# Identification of homogenized equivalent materials for the modal analysis of composite structures in electrical machines

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**ABSTRACT**

This paper addresses the issue of determining homogenized equivalent mechanical properties of composite structures in electrical machines. A procedure consisting of two operations, weight measurement and modal testing is proposed. These operations are performed repeatedly after each fabrication step. Therewith mass density and stiffness properties are determined and can be used for the numerical simulation. This procedure is applied by way of illustration for the stator of a 550W induction machine and experimental results are presented.

## 1 INTRODUCTION

Estimation and calculation of the vibration and acoustic noise in electric machinery is of high interest. Significant amount of research in this area has been done throughout the past century (1), (2), (3). Such analyses focus mainly on two aspects: mechanical excitation of the stator by the harmonics of the magnetic air gap field, and the mechanical properties of the stator. Classical approaches are mostly based on analytical models and experimental analysis. As substantial computational power is available nowadays, more accurate numerical simulation are performed (4).

Since the stator and the housing of induction machines are the most important generators of audible noise towards environment, the analysis concentrates on those components and the rotor can be disregarded in the structural modal analysis. Electrical machine stators are manufactured in several successive steps:

1. Stacking of sheets and backing them together under pressure
2. Cutting of the stator contour by means of wire-electro discharge machining
3. Winding of insulated copper wires into the stator slots
4. Impregnation of the winding with resin
5. Pressing in of the stator into frame
6. Final assembly of stator, housing, rotor and end shields

In consequence, the system stator-housing constitutes a complex heterogeneous structure, not only because it is composed of intertwined elements with different material properties (steel, copper, insulator, resin), of which the mechanical cohesion might moreover be imperfect, but also because different steps in the fabrication process are known to deteriorate locally the microstructure of steel laminations in a significant but unknown way.

The structure under analysis for the study of radiated noise is therefore a composite object whose properties cannot be derived from the knowledge of those of the individual constitutive elements (manufacturer's data sheet for instance) (5), (6). For this reason, it is practically impossible to obtain the complete input dataset that is required for undertaking any numerical analysis. This paper presents a methodology to overcome this fundamental design difficulty.

The basic idea is to devise identification experiments that allow acquiring the missing or unavailable information about the composite structure. On the basis of those specific measurements and by means of numerical identification methods, equivalent homogenized material parameters are determined by means of an iterative procedure. When a sufficient accuracy is obtained, the equivalent characteristics can be used for performing more elaborate numerical simulations. Approaches to measure Young's modulus of electrical machine's stators have been developed, but either have not been applied to the individual composite materials of the stator separately (7), or the influences of fabrication have not been included (8). Modal analysis has been used after different fabrication steps, however the procedure has not been used to determine homogenized material parameters (4), (9).

In parallel, the homogenized characteristics may also estimate by standard analytical methods (10), for control purposes and to provide a more suitable starting point for the optimization process.

The identification is carried out repeatedly after each of the fabrication steps listed above, so as to distinguish the influence of the different constitutive elements on the final overall vibrational dynamics of the machine.

Although the homogenized material parameters are by construction intrinsically model-dependent, i.e. they cannot be transferred to other machine types; their comparison with standard material characteristics provides for each individual machine a measure of the manufacturing parasitic effects. This measure is very useful from a technical point of view, as it contributes to build up a body of knowledge about fabrication effects in general.

The outline of the paper is as follows: After a short introduction into the structural dynamic simulation of electrical machines, the general concept of parameter identification by means of modal analysis at each manufacturing step is introduced. Afterwards, the described procedure is applied to an induction machine prototype and experimental results are given. A summary and a perspective for future work conclude the paper.

## 2 PARAMETER IDENTIFICATION METHOD

### 2.1 Structural dynamic simulation of electric machines

The purpose of numerical structural dynamic simulation is to predict the vibrational behavior of a given structure by means of a numerical model. Several approaches exist. In most cases however, a matrix equation of the form

$$\mathrm{Kx} + \mathrm{C}\frac{d}{dt}\mathrm{x} + \mathrm{M}\frac{d^2}{dt^2}\mathrm{x} = \mathrm{f} \qquad [1]$$

is composed, by means of e.g. a finite element (FE) formulation. **K** is the stiffness matrix, **C** the damping matrix, **M** the mass matrix and **x** and **f** the displacement and force vector, respectively. Assuming a harmonic response, the equation is transformed into

$$\left(\mathrm{K} + i\omega\mathrm{C} - \omega^2\mathrm{M}\right)\mathrm{x}e^{i\omega t} = \mathrm{f} \qquad [2]$$

with the angular frequency $\omega$. Modal analysis seeks for the eigenvalues $\omega^2$ and eigenvectors **x** (mode shapes) of the free vibrational problem, i.e. for **f** = 0 (11). The constitution of the stiffness, mass and damping matrices depend on the material distribution, which is a combination of geometry and material properties.

The numerical structural dynamic analysis of electrical machines is either done as modal analyses (3), (12) or as forced vibration analyses (13). In either case, the harmonic displacement during operation is the quantity of interest. It is typically calculated with electromagnetic forces as the excitation. A subsequent step, which may be undertaken to determine the radiated audible noise, is an acoustic boundary element simulation, taking the harmonic displacement of the stator housing as excitation (14).

Due to practical limitations in the number of elements and therefore in the degree of detail used in the model, the micro structural properties, such as filaments of the winding or individual sheets cannot be modeled. Furthermore, the fabrication process incorporates additional unknowns into the mechanical properties. Therefore, a realistic modeling approach consists in finding homogeneous material parameters, which yield the same macroscopic behavior as the real structure. The objective of this study is to define simple identification experiments for such parameters.

### 2.2 Mass density

The identification is done in parallel with the subsequent fabrication steps. After each fabrication step the same operations are performed: Determination of mass densities and modal analysis.

Firstly, the mass density of the different material regions of the FE simulation are determined. Because of the imperfectly known heterogeneous composition of the composite structure under analysis, the values of mass density cannot be directly taken from literature. A characteristic value for the portion of copper per stator cross-sectional slot area is the so called fill factor. Typical values of approximately 40% are achieved according to the winding technique. Therefore, the stator is weighed after each fabrication step and the increase of mass **Δ** $m_k$ is recorded. Using the FE model, the volume $v_k$ of the added component is calculated and the mass density $\rho_k$ of composite material $k$ is simply given by

$$\rho_k = \frac{\Delta m_k}{v_k} \,. \qquad [3]$$

## 2.3 Modal analysis procedure

The second operation consists in performing a modal analysis after each fabrication step. The modal parameters obtained are then used together with the numerical simulation of that step to identify the stiffness and damping properties of the composite material.

The modal parameter extraction method used in this study is a classical single degree-of-freedom (SDOF) circle-fit approach (11). Since this is only one among several possible modal analysis approaches, and because more complex methods, such as multi degree-of-freedom (MDOF) and multiple frequency response functions (FRF) analysis methods can be used, the SDOF extraction method is only reviewed briefly here (11).

The time signal of the force excitation $f_j$ ($t$) at measurement point $j$ and the acceleration $a_k$ ($t$) at measurement point $k$ are sampled. The FRFs of both $F_j$ ($\omega$) and $A_k$ ($\omega$) signals are obtained, by means of the discrete Fourier transform (DFT). The transfer function of displacement, called receptance, is defined as:

$$H_{j,k}\left(\omega\right) = -\frac{A_k\left(\omega\right)}{\omega^2 F_j\left(\omega\right)} \,. \qquad [4]$$

The complex receptance $H_{j,k}$ ($\omega$) is then plotted in the complex plane for a frequency interval around the resonance and a circle is sought that fits the measured data points in the least-square sense. The interval is determined visually around the peaks of the receptance in the bode plot. This approach requires the assumption that in this interval the FRF is dominated by a single mode.

The angle $\theta$ between the radius to each data point and the imaginary axes depends on frequency and the maximum of the frequency sweep $d\omega/d\theta$ indicates the resonance frequency $\omega_r$ of that mode. The structural damping ratio is determined by any pair of data points, one before the resonance $\omega_b$ and one after the resonance $\omega_a$, by

$$\eta_r = \frac{\omega_a^2 - \omega_b^2}{\omega_r^2\left(\tan\left(\theta_a/2\right) + \tan\left(\theta_b/2\right)\right)} \,. \qquad [5]$$

The complex modal constant $A_{j,k,r}$ is now given by the calculated radius $R$

$$A_{j,k,r} = 2R \cdot \omega_r^2 \eta_r \cdot e^{i\theta r} \,, \qquad [6]$$

where $\theta_r$ is the polar angle at the resonance. After extracting the parameters of all $N$ significant modes of the system can be represented by the following analytical model

$$\tilde{H}_{j,k}(\omega) = \frac{1}{-\omega M} + \sum_{r=1}^{N} \frac{A_{j,k,r}}{\omega_r^2 - \omega^2 + i\eta} + \frac{1}{K}, \qquad [7]$$

where the scalar parameters $M$ and $K$ are determined by the modes that lay outside the measurement range and are calculated from the measured FRF at the low frequency and high frequency boundary, respectively.

In this study, two methods have been used to determine the shapes of the individual modes. The first one is a visual determination, which uses the displacement at a single frequency $\omega_m$ and shows an animation of the time varying deformation on the computer screen using the visualization toolkit (VTK) (15). The radial displacement $d_l$ of node $l$ at time $t$ and at the frequency $\omega_m$ is computed to

$$d_k = s\left(\Re\left\{H_{j,k}(\omega_m)\right\}\cos(\omega_s t) + \Im\left\{H_{j,k}(\omega_m)\right\}\sin(\omega_s t)\right), \qquad [8]$$

where the factor $s$ and the frequency $\omega_s$ are adapted to visualize the desired effect in an adequate speed and scale.

Since an equidistant mesh of the stator surface is used, the second method to determine the mode shape numerically is to perform a spatial DFT along the circumference. This gives a spectral decomposition of the mode shapes into a series of fundamental radial modes shown in Figure 1. This procedure is then repeated for each axial plane.

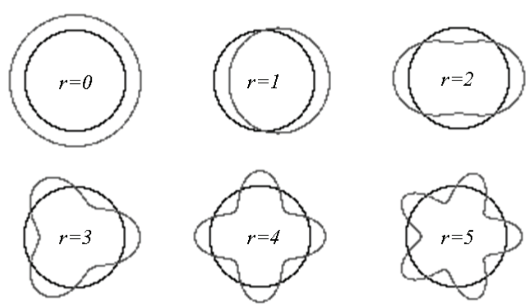


**Figure 1: Fundamental radial mode shapes**

The experimental results of this study have shown, that the results obtained from the modal analysis of the copper winding without resin are limited, since the missing resin seams to provoke dry friction, which leads to non-linear behavior contradicting the assumptions of modal analysis. Therefore, this fabrication step should only be used to determine the ratio between matrix (resin) and filaments (copper). Note, that the concept of filaments in a matrix is only valid for small to medium sized machines, since large machines do not have a random wire distribution. In that case, the concept of periodic copper bars seams to be more appropriate. This, however, does not affect the validity of the proposed approach, because this is independent from the underlying model of the equivalent material.

### 2.4 Ansys numerical model

Using the software package ANSYS, a FE model based on hexahedrons is constructed to perform the 3D numerical modal analysis. As a simple example of the proposed procedure, the experimental modal analysis and mass data is used to adjust the eigenfrequencies of the stator system. The model with and without winding is shown in Figure 2. The first one contains 47233 elements and the second one contains 91584 elements. The element type is solid185, which is a first order element. ANSYS is used to solve the undamped eigenvalue problem from [2]. Clamping is implemented as zero-value Dirichlet boundary condition.

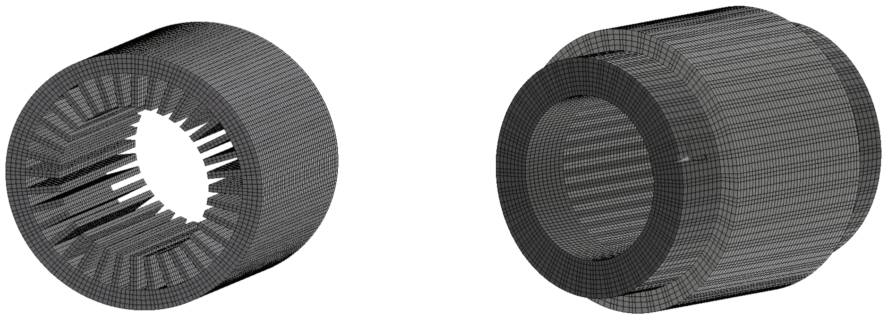

**Figure 2: FEM model, homogenized sheet stack (left) and stack with winding (right).**

The numerical simulation is used to identify the stiffness properties of the composite material. The easiest approach consists in adapting the Young's modulus of the material so that the numerically obtained resonance frequencies fit the experimentally obtained values. For this purpose, a statistical optimization method, such as Simulated Annealing or Threshold Accepting, may also be used (16).

Note, that the chosen modal analysis method depends on the desired accuracy and whether one wishes to obtain isotropic or anisotropic stiffness properties. If, for instance, only an isotropic identification of the stiffness properties is sought, then a simple identification by means of eigenfrequencies is sufficient. If, however, a highly accurate description of the mechanical properties is desired, and the full anisotropic character of the material should be taken into account, then all modal constants, as well as the corresponding mode shapes are to be measured. Note, that the proposed approach is a general concept, which indeed is applied as a single Young's modulus estimation here, and can be used to determine isotropic as well as anisotropic mechanical properties.

In addition to the identification method discussed so far, additional information can help achieving a better estimation of the mechanical properties of the composite materials by means of homogenization techniques. The knowledge of the thickness of the steel sheet $h_1$ and its lamination $h_2$, or the copper cross-section of one wire, defined by radius $r_1$ and the thickness of the isolation layer on the conductor's surface defined by the radius $r_2$ for instance, can be exploited to improve the method. The complete parameter identification method is summarized in Table 1, where **H** indicates Hooke's matrix representing the anisotropic stiffness properties, $m$ the mass and $\rho$ the mass density of the composite materials: lamination (lam), copper (cu) and resin (res) and aluminum of the housing (al). It may also benefit the analysis to know the mass of the tape material $m_{tape}$ used to tie the winding together.

**Table 1: Summary of parameter identification method.**

| Component | Mass Densities | Modal Parameters | Additional Info |
|---|---|---|---|
| Laminated sheet stack after cutting | $m_{lam} \rightarrow \rho_{lam}$ | $\mathbf{H}_{lam}$, $\eta_{lam}$ | $h_1 / h_2$ |
| Sheet stack with copper wires | $m_{cu} \rightarrow \rho_{cu}$ | ($\mathbf{H}_{cu}$, $\eta_{cu}$) | $r_1/r_2$, $m_{tape}$ |
| Sheet stack with impregnated winding | $m_{res} \rightarrow \rho_{res}$ | $\mathbf{H}_{cu,res}$, $\eta_{cu,res}$ | |
| Stator pressed into frame | ($m_{al} \rightarrow \rho_{al}$) | influence of pressure | |

# 3 EXAMPLE AND EXPERIMENTAL RESULTS

## 3.1 Experimental setup for model analysis

The machine under analysis is a 550W six pole induction machine with squirrel-cage rotor. The rating plate data and relevant geometrical parameters of the stator are listed in Table 2. The machine is designed for continuous operation as a pump drive at constant voltage and frequency. The weight after each step is recorded and given in Table 3.

**Table 2: Rating plate data of the machine and geometrical parameters of the stator.**

| | | | |
|---|---|---|---|
| Rated power $P_r$ | 550 W | Stator core inner diameter $D_{1in}$ | 70 mm |
| Rated voltage $V_r$ | 380 V | Stator core outer diameter $D_{1out}$ | 120 mm |
| Rated speed $n_r$ | 927 1/min | Effective length of the stator core $L_i$ | 80.5 mm |
| Rated power factors $\cos\varphi_r$ | 0.68 | Stator tooth width $c_t$ | 3 mm |
| Rated frequency $f_r$ | 50 Hz | Thickness of stator yoke $h_c$ | 9.15 mm |
| Number of stator slots $s_1$ | 36 | Length of winding overhang $h_{ov}$ | 25 mm |

**Table 3: Overall weight at each manufacturing steps.**

| Component | Total Mass (kg) |
|---|---|
| Laminated sheet stack after cutting | 3.08 |
| Sheet stack with copper wires | 4.36 |
| Sheet stack with impregnated winding | 4.41 |
| Stator pressed into frame | 4.90 |
| Complete Machine | 8.64 |

The vibrational response of the studied machine is measured after each manufacturing step in two conditions: freely hanging on a rubber belt and clamped to the ground with a force of approximately 1.4kN via a prism by an aluminum rod (diameter 36mm), such that the stator is supported along two axial lines at the outer surface with a distance of 45mm (approx. 160 and 200 in Figure 3). To protect the lamination during mounting and experiment, a 2mm rubber sheet is placed between the rod and the inner stator surface.

The measurement instrument for the experiments is a type 2032 dual channel signal analyser from Brüel & Kjær (B&K). The structure is excited by means of a type B&K 8202 impulse hammer. The sensor is a type B&K 4375 accelerometer. The structure is always excited at the same location, the accelerometer is moved along the stator outer surface. Figure 3 shows the measurement grid.

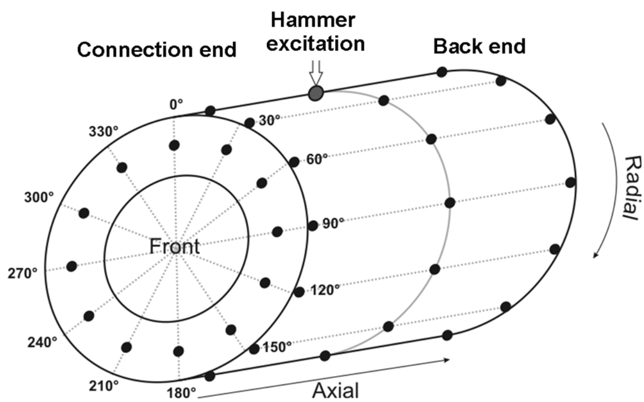


**Figure 3: Measurement grid.**

## 3.2 Experimental results and identified parameters

The results of the experimental modal analysis of the stator system after different fabrication steps are shown in Figure 4 and in Table 4. Looking at the individual FRFs, the influence of the added components on the overall vibrational behavior can be observed. During the identification of modal parameters, it has been noticed that the second mode without resin impregnation, shows a somewhat strong asymmetry, i.e. the higher-frequency flank of the second resonance peak is much steeper than the lower-frequency flank, which is a hint that the system behaves non-linear because of dry friction (11). Physically this makes sense, because the individual wires are more or less in loose friction contact, rather than integrated in a compound. The meaning of a modal analysis without resin is therefore questionable.

From the modal parameters in Table 4, it can be seen that from the pure sheet stack to the copper wound and then impregnated sheet stack, the modal damping increases as a general rule. This is in contradiction with the results given in (3), where only an overall value for the damping, rather than a specific value per material, is given.

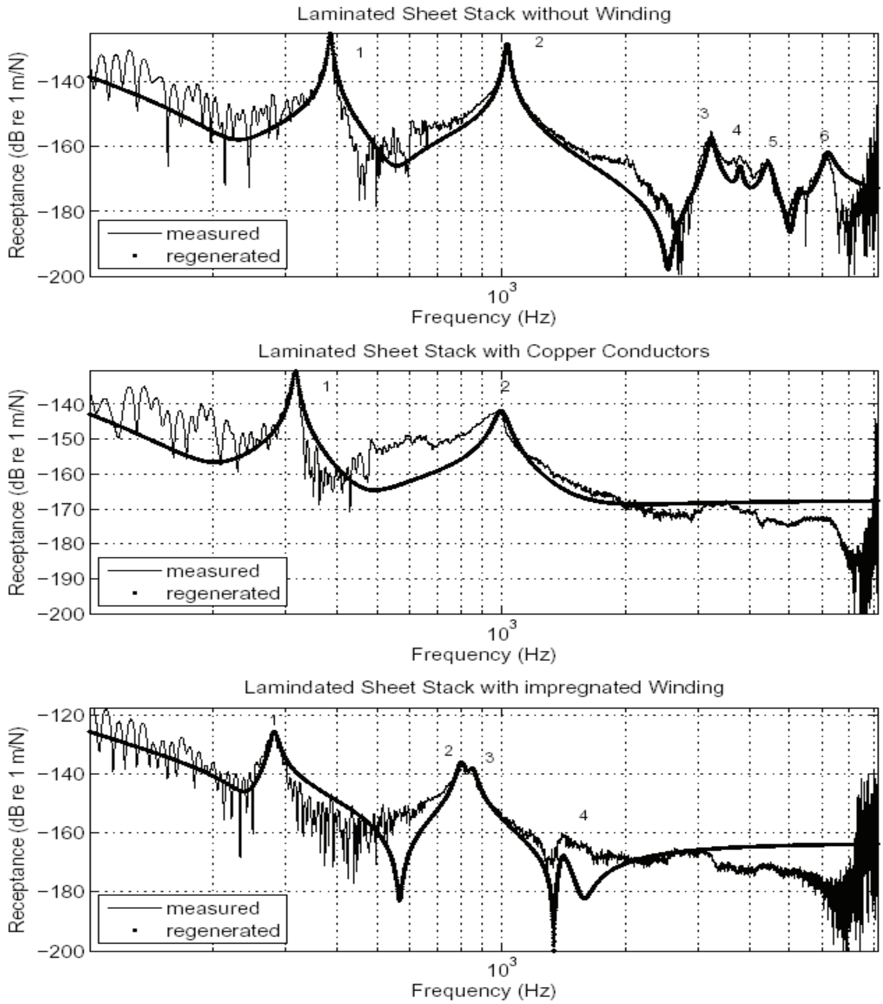


**Figure 4: Experimental and regenerated receptance FRFs for the clamped condition**

The modal parameters are used to determine the homogenized equivalent material properties of the materials involved. Therefore, the same FE model can be used. Only the material characteristics must be modified. The mass density is adapted such that the weight is correct. Afterwards, the simulation is performed repeatedly for different values of the core's Young's modulus until the first resonance mode with $r$ = 2, fits the measured resonance. A mass density of 7810.5 kg/m$^3$ and a Young's modulus of 187GPa is obtained for the laminated sheet stack. Of course, treating the laminated stack as a material having only a single Young's modulus, i.e. assuming the material to be isotopic, is a simplification but that for the flexural modes of the stator which are most strongly excited, it is defensible. Nevertheless, the proposed approach can also be used to determine the individual components of Hooke's matrix for an anisotropic model of the sheet stack.

**Table 4: Results of experimental modal analysis**

| | Sheet Stack without Winding | | | Sheet Stack with Copper Wires | | | Sheet Stack with impr. Winding | | |
|---|---|---|---|---|---|---|---|---|---|
| Mode | $f_r$ (Hz) | $\eta$ (%) | $A$ (1/kg) | $f_r$ (Hz) | $\eta$ (%) | $A$ (1/kg) | $f_r$ (Hz) | $\eta$ (%) | $A$ (1/kg) |
| 1 | 1432 | 0.6 | 0.5556 $e^{-j12^\circ}$ | 1255 | 4.8 | 0.2125 $e^{-j32^\circ}$ | 1170 | 5.13 | 0.3639 $e^{-j15^\circ}$ |
| 2 | 1766 | 0.7 | 0.0196 $e^{-j19^\circ}$ | 1731 | 8.4 | 0.0182 $e^{-j38^\circ}$ | 3502 | 5.0 | 0.153 $e^{-j23^\circ}$ |
| 3 | 3770 | 0.4 | 0.0666 $e^{j84^\circ}$ | 3071 | 24.6 | 0.0344 $e^{j41^\circ}$ | | | |
| 4 | 6148 | 0.8 | 0.4998 $e^{j165^\circ}$ | 6063 | 18.0 | 0.6607 $e^{-j140^\circ}$ | | | |
| 5 | 6550 | 1.0 | 0.0199 $e^{j96^\circ}$ | | | | | | |
| 1 | 384 | 2.0 | 0.0618 $e^{-j21^\circ}$ | 316 | 3.0 | 0.034 $e^{-j31^\circ}$ | 280 | 4.2 | 0.0626 $e^{-j28^\circ}$ |
| 2 | 1033 | 2.4 | 0.3821 $e^{-j34^\circ}$ | 998 | 7.6 | 0.246 $e^{-j73^\circ}$ | 798 | 4.5 | 0.1472 $e^{j14^\circ}$ |
| 3 | 3227 | 3.8 | 0.2131 $e^{-j30^\circ}$ | | | | 849 | 4.8 | 0.1113 $e^{-j47^\circ}$ |
| 4 | 3794 | 2.8 | 0.0464 $e^{-j102^\circ}$ | | | | 1401 | 5.5 | 0.2070 $e^{-j2^\circ}$ |
| 5 | 4438 | 5.2 | 0.2041 $e^{-j160^\circ}$ | | | | | | |
| 6 | 5244 | 7.4 | 0.1036 $e^{-j92^\circ}$ | | | | | | |
| 7 | 6163 | 6.6 | 0.7188 $e^{-j172^\circ}$ | | | | | | |

Then, the same procedure is applied to obtain the values for the impregnated copper winding, by adjusting it to the measurements at that stage. A mass density of 3166.4 kg/ $m^3$ and a Young's modulus of 0.35GPa is obtained for the winding.

All parameters are obtained from the simulation of the freely vibrating structure. As a first check, a simulation with clamped conditions is now performed. The first five computed eigenfrequencies are: {619, 1112, 1289, 1361, 1452}Hz. The deviation with respect to the resonance frequencies in Table 4 are still significant. On the other hand, the adjustment procedure was a simple optimization, whose objective was only to fit the first resonance using isotropic parameters. From this result it is clear that these limitations have to be eliminated in future studies to improve the effectiveness of the method.

## 4 SUMMARY AND CONCLUSION

This paper proposes a procedure to identify the homogenized equivalent materials for the modal analysis of composite structures in electrical machines. This procedure is twofold, determination of mass density and modal analysis, which are repeated after each fabrication step. The mass density, stiffness and damping of each material are determined one after the other, as it is added in the manufacturing process.

Future work will focus on the determination of those parameters by standard analytical techniques and their experimental validation. Such homogenization methods exist, but have not been applied extensively to the composite materials in electrical machines.

The proposed procedure can also be used to identify influences of the different manufacturing process, such as to evaluate e.g. the difference between different cutting technologies.

## REFERENCES


(1) H. Jordan, *Geräuscharme Elektromotoren*, H. Franz, Ed. W. Girardet, November 1950.

(2) L. Timar, P, A. Fazekas, J. Kiss, A. Miklos, and G. Yang, S*, Noise and Vibration of Electrical Machines,* L. Timar, P, Ed. Elsevier, 1989.

(3) J. Gieras, C.Wang, and J. C. Lai, *Noise of Polyphase Electric Motors*. CRC Press Taylor & Francis Group, 2006.

(4) S. Verma and A. Balan, "Experimental investigations on the stators of electrical machines in relation to vibration and noise problems", *Electric Power Applications, IEE Proceedings* -, vol. 145, no. 5, pp. 455–461, 1998.

(5) M. B. G. Reyne, S. Derou, and A. Foggia, "Finite element modeling of a synchronous machine: electromagnetic forces and mode shapes", *Magnetics, IEEE Transactions on*, vol. 29, no. 2, pp. 2014–2018, Mar 1993.

(6) G. Reyne, A. R. de Rochebrune, and A. Foggia, Mechanical homogenization applied to the modeling of static inductors," *Magnetics, IEEE Transactions on*, vol. 28, no. 2, pp. 1283–1286, Mar 1992.

(7) Z. Tang, P. Pillay, A. Omekanda, C. Li, and C. Cetinkaya, "Young's modulus for laminated machine structures with particular reference to switched reluctance motor vibrations," *Industry Applications, IEEE Transactions on*, vol. 40, no. 3, pp. 748–754, 2004.

(8) S. Garvey, "The vibrational behaviour of laminated components in electrical machines," in *Electrical Machines and Drives*, 1989. Fourth International Conference on, 1989, pp. 226–231.

(9) R. Nitzsche, "Elektromagnetisch erzwungene und Eigenschwingungen des Statorgehäuses eines zweipoligen Turbogenerators," Ph.D. dissertation, Fachbereich Elektrotechnik und Informationstechnik der Universität Hannover, November 1998.

(10) M. Christensen, R, *Mechanics of Composite Materials*. John Wiley & Sons, 1979.

(11) D. Ewins, *Modal Analysis*. Baldock, Hertfordshire, England: Research Studies Press LTD., 2000.

(12) C. Neves, R. Carlson, N. Sadowski, J. Bastos, N. Soeiro, and S. Gerges, "Experimental and numerical analysis of induction motor vibrations," *Magnetics, IEEE Transactions on,* vol. 35, no. 3, pp. 1314–1317, 1999.

(13) C. Schlensok, B. Schmülling, M. van der Giet, and K. Hameyer, "Electromagnetically excited audible noise evaluation and optimization of electrical machines by numerical simulation," *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, vol. 26, pp. 727 – 742, 2007.

(14) M. Furlan, A. Cernigoj, and M. Boltezar, "A coupled electromagnetic mechanical-acoustic model of a dc electric motor," *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering,* vol. 22, no. 4, pp. 1155–1165, 2003.

(15) W. Schroeder, K. Martin, and B. Lorensen, *The Visualisation Toolkit*, 3rd ed. Kitware, 2004.

(16) I. Ramesohl, G. Henneberger, S. Kuppers, and W. Hadrys, "Three dimensional calculation of magnetic forces and displacements of a claw-pole generator," *Magnetics, IEEE Transactions on*, vol. 32, no. 3, pp. 1685–1688, 1996.