Comparison of 2-D and 3-D Coupled Electromagnetic and Structure-Dynamic Simulation of Electrical Machines

Michael van der Giet, Christoph Schlensok, Benedikt Schmülling, and Kay Hameyer

Institute of Electrical Machines, RWTH Aachen University, D-52062 Aachen, Germany

For the coupled electromagnetic and structure-dynamic simulation of electrical machines, various 2-D and 3-D techniques are available. This paper reviews and compares them. The strength as well as the weaknesses of these methods are pointed out. Numerical results for the analysis of a switched reluctance machine (SRM) as an example evaluate accuracy and computational effort. It shows that the 2-D simulation gives quite accurate results, as long as the axial effects and 3-D mode shapes are not relevant.

Index Terms—Finite element (FE), finite-element method (FEM), noise and vibration of electrical machines, switched reluctance machine (SRM), 2-D, 3-D.

I. INTRODUCTION

THE simulation of vibration and acoustic noise of electrical machines requires the solution of a multiphysics problem. Therefore, a triple coupling between electromagnetic field calculation, structure-dynamic, and acoustic simulation has to be established.

In both senses, decreasing numerical effort and including more physical aspects, it is desirable to use 2-D techniques, whenever possible. On the other hand, the informational value of a 2-D structure-dynamic simulation of electrical machines may be limited due to 3-D effects that cannot be captured in 2-D. Both simulation approaches 2-D and 3-D have been widely used [1], [2], [3]. However, a detailed evaluation of both methods, a comparison of their strength and weaknesses, as well as a numerical comparison of accuracy and computational effort are still missing.

Therefore, this paper compares 2-D and 3-D coupled electromagnetic and structure-dynamic simulation of electrical machines. The study is carried out for a switched reluctance machine (SRM), exemplarily, which has already been studied in [4].

II. COUPLED SIMULATIONS

It can be distinguished between a numerically weak or strong coupling. Both approaches have its advantages and disadvantages. Numerical weak coupling allows for using different grids, on which the different problems are solved. The solution from one ore more 2-D electromagnetic finite element (FE) simulations can be projected onto a 3-D structure-dynamic model, which includes the housing of the machine and its mounting [1], [2].

On the other hand, a strong coupling, i.e., coupling the simulations on matrix level, allows for an efficient implementation of reaction and close interaction between the solution quantities. Using this technique, additional aspects, such as magnetostriction and the influence of the deformation on the electromagnetic excited forces, can be taken into account [5]. Strong coupling of electromagnetic and structure-dynamic simulation including



Fig. 1. Overview of simulation methods.

the aforementioned aspects has only been implemented in 2-D so far [5] and is not part of the studies here. Fig. 1 gives an overview of the available simulation methods.

A. Electromagnetic Simulation

For electrical machines, which are homogenous in the axial direction and of which the axial dimension is sufficiently large compared to its diameter, it is possible to use 2-D electromagnetic finite-element method (FEM) to obtain the field distribution. For machines, of which the cross section is only varying slightly with respect to the axial direction, the multislice method (MSM) can be applied [6].

Due to a high ratio between accuracy and computational effort, the 2-D or 2-D MSM has become a standard for the electromagnetic simulation of electrical machines.

B. Structure-Dynamic Simulation

The time harmonic electromagnetic force is used as an excitation for the structure-dynamic simulation, which is governed by

$$(K - \omega^2 M)u = F \tag{1}$$

where K is the stiffness matrix, M is the mass matrix, u is the time harmonic displacement vector, and F is the load vector, which is computed by projecting the electromagnetic forces from the 2-D electromagnetic mesh to an either 2-D or 3-D structure-dynamic mesh [7].

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Fig. 2. Exploded view of the structure-dynamic model.

In the 2-D case, the mesh generation can be easily performed by taking the same mesh as in the electromagnetic case, or at least using the same routines with different element sizes. The 3-D approach, however, requires a complete 3-D model of the machine, such as in Fig. 2, which takes significant effort to construct. For simplicity, the end-shields of the SRM are not shown in this figure, though they are considered in the model.

For the 2-D case, there are two approximations of the missing third dimension in the constitution of K. One, called *plane stress*, assumes the stress in the z-direction to be zero, and the other, called *plane strain*, assumes the strain in the z-direction to be zero. Plane stress corresponds to an infinitely thin sheet and plane strain to a slice out of a infinitely long structure [8].

For the 3-D case, the constitution of K depends solely on the used Hooke's matrix of the material. For pure materials, such as the aluminum of the housing, Hooke's matrix has an isotropic structure and values can be taken from the literature. For composite materials, such as the stator, consisting of laminated sheets, or the winding, Hooke's matrix is no longer isotropic, but its values correspond to equivalent material parameters, which may be identified by measurements and an optimization procedure [9]. Due to the symmetry in the model plane of a 2-D model, it is not reasonable to consider an anisotropic Hooke's matrix for the 2-D simulation. The use of anisotropic material parameters for the 3-D simulation may increases accuracy; on the other measurements are required.

Due to its computational speed, the 2-D method can be used to evaluate the structure-dynamic behavior over an entire frequency band, whereas the 3-D approach, due to its high computational effort, is typically used for only several selected frequencies, which can be identified by modal analysis, analytical considerations of the specific machine type, and considering the excitation spectrum. Also, the 2-D structuredynamic simulation can be used advantageously to identify relevant frequencies and to give a first overview of the deformation spectrum.

The 2-D approach cannot consider vibrations in the axial direction of the machine. Therefore, all 3-D mode shapes are neglected. They may be because of a nonuniform force excitation or the nonuniform structure of the machine. This can be, for example, the skewing of the rotor bars of an induction motor or the mounting of the machine at the front face.

In addition to neglecting 3-D mode shapes, the 2-D simulation is not capable of considering the effect of the vibrating

 TABLE I

 COMPARISON OF 2-D/3-D SIMULATION

Structure-Dynamic-Simulation	
2D	3D
fast	slow
easy mesh generation	more complicated mesh generation
frequency band	selected frequencies
no 3D mode shapes	full 3D mode shapes
no consideration of rotor	consider rotor vibration
no coupling to acoustics	coupling to acoustic simulation
strong coupling and	only weak coupling implemented
magnetostriction possible	so far
isotropic use	transverse isotropic,
	with parameter identification

rotor. In theory, the rotor can be included in the model, however, because it is not connected to the rest of the model, there would be no influence.

As one further advantage of the 3-D method, the capability to couple in a next simulation step to an acoustic boundary element calculation may be given, where the meaning of doing this in 2-D would be very limited.

Because a strong coupling and the consideration of magnetostriction have only been implemented in 2-D so far, this may also be called an advantage of this method. Table I summarizes this comparison.

III. NUMERICAL RESULTS

Because the objective of a structure-dynamic simulation is to find the amount of body sound that is either transmitted to other mechanical parts or that is radiated by the housing as airborne sound, the body sound index is introduced. It is defined as

$$L_{s} = 10 \log \left(\frac{\sum_{p=1}^{N} \int_{S_{p}} \omega^{2} |\underline{u}_{p} \cdot \vec{n_{p}}|^{2} dS}{S_{0} \cdot h_{U_{0}}^{2}} \right)$$
(2)

where N is the number of elements of the surface mesh, ω is the angular frequency of the problem, $\underline{\vec{u}_p}$ is the deformation in the *p*th element, $\vec{n_p}$ is the normal vector, and S_p is the surface of the same element. In 2-D, the integral is one dimension lower, and therefore, it has to be multiplied by the length of the machine. The reference values $S_0 = 1 \text{ m}^2$ and $h_{U_0} = 25 \cdot 10^{-16} \text{ m}^2/(\text{s}^2)$ can be taken from the literature [10].

The surface, on which the body sound index is calculated, would typically be chosen either to the entire housing of the machine, which radiates the airborne sound, or the mounting interface, which transmits the body sound. For the second to apply, it would be necessary to simulate the machine as part of a mechanical system, where the mounting interface does not feature a zero deformation Dirichlet boundary condition (BC), as it is done in this study. To be able to compare different modeling stages, the stator surface is used to compute the body sound index.

A. Different Modeling Stages

To evaluate the differences between 2-D and 3-D simulation, the studied SRM is considered at different modeling stages: only stator, stator with winding, stator with winding, and housing. This is the maximum parts that can be taken into account in the



Fig. 3. Different modeling stages at 3333 Hz.



Fig. 4. Different modeling stages at 6000 Hz.

2-D case. For the 3-D case, the complete model as shown in Fig. 2 situates the full model capabilities.

The simulation result of different modeling stages are shown in Fig. 3 for 3333 Hz and in Fig. 4 for 6000 Hz. The BC of the first three models, i.e., stator, with winding, and housing, is set such that the deformation of the nodes at 0° , 90° , 180° , and 270° is forced to zero by a Dirichlet BC only in radial direction. This means that the model is able to vibrate freely in radial direction, as it is natural for this type of excitation. In the 3-D case, the axial component of the nodes in the middle of the model are also forced to have zero deformation. Due to the parameterization by measurements, the full 3-D model was only simulated with anisotropic material properties in the stator and is equipped with a zero deformation Dirichlet BC at the mounting wholes.

From Figs. 3 and 4, it can be seen that the overall correlation between 2-D and 3-D simulation is much better for 3333 Hz. The reason for that is the different mode shape of this vibrational mode, as it will be shown in Section III-C.

At 3333 Hz, the influence on the body sound index of the isotropic or anisotropic nature of Hooke's matrix is small for the simulation of only stator and winding. The correlation of



Fig. 5. Comparison of body sound index at 3333 Hz.

2-D and 3-D for only winding and housing is good as expected, because the model has the same components. Because the 2-D case is inherently isotropic, its results have to be compared to that of 3-D isotropic, which shows a large difference. The fact that the anisotropic 3-D approach yields simulation results that are closer to the 2-D approach can only be a matter of cancellation of contradicting effects rather than a true correlation.

B. Numerical Costs

Both simulations, electromagnetic and structure-dynamic, are implemented using iMOOSE [11] and PETSc [12]. The solver used for all problems was the conjugate gradient (CG) method. The machine, on which the problems are benchmarked, is an AMD Athlon(tm) 64 X2 dual core processor 4600+ with a total of 4-GB RAM. Because the focus on this study lies in the comparison of 2-D and 3-D approaches, and not on the optimization of one of these methods, the code was only implemented in a serial processing way.

Fig. 5 shows the convergence behavior of the two approaches. For both approaches, it can be seen that even for very large models the body sound index does not seem to converge. Only in the case of second-order triangles a converging behavior may be observed. It has to be noted that the focus of this study is the comparison of both approaches rather than a detailed study of one of the approaches. Therefore, the discretization of the electromagnetic simulation has been kept constant. Additional studies on the projection method, i.e., mapping of the electromagnetic forces onto the mechanical mesh, may reveal more insight into the convergence behavior of the coupled problem.

Fig. 6 gives an overview of the computational costs of the structure-dynamic simulation versus the number of degrees-of-freedom (DOF) in the model. The computational costs of the electromagnetic simulation are not included, because they are constant for all simulations. It can be seen that the memory usage scales almost quadratically with the number of DOFs with the constant offset that is because of the memory used for the program structure. The computational time scales quite linearly with the number of DOFs.



Fig. 6. Comparison of computational time and memory.



Fig. 7. Real part of the deformation at 3333 Hz.



Fig. 8. Real part of the deformation at 6000 Hz.

C. Comparison of Mode Shapes

The better correlation of the body sound index for 3333 and 6000 Hz can be understood looking at the mode shapes at both frequencies in Figs. 7 and 8. It is clearly seen that the deformation correlates better in terms of shape and magnitude in the 3333-Hz case than in the 6000-Hz case. A closer look at the 3-D solution shows that the 3333-Hz case is an almost pure 2-D mode shape, where the deformation at 6000 Hz varies strongly along the axial direction of the machine. A previous study [4], however, showed that 6000 Hz was one of the most dominant

frequency regarding the acoustic noise radiated by this SRM. Therefore, it can be concluded that the particular caution has to be taken when analyzing electrical machines by means of 2-D structure-dynamic simulation, especially if the machine is face-mounted.

IV. CONCLUSION

This paper evaluates the differences between 2-D and 3-D coupled electromagnetic and structure-dynamic simulation of electrical machines. State-of-the-art simulation methods are compared and strengths and weaknesses are pointed out. Numerical results for an SRM are presented, exemplarily. An acceptable correlation between 2-D and 3-D structure-dynamic simulation is only found if the resulting mode shape does not have an intensive axial component. Future work will be spent on the projection method and on the mutual convergence behavior of the coupled simulation.

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Manuscript received June 24, 2007. Corresponding author: M. van der Giet (e-mail: Michael.vanderGiet@IEM.RWTH-Aachen.de).