Field and Field-Circuit Description of Electrical Machines

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***Abstract*—The field and coupled field–circuit models of electrical machines are presented. The field model consists of: (a) finite element (FE) equations for the magnetic field, (b) equations describing eddy-currents and (c) equations, which describe the currents in the machine's windings. Moreover the FE equations are coupled by the electromagnetic torque to the differential equations of motion. In the presented field-circuit model the flux linkages with the windings are expressed by two components. One component with inductances and the other described by edge or nodal values of the magnetic potential. The FE equations are derived by using the notation of circuit theory. The approach to consider the differential equation of motion in the simulation is discussed in the paper.**

***Keywords*—electrical machines, finite element method, magnetic field, eddy currents, field-circuit coupling.**

I. Introduction

Since several years now, numerical methods to simulate the electromagnetic field are applied in the analysis and synthesis of electrical machines. The classical equivalent circuit methods are supported by additional procedures of field analysis. As a result we obtain the so called field-circuit model [31, 33]. The machine parameters and characteristics can be calculated on the basis of field formulation with omission of circuit equations. This description is treated as in fully field.

In the paper we try to systemize the applied models of electrical machines and actuators. We will consider the electrical machine models which are used to describe the electromagnetic phenomena and characteristics for the steady states and transients operation of the device.

It seems that the most distinctive feature of the discussed models is the method of flux linkage calculation. In the typical approaches the flux linkage with the winding is determined by inductances. The inductances represent coefficients or functions that express dependence between the flux linkages with windings or parts of it and the field exciting winding currents. For example, the dependence between the flux associated with the end of the q^{th} winding and the current in the p^{th} winding expresses mutual inductance of the winding ends.

The models, described by the systems of ordinary differential equations with inductances, will be considered as the circuit models. In the field models winding inductances do not exist. Flux linkages and electromagnetic torques and forces are calculated using field quantities, e.g. the magnetic vector potential. The field equations are then coupled to the equations that describe the winding connections and contain terms defined by field quantities and lumped parameters [1, 10, 24]. The models of this type can be considered as the field-circuit [7, 8, 21, 28].

In the paper particular attention is paid to the field and field-circuit models of typical electromechanical converters (electrical machines and actuators). Therefore, it is assumed that the winding voltages and speed are the input quantities. However, winding currents and electromagnetic torque are the output quantities.

The field models of electrical machine consist of the following equations: (a) finite element equations of magnetic field; (b) finite element equations that describe eddy currents distribution, (c) equations that describe currents in the winding. Moreover the field and circuit equations are coupled via the electromagnetic torque or force to the equation of motion.

Most often, field models are formed using magnetic and electric potentials. Therefore the FE equations (a) describe magnetic field in terms of magnetic scalar potential Ω or magnetic vector potential $\boldsymbol{A}$. Moreover, equations (b) express eddy currents by electric scalar potential V or electric vector potential $\boldsymbol{T}$. Equations (c) are related to the loops of multiply connected conductors. In the notation of field theory these equations describe magnetic vector potential $\boldsymbol{T}_0$ [2]. Edge values of $\boldsymbol{T}_0$ represent loop currents in the loops around 'holes' in multiply connected conductors, e.g. the loop currents in the winding composed of stranded conductors or the loop currents around the holes in regions with eddy currents [15].

Because magnetic field and electric field of conducting currents can be expressed by scalar or vector potentials there are three formulation of equation (a)–(b)–(c). We can apply Ω–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation, or $\boldsymbol{A}$–V–$\boldsymbol{T}_0$ formulation, or description using $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$. Of course, in specific case field model can be simplified, e. g. when eddy currents are negligible we consider only equations (a)–(c) that are represented by formulation Ω–$\boldsymbol{T}_0$ or $\boldsymbol{A}$–$\boldsymbol{T}_0$.

The presented field model is formed with the intention to join it with equation of supply and control circuit. Therefore, it is advantage to express equations (a)–(b)–(c) using the language of circuit theory. First the FE equations of magnetic field will be presented.

II. Finite Element Equations of Magnetic Field

Two most popular FE formulations are discussed: (α) the formulation using scalar potential Ω and nodal elements, and (β) the formulation using vector potential and edge elements. In the formulation (α) polynomial interpolating function $\Omega(x,y,z)$ is constructed on the nodal values of Ω, i.e. on the nodal potential. However, the

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formulation (β) applies edge value of $\boldsymbol{A}$. For oriented edge P_1P_2 the edge value of $\boldsymbol{A}$ is equal to the line integral of $\boldsymbol{A}$ on P_1P_2 [15]. The edge value of $\boldsymbol{A}$ for edge P_1P_2 can be considered as a loop flux in the loop around P_1P_2. Thus in the formulation (α) magnetic field is defined by the nodal magnetic potentials and in the case of (β) formulation magnetic field is described by thy loop fluxes.

It has been shown [14, 15] that FE equations represent nodal and loop equations of two types of networks: 'edge networks' (EN) where branches are associated with edges of the elements, and 'facet networks' (FN) with branches connecting the centers of the relevant facets with the centre of the element volume. FE model composed of 8-node hexahedrons is used to illustrate these networks - see Figs. 1, 2. The structural matrices of the networks are the FE representation of differential operators. For example, the transposed nodal incidence matrix $\boldsymbol{k}_n$ of EN represent 'grad' operator and the transposed loop matrix $\boldsymbol{k}_e$ for FN is the network representation of 'curl' operator. The nodal equations for EN are equivalent to the nodal finite element formulation using scalar potential Ω, whereas loop equations for FN refer to the edge element formulation based on vector potentials $\boldsymbol{A}$.

Table I summarises the equations for both models and shows: (a) equations that describe branch fluxes ϕ_b in EN and node-to-node magnetic 'voltages' $\boldsymbol{u}_{\Omega f}$ for FN, and (b) FE equations using the notation of equivalent networks.

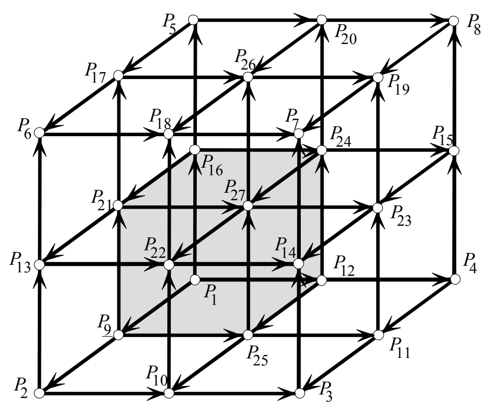

Fig. 1. Edge graph of 8 hexahedrons.

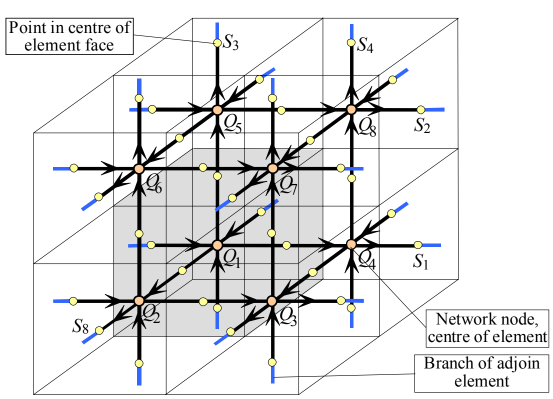

Fig. 2. Facet graph of 8 hexahedrons.

TABLE I.
EQUATIONS OF EQUIVALENT MAGNETIC NETWORKS

Network	Branch equation	Substitutions	FE equations
Edge	$\phi_b = \Lambda(\boldsymbol{u}_\Omega + \Theta_{be})$	$\boldsymbol{u}_\Omega = \boldsymbol{k}_n \boldsymbol{\Omega}$	$\boldsymbol{k}_n^T \Lambda \boldsymbol{k}_n^T \boldsymbol{\Omega} = -\boldsymbol{k}_n^T \Lambda \Theta_{be}$ *
Facet	$\boldsymbol{u}_{\Omega f} = \boldsymbol{R}_\mu \phi_f - \Theta_{bf}$	$\phi_f = \boldsymbol{k}_e \phi_e$	$\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e \phi_e = \boldsymbol{k}_e^T \Theta_{bf}$ **
Comments: $\boldsymbol{\Omega}$ is the vectors of nodal potentials, Λ is the matrices of branch permeances, Θ_{be}, Θ_{bf} are the vectors of branch *mmf*s, ϕ_e is the vectors of loop fluxes; $\boldsymbol{R}_\mu$ is the matrices of branch reluctances, *nodal equations of EN, **loop equations of FN			

It should be noted that in equivalent models of element there exist inter-branch couplings [14, 15].The matrices of branch parameters are not diagonal.

In the presented equations the branch *mmf*s are establish from loop currents (ampere-turns) in the loops around branches. However, when using the FN it is not necessary to know the branch sources, instead the loop sources are needed. The loop *mmf*s are represented by the current passing through the loops of FN. Thus, for scalar potential formulation we should define loop currents to determine the right hand side (RHS) vector of FE equations. Whereas, for vector potential method the RHS vector can be calculated using the current passing through the loops.

In general, we can consider three categories of currents that are the source of magnetic field: (i) conducting currents, (ii) magnetizing current in the region with permanent magnets, (iii) displacement current. In the discussed model displacement current are neglected. Magnetizing currents are assumed to be known. These currents are determined by magnetization vector $\boldsymbol{H}_m$. The edge value of $\boldsymbol{H}_m$ represents the magnetizing current in the loop around edge. The most complicated is to establish the *mmf*s caused by the conducting currents. It is advantage to separate two kinds of conducting currents: eddy currents and currents inducted in windings. The eddy currents are calculated using FE method.

III. FINITE ELEMENT EQUATIONS OF EDDY CURRENTS

System containing region with eddy currents field may be described using electric scalar potential $\boldsymbol{V}$ or electric vector potential $\boldsymbol{T}$. FE equations for these formulations are constructed in the way similar to the FE for magnetic field. The FE equations for scalar potential formulation and nodal elements represent nodal equations of edge electric network. Whereas, the FE equations for vector potential $\boldsymbol{T}$ and edge elements are equivalent to the loop equations of facet electric network with loops around element edges. The edge value of $\boldsymbol{T}$ represents loop current that may be considered as the eddy current.

The equations of electric network are shown in Table II. The branch equations express the branch currents $\boldsymbol{i}_b$ in EN and the node to node potentials $\boldsymbol{u}_{Vf}$ in FN.

In the electric equivalent networks, similar to magnetic ones, inter-branch coupling exists. For example, the current in the i^{th} branch of the edge element model depends on the voltage across the conductance of the j^{th} branch, whereas the voltage of the branch q in the facet model is linked to the flux in the branch p.

When formulating equations presented in Table II, branch *emf*s are expressed by time derivative of magnetic flux in the loops around the network branches. Therefore, the branch *emf*s in the EN are calculated using the loop fluxes in the facet magnetic network. In the case of the FN

TABLE II.
EQUATIONS OF EQUIVALENT ELECTRIC NETWORKS

Network	Branch equation	Substitutions	FE equations
Edge	$\boldsymbol{i}_b=\boldsymbol{G}(\boldsymbol{u}_V+\boldsymbol{e}_{be})$	$\boldsymbol{u}_V=\boldsymbol{k}_n\boldsymbol{V}$	$\boldsymbol{k}_n^T\boldsymbol{G}\boldsymbol{k}_n^T\boldsymbol{V}=-\boldsymbol{k}_n^T\boldsymbol{G}\boldsymbol{e}_{be}$ *
Facet	$\boldsymbol{u}_{Vf}=\boldsymbol{R}\boldsymbol{i}_f-\boldsymbol{e}_{bf}$	$\boldsymbol{i}_f=\boldsymbol{k}_e\boldsymbol{i}_e$	$\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e\boldsymbol{i}_e=\boldsymbol{k}_e^T\boldsymbol{e}_{bf}$ **
Comments: $\boldsymbol{V}$ is the vectors of nodal potentials, $\boldsymbol{\Lambda}$, is the matrices of branch conductances, $\boldsymbol{e}_{be}$, $\boldsymbol{e}_{bf}$ are the vectors of branch *emf*s, $\boldsymbol{i}_e$ is the vectors of loop currents; $\boldsymbol{R}$ is the matrices of branch resistances, *nodal equations of EN, **loop equations of FN			

that is analysed using the loop method it is not necessary to establish the branch *emf*s $\boldsymbol{e}_{bf}$. The RHS vector $\boldsymbol{e}_{mf}$ in loop equations is represented by the loop *emf*s, $\boldsymbol{e}_{bf}=\boldsymbol{k}_e^T\boldsymbol{e}_{bf}$. Thus, we need the loop *emf*s. The loop *emf*s in FN are expressed by time derivative of flux passing through the loops, i.e. flux associated with branch of EN.

The important disadvantage of formulation using vector potential $\boldsymbol{T}$ is that the method can only be applied to simply connected conductors, e.g. solid parts of a core with no 'holes'. Whereas, electrical machine windings must be considered as a multiply connected regions. The FE equations for the classical $\boldsymbol{T}$ formulation refer to loops around the element edges, see loops with current i_{mi}, i_{mk} in Fig. 3. The loops around the 'holes' do not exists. It is thus necessary to modify classical $\boldsymbol{T}$ method and to introduce additional equations describing the loop currents flowing around the 'holes', e.g. currents i_{ci} (i=1, 2, 3) in Fig. 3. These currents are circuit representation of the edge values of $\boldsymbol{T}_0$ introduced in [2, 4]. The edge values of $\boldsymbol{T}_0$ describe also currents in the windings composed of thin stranded conductors with negligible skin effect.

IV. EQUATIONS OF WINDING CURRENTS

In the presented approach the winding currents represent loop currents in the determined loops of multiply connected conductors, see currents i_{ci} in Figs. 3, 4. The winding outlets are considered to be out of the region divided into the FEs. In general the winding equations can be connected with the equations of supply system. Here, to simplify description it is assumed that the terminal voltages are given and the loop *emf*s $\boldsymbol{e}$ produced by external sources are known.

The procedure of winding equation formulation starts with the description of winding loops in the finite element space. As a result we get matrices that transform winding currents into magnetic field sources and transform fluxes ϕ_b, ϕ_e (see Table I) into the flux linkages with the winding loops. There are two methods of winding description in the finite element space. The method based on the definition of intersection points between the winding loops and the finite element facets [13, 16]. This method describes windings in the facet element space. The more universal is method that relies on the calculation of intersection points between the finite element edges and the surfaces of winding loops. This method describes the winding in the edge element space [13].

The path of the i^{th} winding loop L_i can be defined by the parametric equations of oriented curves, $\boldsymbol{r}=\boldsymbol{r}_i(t)$. For a real winding these equations have a very complicated form. Therefore, it is suggested each loop L_i be replaced by a set of m_i closed plane curves (sub-loops) $L_{i,j}$, e.g. by triangles or parallelograms (Fig. 5). In order to describe sub-loops in the edge element space we should define the oriented surfaces $\boldsymbol{S}_{i,j}$ of $L_{i,j}$ – see Fig. 5.

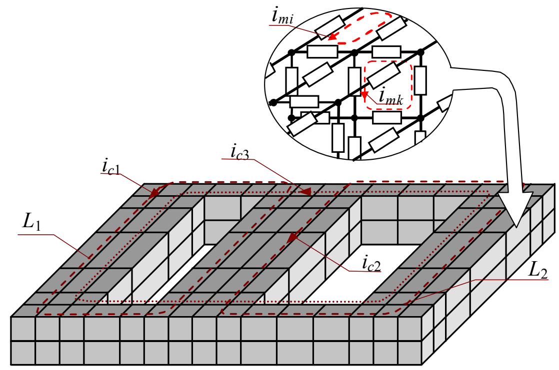

Fig. 3. Multiply connected conducting regions with eddy currents i_m.

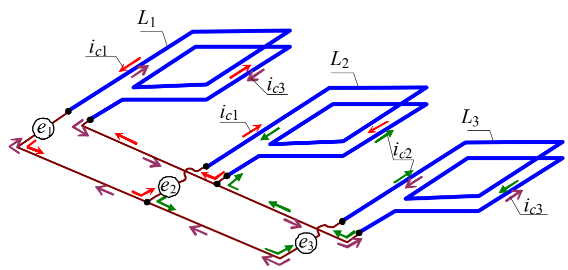

Fig. 4. Loops of 3-phase winding composed of stranded conductors.

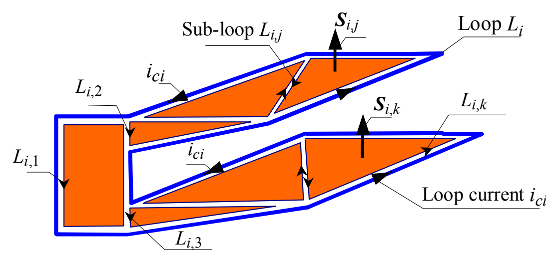

Fig. 5. Sub-loops $L_{i,j}$ of loop L_i with current i_{ci}.

As a result the winding loops are represented by a set of closed oriented plane curves of parametric equations $\boldsymbol{r}=\boldsymbol{r}_{i,j}(t)$ and by plane oriented open surfaces $\boldsymbol{S}_{i,j}$ of parametric equations $\boldsymbol{r}=\boldsymbol{r}_{i,j}(u,v)$. Usually, the description of winding loops may be simplified by ignoring very small sub-loops. For example, the loop L_i in Fig. 5 may be represented by 5 loops only. The flux that penetrates loops $L_{i,2}$, $L_{i,3}$ is very small and these loops may be ignored.

The presented representation of winding by sub-loop makes easy the process of forming matrix $\boldsymbol{N}_e$ that describes windings in the edge element space. The number of intersection points between the edge $\boldsymbol{K}_{p,q}$ (going from P_p to P_q) and surfaces $\boldsymbol{S}_{i,j}$ of L_i represent the entry of $\boldsymbol{N}_e$ that is related to the loop L_i and to the edge $\boldsymbol{K}_{p,q}$. In the calculation of intersection points the sense of $\boldsymbol{K}_{p,q}$ in relation to the sense of $\boldsymbol{S}_{i,j}$ should be taken into account. The entry $N_{eK_{p,q},i}$ is equal to the difference between the numbers of intersection point of negative and positive scalar product $\boldsymbol{S}_{i,j}\,\boldsymbol{K}_{p,q}$ [13]. Therefore, in Fig. 6, $N_{eK_{4,5},i}=0$.

The product of matrix $\boldsymbol{N}_e$ and vector $\boldsymbol{i}_c$ is equal to the ampere-turns θ_{be} around the edges. The ampere-turns around the edges represent the branch *mmf*s in magnetic EN. Thus, the vector Θ_{be} can be defined as follows

$$\Theta_{be} = \boldsymbol{\theta}_{be} = \boldsymbol{N}_e \boldsymbol{i}_c \,. \tag{1}$$

The vector $\boldsymbol{\theta}_{be}$ of ampere-turns in the loops around the branches of EN can be transformed into the ampere-turns $\boldsymbol{\theta}_{bf}$ in the loops around the branches of FN [15]. Fig. 7 explains the transformation in the case of network model of hexahedra. The transformation matrix $\boldsymbol{K}$ consists of weighted average factors. The product of matrix $\boldsymbol{K}$ and vector $\boldsymbol{\theta}_{be}$ gives the vector $\boldsymbol{\theta}_{bf}$ and the branch *mmf*s in FN,

$$\Theta_{bf} = \boldsymbol{\theta}_{bf} = \boldsymbol{K}\boldsymbol{\theta}_{be} = \boldsymbol{K}\Theta_{be} \,. \tag{2}$$

In the formulations using potential $\boldsymbol{A}$ it is not necessary to know the branch sources. The loop sources should be determined. The loop *mmf*s Θ_{mf} in FN are equal to the ampere-turns (total currents) θ_{mf} that penetrate the loops of FN. The ampere-turns θ_{mf} are obtained by multiplication of the loop matrix $\boldsymbol{k}_e^T$ and the vector θ_{bf},

$$\Theta_{mf} = \boldsymbol{\theta}_{mf} = \boldsymbol{k}_e^T \boldsymbol{\theta}_{bf} = \boldsymbol{k}_e^T \boldsymbol{K} \boldsymbol{N}_e \boldsymbol{i}_c \,. \tag{3}$$

The total currents that pass through the loops of FN may also be calculated from the currents (ampere-turns) $\boldsymbol{\theta}_{me}$ that cross the element faces, i.e. that cross the loops of EN and represent the facet values of current density. Matrix $\boldsymbol{k}_e$ transposes the currents in the loops around the edges into the currents in the branch of FN, i.e. into the total currents $\boldsymbol{\theta}_{me}$. Thus, using (1), we can find that the vector $\boldsymbol{\theta}_{me}$ of ampere-turns that cross the element facets is

$$\boldsymbol{\theta}_{me} = \boldsymbol{k}_e \boldsymbol{\theta}_{be} = \boldsymbol{k}_e \boldsymbol{N}_e \boldsymbol{i}_c \,. \tag{4}$$

The matrix product $\boldsymbol{k}_e \boldsymbol{N}_e$ is equal to the matrix $\boldsymbol{N}_f$ that describes winding loops in the facet element space.

$$\boldsymbol{N}_f = \boldsymbol{k}_e \boldsymbol{N}_e \tag{5}$$

The transposition matrix $\boldsymbol{N}_f$ can be easy determined by the calculation of intersection points between the loops $L_{i,j}$ and element facets $\boldsymbol{F}_q$. Fig. 8 illustrates the method of matrix $\boldsymbol{N}_f$ calculation. The winding loop L_i intersects two times facet $\boldsymbol{F}_q$. The scalar products of $\boldsymbol{F}_q$ and edges of $L_{i,j}$, $L_{i,k}$ are negative. Therefore the entry $N_{fq,i}$ is equal to –2.

When matrix $\boldsymbol{N}_f$ and currents $\boldsymbol{i}_c$ are given it is easy to calculate the vector $\boldsymbol{\theta}_{me}$. Then, using matrix $\boldsymbol{K}$, the vector $\boldsymbol{\theta}_{mf}$ and loop *mmf*s Θ_{mf} in facet network can be found [15].

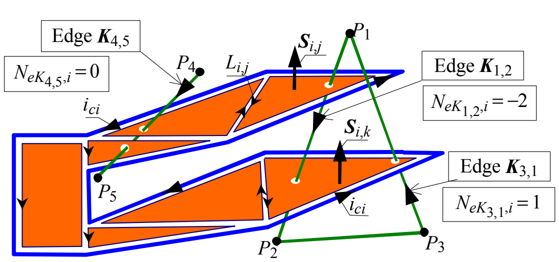

Fig. 6. Winding loop in the edge element space.

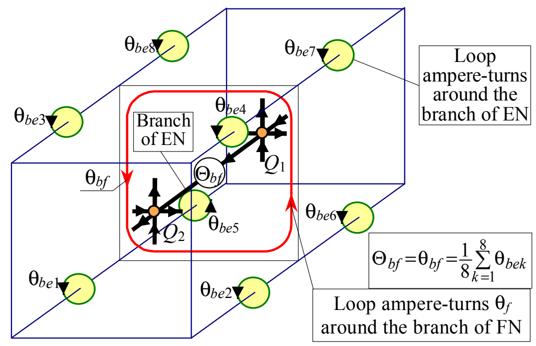

Fig. 7. Transformation of ampere-turns around the branches of EN into the ampere-turns around the branches of EN.

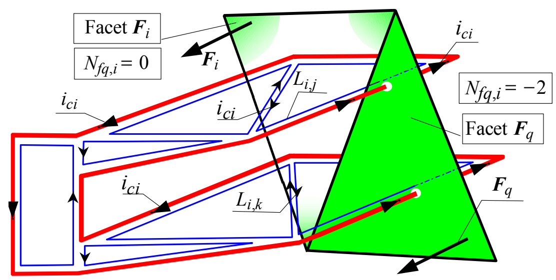

Fig. 8. Winding in the facet element space.

For given matrix $\boldsymbol{N}_f$ the vector Θ_{mf} may be expressed in the following form

$$\Theta_{mf} = \boldsymbol{\theta}_{mf} = \boldsymbol{K}^T \boldsymbol{\theta}_{me} = \boldsymbol{K}^T \boldsymbol{N}_f \boldsymbol{i}_c \,. \tag{6}$$

It is interesting to notice that there are two methods of forming the loop *mmf*s in FN: (a) the method using description of winding by the matrix $\boldsymbol{N}_e$ – see (3), and (b) the method using matrix $\boldsymbol{N}_f$ – see (6).

The presented above descriptions of winding can be used in the calculations of flux linkages with the loops L_i. For EN of branch fluxes ϕ_b the vector $\boldsymbol{\Psi}$ of flux linkages is

$$\boldsymbol{\Psi} = \boldsymbol{N}_e^T \boldsymbol{\phi}_b \,. \tag{6}$$

The description of winding loops in the edge element space is not unique. Thus matrix $\boldsymbol{N}_e$ is not unique either. The set of surfaces $\boldsymbol{S}_{i,j}$ with the total boundary L_i is not unique. However, the results of vector $\boldsymbol{\Psi}$ calculation are independent of the choice of $\boldsymbol{S}_{i,j}$.

In the case of the magnetic vector potential method and the facet magnetic network two formulas can be applied

$$\boldsymbol{\Psi} = \boldsymbol{N}_e^T \boldsymbol{K}^T \boldsymbol{k}_e \boldsymbol{\phi}_e \,, \tag{7a}$$

$$\boldsymbol{\Psi} = \boldsymbol{N}_f^T \boldsymbol{K} \boldsymbol{\phi}_e \,. \tag{7b}$$

A comparison of equations (7) with respect to (5) leads to the identity $\boldsymbol{K}^T \boldsymbol{k}_e = \boldsymbol{K} \boldsymbol{k}_e^T$.

The presented above descriptions of branch and loop mmfs and flux linkages are summarized in Table III. Formulas shown in Table III have been used in the formulation of winding equations, i.e. equations that describe loop currents $\boldsymbol{i}_c$.

TABLE III.
DESCRIPTIONS OF MMFS AND FLUX LINKAGES ELECTRIC NETWORKS

Network	Branch *mmf*s	Loop *mmf*s	Flux linkages
Edge	$\Theta_{be} = \boldsymbol{N}_e \boldsymbol{i}_c$	$\Theta_{me} = \boldsymbol{N}_f \boldsymbol{i}_c$	$\boldsymbol{\Psi} = \boldsymbol{N}_e^T \boldsymbol{\phi}_b$
Facet	$\Theta_{bf} = \boldsymbol{K}\boldsymbol{N}_e \boldsymbol{i}_c$	$\Theta_{mf} = \boldsymbol{K}^T \boldsymbol{N}_f \boldsymbol{i}_c$	$\boldsymbol{\Psi} = \boldsymbol{N}_f^T \boldsymbol{K}\boldsymbol{\phi}_e$
Comments: matrix $\boldsymbol{N}_e$ describes windings in the edge element space, matrix $\boldsymbol{N}_f$ describes windings in the edge element space $\boldsymbol{i}_c$ is the vector of currents in winding loops $\boldsymbol{K}^T \boldsymbol{k}_e = \boldsymbol{K}\boldsymbol{k}_e^T$, $\boldsymbol{N}_f = \boldsymbol{k}_e \boldsymbol{N}_e$			

The winding equations can be described in the following unified form

$$\boldsymbol{R}_m \boldsymbol{i}_c + \mathrm{p}\boldsymbol{\Psi} = \boldsymbol{e}\ , \qquad (8)$$

where $\boldsymbol{R}_m$ is the matrix of loop resistances, p=d/dt and $\boldsymbol{e}$ is the vector of external *emf*s, see Fig. 4. It should be notice that (8) also relates to the loop around the holes in multiply connected conductors with eddy currents, i.e. to the loops with currents i_{ci} in Fig.3. In equations for system in Fig. 3 the vector $\boldsymbol{e}$ is equal the voltages produces by eddy currents; i.e. $\boldsymbol{e} = -\boldsymbol{N}_e^T \boldsymbol{k}_e^T \boldsymbol{R}\boldsymbol{k}_e \boldsymbol{i}_e$.

V. EQUATIONS OF COUPLED MAGNETO-ELECTRIC MODEL

In the preceding section it has been discussed the coupling between the FE equations of magnetic field and the equations of winding currents. In order to form the complete field model of machine the links between the magnetic field and eddy current field must be consider. It has been shown [15] that branch sources in FN are established from loop quantities in EN, and – by symmetry – branch sources in EN are found from loop quantities in FN. Branch *mmf*s Θ_{be} in EN correspond to loop currents. Branch *emf*s $\boldsymbol{e}_{be}$ in EN are found as time derivatives of loop fluxes ϕ_e in FN. Using the symbols in Tables I, II, the branch sources of EN can be written as follows

$$\Theta_{be} = \boldsymbol{i}_e\ ,\ \boldsymbol{e}_{be} = -\mathrm{d}\phi_e/\mathrm{d}t\ . \qquad (9)$$

In the analysis of FN we need loop sources. The loop *mmf* is equivalent to the current passing through the loop of the magnetic network, thus loop *mmf*s Θ_{mf} in the FN correspond to the branch currents $\boldsymbol{i}_b$ in the EN, see Table II. In the FN models of eddy current regions the loop *emf*s may be found by taking time derivatives of branch fluxes in the magnetic network passing through the loops of the electric network, i.e. fluxes ϕ_b in the branches of EN, see Table I. Thus, the loop sources in FN can be express in the following form

$$\Theta_{mf} = \boldsymbol{i}_b = \boldsymbol{G}(\boldsymbol{k}_n \boldsymbol{V} - \mathrm{p}\phi_e)\ , \qquad (10a)$$

$$\boldsymbol{e}_{mf} = -\mathrm{p}\phi_b = -\mathrm{p}(\Lambda(\boldsymbol{k}_n \boldsymbol{\Omega} + \Theta_{be}))\ . \qquad (10b)$$

The field sources in FN can be also calculated using the relations presented in section IV. The winding loops should be assumed to be eddy current loops, i.e. $\boldsymbol{i}_c = \boldsymbol{i}_e$ and $\boldsymbol{N}_e = \boldsymbol{1}$. As a result we obtain

$$\Theta_{mf} = \boldsymbol{k}_e^T \boldsymbol{K}\boldsymbol{i}_e\ ,\ \boldsymbol{e}_{mf} = -\mathrm{p}\boldsymbol{K}^T \boldsymbol{k}_e \phi_e\ . \qquad (11)$$

On the basis of the presented above equations the field model of electrical machine may be constructed. Further on we will show the equations of the model for the formulations described in Section I, i.e. for Ω–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation, for $\boldsymbol{A}$–V–$\boldsymbol{T}_0$ formulation, and for description using $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$.

The FE equation for formulation Ω–$\boldsymbol{T}$–$\boldsymbol{T}_0$ are represented by nodal equations of edge magnetic network coupled with the loop equations that describe eddy currents in electric FN and currents in winding loops. These equations can be written in the following matrix form

$$\begin{bmatrix} \boldsymbol{k}_n^T \Lambda \boldsymbol{k}_n & \boldsymbol{k}_n^T \Lambda & \boldsymbol{k}_n^T \Lambda \boldsymbol{N}_e \\ \mathrm{p}\Lambda \boldsymbol{k}_n & \boldsymbol{R}_e + \mathrm{p}\Lambda & (\boldsymbol{R}_e + \mathrm{p}\Lambda)\boldsymbol{N}_e \\ \mathrm{p}\boldsymbol{N}_e^T \Lambda \boldsymbol{k}_n & \boldsymbol{N}_e^T(\mathrm{p}\Lambda + \boldsymbol{R}_e) & \boldsymbol{R}_m + \boldsymbol{N}_e^T \mathrm{p}\Lambda \boldsymbol{N}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega} \\ \boldsymbol{i}_e \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{k}_n^T \Lambda \theta_b \\ \boldsymbol{0} \\ \boldsymbol{e} \end{bmatrix}. \qquad (12)$$

Here $\boldsymbol{R}_e$ is the matrix of loop resistances for loops with eddy currents, $\boldsymbol{R}_e = \boldsymbol{k}_e^T \boldsymbol{R}\boldsymbol{k}_e$, and θ_b is the vector of additional branch *mmf*s in the permanent magnet region. These *mmf*s represent the edge values of magnetization vector. In the above equation vector $\boldsymbol{i}_c$ describes the winding currents and the currents in the loops around the 'holes' in the region with eddy currents. For systems without eddy currents equation (12) becomes simpler and have the following matrix form

$$\begin{bmatrix} \boldsymbol{k}_n^T \Lambda \boldsymbol{k}_n & \boldsymbol{k}_n^T \Lambda \boldsymbol{N}_e \\ \mathrm{p}\boldsymbol{N}_e^T \Lambda \boldsymbol{k}_n & \boldsymbol{R}_m + \boldsymbol{N}_e^T \mathrm{p}\Lambda \boldsymbol{N}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega} \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{k}_n^T \Lambda \theta_b \\ \boldsymbol{e} \end{bmatrix}. \qquad (13)$$

It seems that for eddy currents calculation the most convenient is $\boldsymbol{A}$–V–$\boldsymbol{T}_0$ formulation. This formulation is equivalent to the loop analysis of facet magnetic network, coupled with nodal analysis of EN for eddy currents and with the loop description of winding with stranded conductors. The FE equations for $\boldsymbol{A}$–V–$\boldsymbol{T}_0$ formulation are

$$\begin{bmatrix} \boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e + \boldsymbol{G}\mathrm{p} & -\boldsymbol{G}\boldsymbol{k}_n & -\boldsymbol{K}^T \boldsymbol{N}_f \\ -\mathrm{p}\boldsymbol{k}_n^T \boldsymbol{G} & \boldsymbol{k}_n^T \boldsymbol{G}\boldsymbol{k}_n & \boldsymbol{0} \\ \mathrm{p}\boldsymbol{N}_f^T \boldsymbol{K} & \boldsymbol{0} & \boldsymbol{R}_m \end{bmatrix} \begin{bmatrix} \phi_e \\ \boldsymbol{V} \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \theta_m \\ \boldsymbol{0} \\ \boldsymbol{e} \end{bmatrix}, \qquad (14)$$

where θ_m is the vector of loop *mmf*s in the regions with permanent magnets; $\theta_m = \boldsymbol{k}_e^T \boldsymbol{K}\theta_b$.

In the analysis of electrical machines we can also apply $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation. The FE equations for this formulation represent the loop equations for magnetic and electric FNs. These equations can be expressed as follows

$$\begin{bmatrix} \boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e & -\boldsymbol{k}_e^T \boldsymbol{K} & -\boldsymbol{K}^T \boldsymbol{N}_f \\ \mathrm{p}\boldsymbol{K}^T \boldsymbol{k}_e & \boldsymbol{R}_e & \boldsymbol{k}_e^T \boldsymbol{R}\boldsymbol{N}_f \\ \mathrm{p}\boldsymbol{N}_f^T \boldsymbol{K} & \boldsymbol{N}_f^T \boldsymbol{R}\boldsymbol{k}_e & \boldsymbol{R}_m \end{bmatrix} \begin{bmatrix} \phi_e \\ \boldsymbol{i}_e \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \theta_m \\ \boldsymbol{0} \\ \boldsymbol{e} \end{bmatrix}. \qquad (15)$$

In the case of electrical machine of negligible eddy currents both (14) and (15) are simplified into the following matrix equations

$$\begin{bmatrix} \boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e & -\boldsymbol{K}^T \boldsymbol{N}_f \\ \mathrm{p}\boldsymbol{N}_f^T \boldsymbol{K} & \boldsymbol{R}_m \end{bmatrix} \begin{bmatrix} \phi_e \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \theta_m \\ \boldsymbol{e} \end{bmatrix}. \qquad (16)$$

These equations are the FE representation of $\boldsymbol{A}$-$\boldsymbol{T}_0$ formulation and seems to more simpler than (13) where there is the time derivative of permeance matrix $\boldsymbol{\Lambda}$.

VI. MOVEMENT SIMULATION

Movement simulation is one of the most important problem in the FE analysis of electrical machines. There are a lot of techniques of modelling the changes of rotor position in the FE description of electrical machines [6, 11, 26, 30].

The FE methods taking into account the movement can be divided into two categories: (a) techniques with the fixed grid independent of the moving region position (FG), and (b) the techniques with the moving grid (MG) [26, 29, 33]. The FG methods have been successfully applied in the analysis of the systems with the homogenous moving part and constant speed. In the specific cases, the problems with the changes of speed can also be solved by the fixed grid methods. The FG method needs the suitable discretization of the region considered and the close correlation between the value of rotor speed and the value of element side dimensions. For example, in the case of time step method application, the time step length must be controlled to get the displacement "for element to element" in each time step.

The moving grid methods are more universal. In these techniques the grid is divided into two parts: the moving part associated with the moving elements, i.e. with the rotor of electric machine, and the fixed part associated with the other elements, e.g. with the stator of electric machine. Between these parts, the interconnecting band or slip surface is created. The most popular methods of coupling the fixed and moving part through the band or the slip surface can be easy explain using the notation of equivalent edge and facet networks.

In the scalar potential method, i.e. method equivalent to the nodal analysis of EN, the changes of rotor position are modelled by the changes of nodal permeances matrix $\boldsymbol{k}_n^T \boldsymbol{\Lambda} \boldsymbol{k}_n^T$. For vector potential method that corresponds to the loop analysis of FN the changes of rotor position are represented by the changes of loop reluctance matrix $\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e$. At first we will explain the moving grid method for the formulation using magnetic vector potential $\boldsymbol{A}$. In this formulation the changes can be related to the following matrices of product $\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e$: (a) the structural matrix $\boldsymbol{k}_e$, (b) the matrix $\boldsymbol{R}_\mu$ of branch reluctances or (c) both the matrix $\boldsymbol{k}_e$ and $\boldsymbol{R}_\mu$. In the methods (a) we consider discrete position of rotor that differs on the distribution of nonzero elements of loop matrix $\boldsymbol{k}_e$. As a result we obtain a set of matrices $\boldsymbol{k}_e$ for successive rotor positions. This set represents the data points for interpolation functions that express the dependence of $\boldsymbol{k}_e$ on the angle α that describes the rotor position [11]. It can be proved that if the trigonometric interpolation is applied then the method is equivalent to the method of harmonic weighting functions presented in [9]. When a suitable interpolation of matrix $\boldsymbol{k}_e$ is applied the method of category (a) guarantees high accuracy. However due to the increase of matrix $\{\boldsymbol{k}_e(\alpha)\}^T \boldsymbol{R}_\mu \boldsymbol{k}_e(\alpha)$ density the procedure of solving the FE equations becomes complex and time-consuming. Also, in the case of methods of category (b) with $\boldsymbol{R}_\mu = \boldsymbol{R}_\mu(\alpha)$, the matrix of loop reluctances is dense what results in the increase of computation time. The most representative method of category (b) is the air-gap element method.

The simplest methods of movement simulation belong to category (c). A typical representative of these methods is moving band method with remeshing of FE network. In this method**,** at each time step of the rotor displacement, the finite elements are distorted and $\boldsymbol{R}_\mu = \boldsymbol{R}_\mu(\alpha)$ or, if the distortion is large, the band is remeshed, i.e. the nonzero elements of matrix $\boldsymbol{k}_e$ change their position [6]. In the matrix product $\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e$, function $\boldsymbol{R}_\mu(\alpha)$ is a piecewise continuous and the matrix $\boldsymbol{k}_e$ is represented by discrete set of its values. As a result the derivative of matrix $\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e$ entries with respect to angle α is not continuous that produces the ripple in calculated quantities [11]. However, due to the simplicity, the moving band method belongs to the most popular methods of movement simulation.

The presented method of movement simulation for vector potential formulation can be easy adopted for scalar potential method. Changes of matrices $\boldsymbol{R}_\mu$, $\boldsymbol{k}_e$ are represented by changes of matrices Λ, $\boldsymbol{k}_n$ in the scalar potential methods

VII. ELECTROMAGNETIC TORQUE

One of the most important advantages of the FE analysis of rotating electrical machines is ability to calculate the electromagnetic torque with high accuracy. The relationships that describe electromagnetic torque are formed on the basis of virtual work principle. The obtained formulas can be divided into 2 categories: (a) the force density formulas, e.g. Lorenz formula, the method of magnetizing currents, and (b) stress tensor formulas, e.g. the Maxwell stress tensor formula [25]. Of course, for the exact solution of Maxwell's equations the formulas of both categories give the identical result. However, in the case of FE models the results of different methods of torque calculation are not identical. Moreover, the position of integration surface has an effect on the result of stress tensor method. Therefore, very often in the FE models the electromagnetic torque is calculated using the formulas that are obtained from virtual work principle implemented to discrete systems [5, 12, 25, 26].

In accordance with virtual work principle, for scalar potential method, the torque is equal to the magnetic coenergy derivative versus the virtual moving. An interpolation function can be applied to describe this derivative. The interpolation gives

$$T(\alpha) = \left.\frac{\partial W_c(\alpha+\Delta\alpha)}{\partial(\Delta\alpha)}\right|_{\Delta\alpha=0} = \frac{W_c(\alpha+\beta) - W_c(\alpha+\beta)}{2\beta}, \quad (17)$$

where $W_c(\alpha\pm\beta)$ is the magnetic coenergy for rotor position $\alpha\pm\beta$, see Fig 9. From (17), using symbols in Table 1 we obtain

$$T(\alpha) = \frac{1}{2\beta} \boldsymbol{\Omega}^T \left[\left.(\boldsymbol{k}_n^T \boldsymbol{\Lambda} \boldsymbol{k}_n)\right|_{\alpha+\beta} - \left.(\boldsymbol{k}_n^T \boldsymbol{\Lambda} \boldsymbol{k}_n)\right|_{\alpha-\beta} \right] \boldsymbol{\Omega}. \quad (18)$$

For vector potential method the magnetic energy derivative versus the virtual moving is considered [12]. The derivative can be approximated by the finite differences and as a result we have

$$T(\alpha) = -\frac{1}{2\beta} \boldsymbol{\phi}_e^T \left[\left.(\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e)\right|_{\alpha+\beta} - \left.(\boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e)\right|_{\alpha-\beta} \right] \boldsymbol{\phi}_e. \quad (19)$$

In the literature, there are also other methods of work principle representation. Particularly noteworthy are the

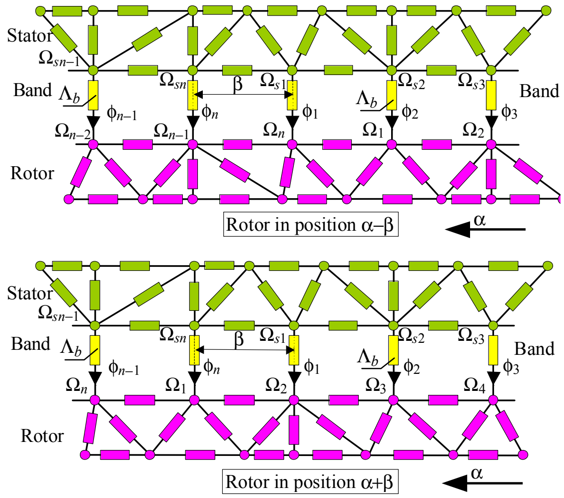

Fig. 9. A part of magnetic EN for 2 discrete rotor positions

methods adapted to the applied techniques of movement simulation [5, 11]. For example, if in the procedure of movement simulation, $\boldsymbol{R}_\mu=\boldsymbol{R}_\mu(\alpha)$ then

$$T(\alpha)=-0.5\phi_e^T\{\boldsymbol{k}_e^T(\mathrm{d}\boldsymbol{R}_\mu/\mathrm{d}\alpha)\boldsymbol{k}_e\}\phi_e. \tag{20}$$

This formula and (18), (19) can be considered as the FE representation of Maxwell stress formula with integration surface related to the band between the stator and rotor.

VIII. Field-Circuit Model

The presented below field-circuit (F-C) model of electrical machine should not be mixed up with the field-circuit model of the electric drive. The field-circuit model of electric drive is constructed by joining FE equations given in section V with equations that describe supply system [20, 23, 27]. Moreover in the F-C model of drives the field and circuit equations are coupled through the expressions that define the electromagnetic and opposite torque to the equation of motion [3, 10, 12].

In this paper the term field-circuit model of electrical machine relates to the approaches that express the flux linkages $\boldsymbol{\Psi}$ with the windings by two components: (a) components defined by field quantities and (b) component represented by inductances. Thus, expressions in Table III become

$$\boldsymbol{\Psi}=\boldsymbol{N}_f^T\boldsymbol{K}\phi_e+\boldsymbol{L}_e\boldsymbol{i}_c\,,\quad \boldsymbol{\Psi}=\boldsymbol{N}_e^T\phi_b+\boldsymbol{L}_e\boldsymbol{i}_c\,, \tag{21a,b}$$

where $\boldsymbol{L}_e$ is the matrix of equivalent inductances. The equivalent inductances relate to this part of winding that is out of the region divided into the finite elements.

The F-C model is applied when the magnetic field is assumed to be 2D independent of the coordinate parallel to the rotor shaft. In the 2D model the matrix $\boldsymbol{L}_e$ describes the inductances of winding ends.

It can be seen that equations of F-C model are similar to the equations presented in section V. To obtain equations of F-C model matrix $\boldsymbol{R}_m$ in (12) – (16) should be replaced by sum $\boldsymbol{R}_m+\mathrm{p}\boldsymbol{L}_e$.

The F-C model with the 2D description of magnetic field is simpler than 3D. The number of unknowns in FE equations is considerably smaller. Therefore, although the software for 3D calculation is commonly available the 2D F-C models are still in wide use.

Some authors use the phrase 'field-circuit' for the circuit equivalent model with the FE calculations of circuit parameters, mostly inductances. The method based on the FE calculations of circuit parameters is effective in specific cases only. Three factors determine the method efficiency: (a) number of coupled windings, (b) saturation ratio, (c) influence of eddy currents on the flux linkages [16, 18, 22]. The circuit model with the FE calculation of circuit parameters can be successfully used in the analysis of systems without eddy currents with non saturated core. The circuit model aided by the FE method can also be effectively applied in the description of system with saturated core but with no-mutual coupling between the windings, e.g. to simulate switch reluctance motor. To form the model of this motor we should define self inductance only. The FE calculation self inductance only are not time consuming, even if it is necessary to take into account that the inductance is a function of current and rotor position.

In the FE calculations of self and mutual inductances we may apply presented above equations of field model. The vector of self and mutual inductances of the i^{th} winding is equal to the vector $\boldsymbol{\Psi}$ of flux linkages produced by elementary current in this winding. The i^{th} component of $\boldsymbol{\Psi}$ represents the self inductance, the j^{th} component is equal to the mutual inductance between the i^{th} and j^{th} winding. In the calculation of inductances for machine of saturated core we should take into consideration that matrices $\boldsymbol{R}_\mu$, $\boldsymbol{\Lambda}$ depend on the currents $\boldsymbol{i}_c$. Therefore, in the calculations of inductance for given values of currents $\boldsymbol{i}_c$ the FE equations should be solved two times: first for given vector $\boldsymbol{i}_c$ to determine the values of entries of matrix $\boldsymbol{R}_\mu$ or $\boldsymbol{\Lambda}$, and then for elementary currents to find the vectors of self and mutual inductances.

It can be calculate that to describe the inductances of six coupled windings for 20 values of currents and 20 rotor position the FE equations should be solved more than billion times. Thus, the FE calculations of circuit parameters are very time consuming.

Usually, in the description of electrical machines the equivalent circuits are applied. The equivalent circuits are formed by the application of current transformation; e.g. Clarke and Park transformation. The inductances of equivalent transformed system can be directly calculated using the presented above field model. In the algorithm the transposed currents $\boldsymbol{i}_T$ are assumed to be known. For elementary values of currents $\boldsymbol{i}_T$ the currents $\boldsymbol{i}_c$ in winding loops are calculated, $\boldsymbol{i}_c=\boldsymbol{k}^T\boldsymbol{i}_T$, where $\boldsymbol{k}$ is the transformation matrix. Then, for the currents $\boldsymbol{i}_c$ the FE equations are solved and the vectors $\boldsymbol{\Psi}_T$ of flux linkages for transformed system are determined, $\boldsymbol{\Psi}_T=\boldsymbol{k}\boldsymbol{\Psi}$. These vectors represent the inductances of transformed circuit model.

It should be noticed that the described algorithm differs from the classical approaches. In the classical approaches at first the inductances of real system are determined and then these inductances are transformed [19]. The calculations using the algorithm with direct calculations of equivalent inductances are much less time consuming [3, 17]. In has been shown [17] that in the case of 3-phase balanced system that produces rotating magnetic field in saturated core the procedure of equivalent inductances calculations needs to solve FE equations 6*m* times only, where *m* is the number of given rms values I_s of phase current to form function $\boldsymbol{L}(I_s)$.

IX. Conclusions

The paper presents the so called field and the field-circuit models of electrical machines. The particular approaches have been elaborated to simulate the machine's behavior with the presented models [7, 8, 21]. The developments in these procedures and in the methods to solve large systems of equations enable to apply the presented model in many practical applications of technical importance.

In the discussion on field and field-circuit models practicability the three directions of applications should be separated, namely: (a) design, (b) diagnostics and (c) control. The field methods are commonly used in the design stage of electrical machines. With the application of high performance field models the designing engineer can avoid the costs of building expensive prototypes of the device under study.

Field methods are also applied for diagnostic purpose [32]. However, in diagnostics the field models are not as popular as for the machine design. Recently, the field models become more and more popular in the analysis and synthesis of electrical machine control, even though the control methods are based on the classical circuit approaches. Mostly, the field methods are used to calculate the parameters of the equivalent circuit of considered control system [3, 18]. The field methods can be especially helpful in the case of sensorless control when an accurate description of machine parameters is required. It seems that untill now, due to the high computational costs, the exact 3D field model has not been directly applied in the control systems.

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