Topology Optimization of Magnetothermal Systems Considering Eddy Current as Joule Heat

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This research presents a topology optimization for manipulating the main heat flow in coupled magnetothermal systems. The heat generated by eddy currents is considered in the design domain assuming an adiabatic boundary. For a practical optimization, the convection condition is considered in the topological process of the thermal field. Topology design sensitivity is derived employing the discrete system equations combined with the adjoint variable method. As numerical examples, a simple iron and a C-core design heated-up by eddy currents demonstrate the strength of the proposed approach to solve the coupled problem.

***Index Terms*—Design sensitivity analysis, eddy current, heat flow, topology optimization.**

I. Introduction

MOST researches associated with the optimization of magnetothermal systems have focused on the Joule heat, generated in coils. However, this can be considered a problem due to the fact that optimization regarding only solid conduction does not result in a good design, because eddy current effects are not neglected in a transient magnetic field. The eddy current problem in shape optimization has been studied [1]. Conventional optimal techniques are aimed at the improvement of current designs. On the other hand, topology optimization focuses on obtaining an initial conceptual design. The topology optimization does not require sophisticated initial design and generates some holes on the domain, which the shape optimum does not.

The topology optimization of electromagnetic systems has begun using optimized material distribution (OMD) [2], [3]. As such, applications to practical systems show the possibility of realization [4], [5].

The design optimization of thermal systems has been analytically or numerically studied since the 1950s. Then, in the 1980s shape optimization received a greater amount of attention. The topology optimization of the thermal systems, however, is a relatively recent technique and is only being done by some researchers. In this context, a topology optimization was presented for a heat conduction problem in order to minimize the resistance between input and output points [6], and many researches have been done for coupled problems [7], [8]. However, most of these are based on electrothermal fields, not magnetothermal systems. Thus the optimization regarding the eddy current problem has never been discussed in great detail. Furthermore, convection study has rarely been considered in the thermal optimization problem.

In this paper, topology optimization of magnetothermal systems is presented including eddy currents as the main source of Joule heat. The adjoint variable topology sensitivity equation is derived using the discrete method, and topology optimization program is developed to deal with eddy currents.

The optimization progresses for maximizing heat transfer with volume constraint. Furthermore, for numerical examples, the proposed approach is applied to both, a simple iron design and a C-core actuator. This method can be extended to design of power reactor for industrial applications.

II. Governing Equations

A. Electromagnetic System

A finite element equation of any problem governed by specified differential equations and the boundary conditions can be achieved by the variational method [9].

To this extend, the transient magnetic field can be described using a set of Maxwell's equations. By introducing a complex vector potential, A^*, such that $B = \nabla \times A^*$ and eliminating H, a single governing equation can be expressed as

$$\nabla \times \left(\frac{1}{\mu}\nabla \times A^*\right) + j\omega\sigma A^* = J_s \tag{1}$$

where J_s, μ and σ are the current density vector, the permeability of material and the electric conductivity, respectively.

To obtain the variational equation, multiplying both sides of (1) with the virtual vector potential $\overline{A}^*$, and integrating over the domain yield

$$\iiint_\Omega \left[\nabla \times \left(\frac{1}{\mu}\nabla \times A^*\right) + j\omega\sigma A^*\right] \cdot \overline{A}^* d\Omega = \iiint_\Omega [J_s] \cdot \overline{A}^* d\Omega \qquad \text{for all } \overline{A}^* \in \widetilde{A}^* \tag{2}$$

where $\widetilde{A}^*$ is the space of admissible vector potential.

After applying boundary conditions, the variational equation becomes [10], [11]

$$a_\Omega(A^*, \overline{A}^*) = l_\Omega(\overline{A}^*) \qquad \text{for all } \overline{A}^* \in \widetilde{A}^* \tag{3}$$

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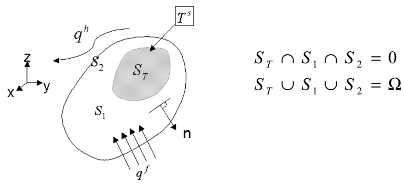


Fig. 1. Body subject to heat transfer.

where

$$a_\Omega(A^*, \overline{A}^*) = \iiint_\Omega \left[(\nabla \times A^*) \cdot \left(\frac{1}{\mu}\nabla \times \overline{A}^*\right) + j\omega\sigma A^* \overline{A}^*\right] d\Omega$$
$$l_\Omega(\overline{A}^*) = \iiint_\Omega [J_s \cdot \overline{A}^*] d\Omega. \quad (4)$$

$a_\Omega(A^*, \overline{A}^*)$ is the energy bilinear form, and $l_\Omega(\overline{A}^*)$ is the load linear form; functions of permeability μ and the system's solution A^*.

Therefore, a matrix form for the finite element solution is expressed as

$$(K_{\text{mag}} + j\omega M_{\text{mag}})\{A^*\} = \{J_S\}. \quad (5)$$

K_{mag} and M_{mag} are magnetic stiffness and magnetic mass matrices, respectively [12].

B. Thermal System

Conduction refers to the energy flow from a high temperature to a low temperature area. For the heat transfer analysis, we assume that the material obeys Fourier's law of heat conduction.

As such, a general equilibrium equation in the steady state can be derived from energy balance and Fourier' law [13]

$$\nabla \cdot (k \cdot \nabla T) = -q^b \quad (6)$$

where k, T and q^b are the thermal conductivity, the temperature and the internal heat generation rate per unit volume, respectively.

By using the Galerkin's method, (6) becomes

$$\iiint_\Omega w \cdot \left[\nabla \cdot (k \cdot \nabla T) + q^b\right] d\Omega = 0 \quad (7)$$

where w is an approximation function.

And integrating parts of (7) yields

$$\iiint_\Omega (k \cdot \nabla w \cdot \nabla T) - w \cdot q^b] d\Omega - \iiint_\Gamma w \cdot k \frac{\partial T}{\partial n} d\Gamma = 0. \quad (8)$$

Natural boundary condition in Fig. 1 is expressed by

$$q^f = k\frac{\partial T}{\partial n} + h_c(T - T_b) \quad (9)$$

where q^f, h_C and T_b are the external heat flux, the convection coefficient and the known environmental temperature, respectively.

By applying (9), the surface integral term of (8) becomes

$$\iiint_\Gamma wk\frac{\partial T}{\partial n} d\Gamma = \iiint_{S1} wq^f d\Gamma - \iiint_{S2} wh_c(T - T_b) d\Gamma. \quad (10)$$

The final form of the variational equation is

$$a_\Omega(T, w) = \iiint_\Omega (k \cdot \nabla w \cdot \nabla T) d\Omega + \iiint_{S2} w \cdot h_c \cdot T d\Gamma$$
$$l_\Omega(w) = \iiint_{S1} w \cdot q^f d\Gamma + \iiint_{S2} w \cdot h_c \cdot T_b d\Gamma + \iiint_\Omega w \cdot q^b d\Omega. \quad (11)$$

The matrix form for the finite element thermal field can subsequently be expressed by

$$[K_{th}]\{T\} = \{Q\}. \quad (12)$$

K_{th} is the finite element thermal stiffness matrix, T is the temperature vector and Q is the main heat vector classified into the Joule heat, generated by the coil and the eddy current, and the heat flowing into or out of the domain.

III. Design Sensitivity Equation

The topology optimization is considered as a heavy computation problem since it deals with a large number of finite elements in the design domain. The adjoint variable method (AVM) is probably the unique alternative to calculate the sensitivities [11].

Consider a measure of the thermal performance as

$$\psi = \psi(T, T(b)) \quad (13)$$

where b is a vector of the design variables.

Taking derivatives of (13) with respect to the design variable yields

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \lambda_T^T\left[\frac{\partial Q}{\partial b} - \frac{\partial}{\partial b}(K_{th}\widetilde{T})\right]. \quad (14)$$

λ_T is a vector of the adjoint variable for thermal systems, and a tilde ($\sim$) indicates a variable that is to be held constant for partial differentiation.

The corresponding adjoint equation to (14) is written as

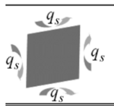
$$K_{th}\lambda_T = \left[\frac{\partial \psi}{\partial b}\right]^T. \quad (15)$$

Note that while performing the thermal adjoint equation, the convection coefficient is kept, and T_b is set to 0.

The heat, Q, contains the Joule heat generated by the applied current in the coil and the eddy current in the body, both of which are associated with the magnetic system. Since the design domain is iron material in the optimization, the final design sensitivity equation is obtained by introducing (5)

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \lambda_T^T\left[\frac{\partial Q_f}{\partial b} + \frac{\partial Q_{\text{conv}}}{\partial b} + \frac{\partial Q_{\text{eddy}}}{\partial b} - \frac{\partial}{\partial b}(K_{th}\widetilde{T})\right]$$
$$+ \lambda_A^T\left[\frac{\partial J_S}{\partial b} - \frac{\partial}{\partial b}(K_{\text{mag}}\widetilde{A}^*) - j\omega\frac{\partial}{\partial b}(M_{\text{mag}}\widetilde{A}^*)\right]. \quad (16)$$

TABLE I
QUADRILATERAL ELEMENT AND MATERIAL INTERPOLATION CRITERIA

		Heat Flow	$b=1$	$b=0$
	Conduction	$q_k = -k\nabla T$	max	0
	Convection	$q_s = h_c A_s (T - T_\infty)$	0	max

Q_f, $Q_{\rm conv}$ and $Q_{\rm eddy}$ are the heat flows interacting with the domain, occurring by convection and by eddy current effects. λ_A is a vector of the adjoint variable of the magnetic system which is calculated by an adjoint equation

$$(K_{\rm mag} + j\omega M_{\rm mag})\lambda_A = \left[\frac{\partial Q_{\rm eddy}}{\partial A^*}\right]^T \lambda_T. \qquad (17)$$

The material interpolation method defines artificial materials such as permeability and electric conductivity for the magnetic domain, and thermal conductivity for the thermal field. In addition, the convection coefficient at the boundary should be considered while the topology commits any hole in the design domain. All components have to be composed by a function of a polynomial of degree p defining the material density, and b to remove the intermediate material density in the optimization result. This yields the modified equivalent material coefficients [14]

$$\mu = \mu_0 + (\mu_0\mu_r - \mu_0)b^P \qquad (18)$$
$$\sigma = b^P \sigma_{\rm initial} \qquad (19)$$
$$k = b^P k_{\rm initial} \qquad (20)$$
$$h_c = h_{c.{\rm initial}}\left(1 - b^{1/p^3}\right) \qquad (21)$$

where subscript "initial" indicates the value prior to the optimization.

Table I shows the quadrilateral element used in finite element (FE) model, and a criteria of the material interpolation for the conduction and the convection. As can be seen, if density of the material properties is 1, the conduction entirely occurs and the convection term disappears on 4 edges. Otherwise, the phenomena are in reverse order.

IV. TOPOLOGY OPTIMIZATION

The topology optimization aims to search for an optimum material distribution that maximizes or minimizes an objective function while satisfying given constraints.

In this paper, the objective is to maximize the nodal temperature on the metal domain, where the eddy current generates the main Joule heat. Maximizing the nodal temperature tends to minimize the thermal resistance toward the target nodal points. This implies that the direction of the main heat flow is intentionally manipulated on the domain in such a way that the optimal design efficiently controls the most heat to be radiated to the outside. And the volume is a constraint used to limit the optimal design. Hence, the topology optimization problem takes the form

$$\text{maximize} \quad \frac{1}{\rm n}\sum_{i=1}^{n} \text{Nodal Temp}_i$$
$$\text{subject to} \quad g = \frac{\iiint_\Omega bAtd\Omega}{V_g \times V_0} - 1 \le 0 \qquad (22)$$

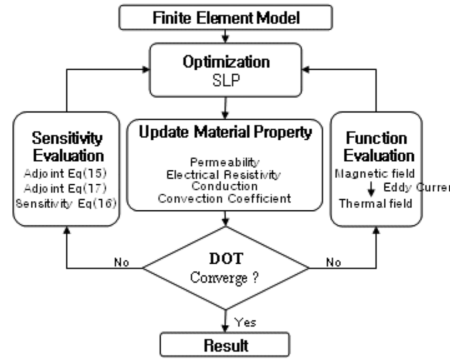


Fig. 2. Flow chart.

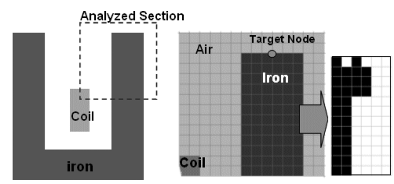


Fig. 3. Numerical model and optimal design.

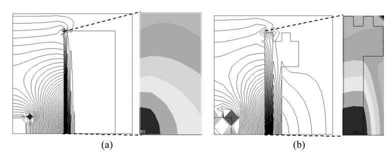


Fig. 4. Magnetic flux line and temperature contour. (a) Original design; (b) optimal design.

bounded to $0 \le b \le 1$ for all $b \in \Omega$.

A is the area, t is the thickness, and V_g, V_0 are the given volume by designer and the initial volume, respectively.

The In-House Code used in this paper has been developed as a controller based on C++ language. The controller calls an analyzer (ANSYS) to evaluate the objective function and iteratively computes the design sensitivity. The design optimization tools (DOT) are used as an optimizer. Fig. 2 outlines the topology optimization procedure.

V. NUMERICAL EXAMPLE

The first numerical model, a particular section indicated in Fig. 3, consists of three materials including coil, air and iron surface. The optimization is performed to find an iron shape satisfying the problem setup. The volume constraint is set to 50% of the initial one ($V_g = 0.5$).

Once the current density with frequency 60 [Hz] is applied in the coil, eddy currents occur inside the iron material. Note that the heat generated by eddy currents is considered in the design domain assuming an adiabatic boundary.

The design domain is optimized to increase the temperature of a target node shown in Fig. 3. Fig. 4 illustrates the magnetic flux line of the analyzed section and temperature contour of the iron domain. The optimal design is determined by eliminating the iron elements holding low density.

The strength of the optimal design is illustrated by comparing the results of the original analysis with the reanalysis of optimal design in Table II.

TABLE II
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Average of Nodal Temperatures [%]	100	175.17
Heat Transfer Rate per Volume [%]	100	258.94
Magnetic Energy of Iron Domain [%]	100	504.08
Volume [%]	100	41.67

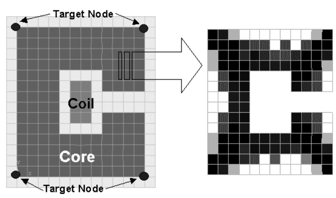
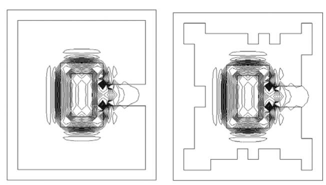
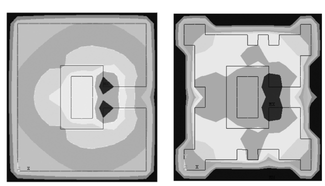


Fig. 5. C-core model with target nodes and optimal design.



(a) (b)

Fig. 6. Plot of magnetic flux line. (a) Original design; (b) optimal design.



(a) (b)

Fig. 7. Plot of temperature contour. (a) Original design; (b) optimal design.

TABLE III
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Average of Nodal Temperatures [%]	100	161.35
Heat Transfer Rate per Volume [%]	100	121.34
Magnetic Energy of Core Domain [%]	100	101.95
Volume [%]	100	70.56

For a global approach, an additional C-core design is implemented with 4 target nodal points. Each target location is selected at vertices of the core domain for such a novel design regarding the thermal field. The 70% of the initial volume is given as a constraint $(V_g = 0.7)$. Fig. 5 illustrates the design model and optimized topologies with gray level.

In order to maximize the average of target nodal temperatures, the thermal resistance is minimized at some regions from the elements of the generated heat to the target spots. Since the heat resistance by the convection term is much higher than the conduction, the conductive material keeps being high density-level around the target spots.

Figs. 6 and 7 present the plot of magnetic flux line and temperature contours, respectively, for comparison between the original and the optimal designs. Table III verifies the performance comparison.

An interesting effect can be observed from Tables II and III. Even though the objective is to maximize the nodal temperature, the optimal design results in maintaining the magnetic energy in comparison with original one. Because the heat source is the eddy current induced by the magnetic field, it turns out that magnetic flux is at least preserved on the process of the optimization. And the temperature distribution is much more sensitive than the magnetic characteristics. From the optimal topologies shown, the optimization regarding target nodes do not generate checkerboard problem.

VI. CONCLUSION

In this paper, the topology optimization is performed regarding eddy currents as the Joule heat. For magnetothermal systems, a topology design sensitivity is derived by employing the discrete system equations combined with the adjoint variable method. By using the OMD, the material interpolation functions are defined for permeability, electric conductivity, thermal conductivity and convection coefficient. The two design models (simple iron and C-core design) heated-up by eddy currents demonstrated the effectiveness of the proposed method. The optimal designs do not only result in an increase of the heat transfer rate per volume, but also maintaining the magnetic energy.

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