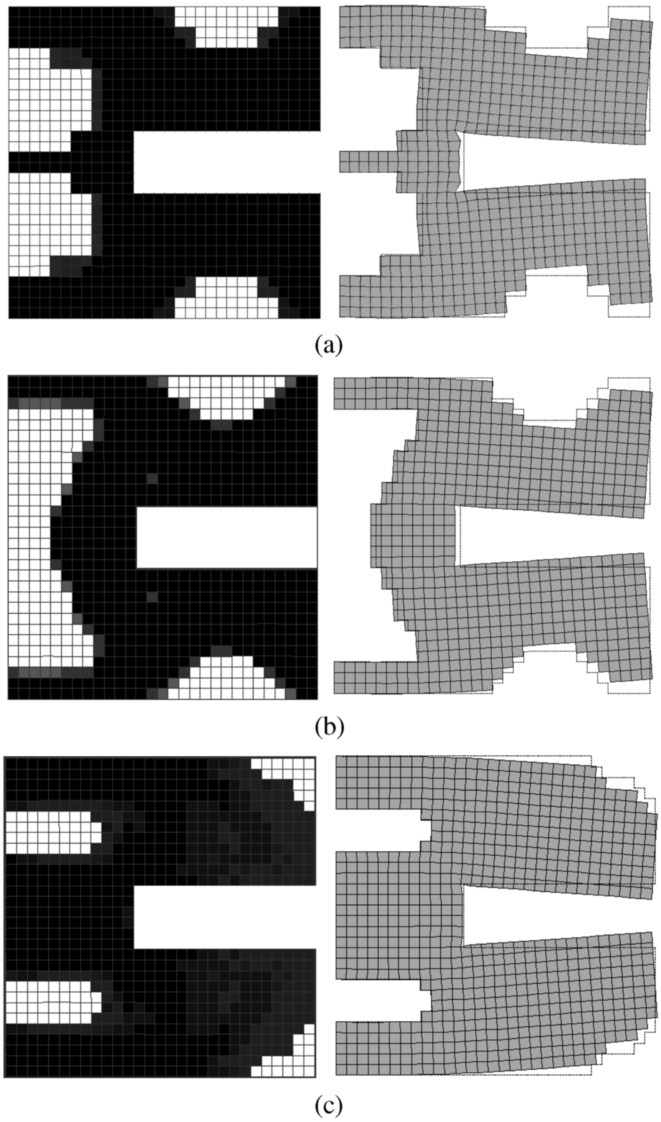

Fig. 4. Final optimal patterns and deformations of reanalysis models. (a) Design A—result of *Problem A*; (b) Design B—result of *Problem B*; (c) Design C—result of *Problem C*.

C. Problem C

The variation of *Problem C* from *Problem B* is that the transverse force, i.e., y-direction force (Fy), is appended to the exciting forces causing core deformation. That is, the total force occurring in the core is taken into account

$$\begin{aligned} \text{Min. } \Psi &= \text{Mechanical Compliance} \\ &\quad \text{caused by Fx + Fy} \\ \text{subject to } g_1 &= Fx_{\text{initial}} - Fx \leq 0 \\ g_2 &= \frac{\iint_{\Omega 1} bA_r t d\Omega}{0.8\, V_0} - 1 \leq 0 \end{aligned} \tag{28}$$

bound to $0 < b_{\min} \leq b \leq 1$ for all $b \in \Omega$.

Fig. 4 illustrates the final optimal patterns with gray areas for each problem and the structural deformations of the reanalysis models without gray areas when the magnetic nodal forces are obtained in the reanalysis models and applied in the model. The reanalysis models are obtained from the final optimal patterns by choosing a threshold larger than 0.5. In order to compare the magnetic properties in a magnetic analysis, plots of the magnetic flux lines are shown in Fig. 5. The flux lines of the models are concentrated in the inner corner, which implies that magnetic saturation takes place there before anywhere else. Also, in the inner corner, the magnetic losses due to the eddy current and hysteresis are expected to be the largest in the time harmonic field, which is not discussed in the paper. Design A has broader regions of flux saturation than other designs, and therefore, the magnetic force is expected to be less than the others. The effects are identified in Table II, which presents the comparison between the analysis results of the original and the reanalysis designs. Three quantities were chosen for comparison: the mechanical compliance, the force in the x-direction, and the volume used for the designs. Design B, which considers the constraints of both the attractive force and volume, only provides better performance than design A in relation to volume. The force constraint is effective in enhancing the magnetic characteristics. When the attractive force and the transverse force are applied in the model, the deformed amount is larger than that of the design loaded with only the attractive force. For this case, design C is optimized from problem C and is suggested for application in practical systems. Although the mass is reduced for a light structure in the system, the mechanical deformation is almost identical while inducing a better attractive force.

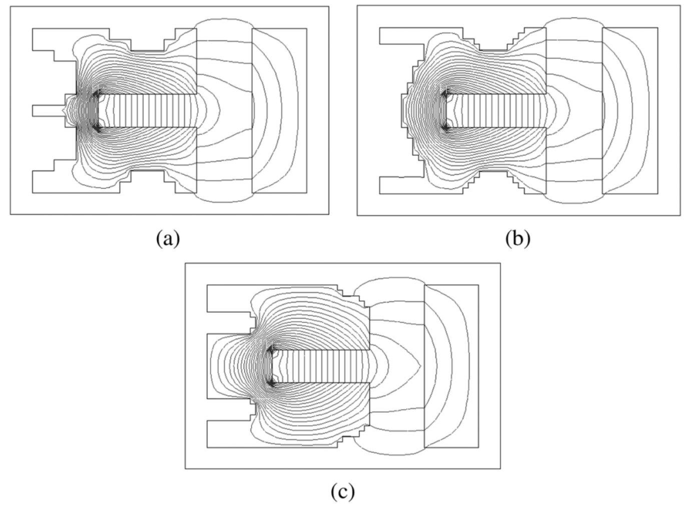

Fig. 5. Plots of magnetic flux lines. (a) Design A—result of *Problem A*; (b) Design B—result of *Problem B*; (c) Design C—result of *Problem C*.

TABLE II
COMPARISON BETWEEN INITIAL AND REANALYSIS MODELS

	Initial	Design A	Design B	Design C
Compliance [%]	100	102.06	101.3	99.85
X-directional magnetic force [%]	100	96.39	100.02	102.88
Used volume [%]	100	73.74	73.86	85.10

VI. CONCLUSION

The topology optimization of the magnetomechanical system is proposed to reduce the structural deformation caused by magnetic force. This paper emphasizes the adjoint design sensitivity analysis derived for the coupled topology optimization by reducing a number of computations. In order to solve the DSA, the adjoint load formula is derived using the magnetic nodal force. The force is calculated by employing the VDF in the magnetic field and applied to the mechanical analysis. The DSA is verified using the FDM in the core design application. The three optimal patterns are presented from three different problem sets. An optimal design is suggested to reduce the mass of the structure and yields less deformation and better attractive force.

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