THE MATHEMATICS OF LUMPED PARAMETER IDENTIFICATION IN ELECTRICAL MACHINES

F. Henrotte, J. Heidt, K. Hameyer

Institute of Electrical Machines – RWTH Aachen University
Schinkelstraße 4, D-52056 Aachen, Germany
E-mail: Francois.Henrotte@IEM.RWTH-Aachen.de

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Abstract

Basic principles and mathematical background of various lumped parameter identification methods for different types of electrical machines (e.g. synchronous machine, induction machine) are discussed in this paper. The idea is to provide a common theoretical framework and clearly differentiate the different approaches commonly encountered in literature.

1 Magnetic energy balance

Considering an electrical machine as a thermodynamic system Ω, one is led to select the vector potential $\mathbf{a}$ as magnetic state variable, and to define the magnetic energy of the system as a functional $\Psi_M(\operatorname{curl}\mathbf{a})$. The variation of that energy is expressed by the chain rule of derivative as

$$\partial_t \Psi_M = \int_\Omega \mathbf{h}_r \cdot \mathcal{L}_\mathbf{v} \operatorname{curl}\mathbf{a} + \int_\Omega \{\mathcal{L}_\mathbf{v}\, \rho_M^\Psi\}(\operatorname{curl}\mathbf{a}) \quad (1)$$

where the co-moving time derivative $\mathcal{L}_\mathbf{v}$ is a time derivative that accounts for a possible motion or deformation of the domain Ω [1]. In the absence of motion, one has $\mathcal{L}_\mathbf{v} \equiv \partial_t$. In (1), $\mathbf{h}_r \equiv \partial_\mathbf{b}\rho_M^\Psi$ is by definition the reversible part of the magnetic field, i.e. the part deriving from a potential (which is precisely the magnetic energy density ρ_M^Ψ), and accounting for the magnetization phenomenon, i.e. the alignment of microscopic magnetic moments. The second terms in (1) represents the variation of magnetic energy with the magnetic fluxes $\mathbf{a}$ held constant, when Ω deforms or when the rotor rotates.
From the point of view of Thermodynamics, the balance of magnetic energy writes

$$\partial_t \Psi_M = \dot{W} + \dot{Q} + \dot{W}_M. \quad (2)$$

The rate of magnetic work

$$\dot{W} = \int_\Omega \mathbf{j} \cdot \mathcal{L}_\mathbf{v}\, \mathbf{a} - \int_{\partial\Omega} \mathbf{n} \times \mathbf{h}_\partial \cdot \mathcal{L}_\mathbf{v}\, \mathbf{a} \quad (3)$$

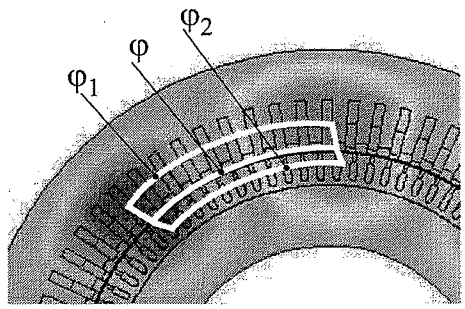

Figure 1: Zoom-in on one pole pair of a typical 2D vector potential plot in an induction machine with the corresponding skeleton (white). The surfaces associated with fluxes φ_k are indicated.

decomposes into a volume term accounting for the currents $\mathbf{j}$ flowing in Ω and a boundary term involving the external magnetic field $\mathbf{n} \times \mathbf{h}_\partial$, which stands for the effect of currents flowing outside Ω. The dissipation functional writes $\dot{Q} = -\int_\Omega \mathbf{h}_i \cdot \operatorname{curl} \mathcal{L}_\mathbf{v}\, \mathbf{a} \leq 0$ with $\mathbf{h}_i$ is the irreversible part of the magnetic field (magnetic hysteresis). Finally, $\dot{W}_M$ represents the power delivered by magnetic forces. Identifying (1) and (2) and factorising the arbitrary $\mathcal{L}_\mathbf{v}\, \mathbf{a}$, one obtains the conservation equations of the system, namely Ampere's law $\operatorname{curl}[\mathbf{h}_r + \mathbf{h}_i] = \mathbf{j}$ and the definition of the electromechnanical coupling in the machine. $\dot{W}_M = \int_\Omega \{\mathcal{L}_\mathbf{v}\, \rho_M^\Psi\}(\operatorname{curl}\mathbf{a})$.

2 The skeleton of $\mathbf{a}$ and the co-skeleton

The definition of the equivalent circuit of an electrical machine is a simplification, whose validity relies on the fact that the induction field exhibits a well-defined and permanent geometrical structure when the machine is operated in steady state operation. This regularity is a redundancy one attemps to get rid of by defining the equivalent circuit. As the equivalent circuit counts a small number of lumped parameters, identification is in general done empirically, as reported in the abundant literature on the subject [2]...[8], and no general (theoretical) definition of the lumped element of the equivalent circuit is available.
As mentioned above, all vector potential plots look more or less the same, Fig. 1. As the vector potential (a differ-

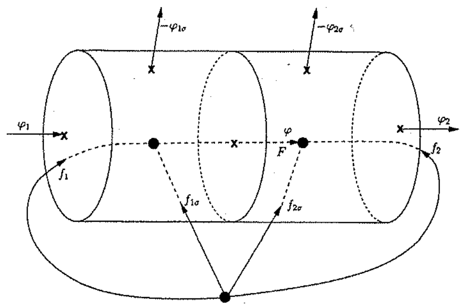

Figure 2: Skeleton (double cylinder) and co-skeleton (pretzel) with representation of the characteristic fluxes φ_k and magneto-motive forces f_k.

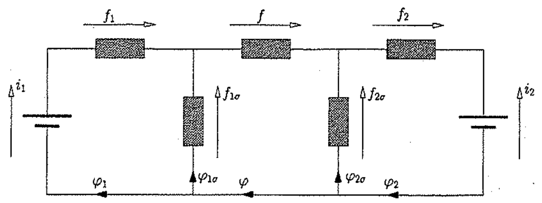

Figure 3: Electric equivalent circuit, fluxes φ_k behave like currents, magnetomotive forces f_k like voltages and the resistances are the lumped reactances of the machine.

ential 1–form) evaluates on curves [1], the idea is to select the characteristic (closed) curves whose associated fluxes give most information about the field. In electrical machines, due to the symmetry, one can restrict the analysis to one pole pair of the machine and it is enough to select the 3 curves C_1, C_2 and C that enclose the maximum fluxes in the stator the rotor and the air gap regions respectively. Those fluxes are accordingly noted φ_1, φ_2 and φ and represented in Fig. 1 and Fig. 2. The 3 characteristic curves define the cylinder-shaped topological structure depicted in Fig. 2, and which can be considered as the skeleton of the a field. This skeleton is fixed with respect to the rotating field, i.e. it rotates at the speed $\omega_0 = 2\pi f_0/p$, where f_0 is the frequency and p the number of pole pairs. By Poincaré duality, the skeleton is associated a dual topological structure, the co-skeleton, which has the shape of a pretzel, Fig. 2. Whereas the characteristic fluxes φ_k in the machine are associated with the closed curves of the skeleton, the characteristic magnetomotive forces f_k and currents i_k are associated with the curves (not necessarily closed) and the surfaces of the co-skeleton. All topological relations associated with the (co-)skeletons can be summarized in circuit form, Fig. 3.
Fluxes embraced by stator coils and stator currents can also be determined by classical methods, See e.g. [3]. As leakages fluxes are clearly defined in the skeleton, the existing relation between stator- and skeleton fluxes is exploited to obtain a definition for the leakage inductances seen form the stator terminals. One can this way provide mathematical definitions for all lumped parameters of the T-circuit of the electrical machine, as will be explained in

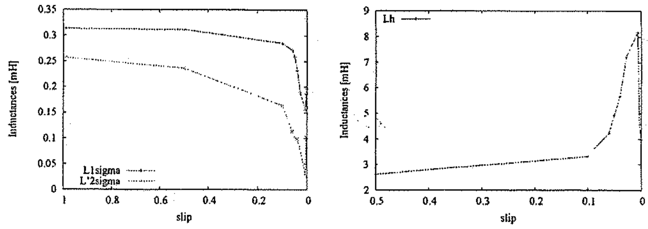

Figure 4: Leakage (left) and magnetizing (right) inductances computed at different slips.

further details in the full paper. Fig. 4 shows leakage and magnetizing inductances computed at different slips. On sees that they are far from being constant, although most empirical approaches would assume they are.

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