Multi-objective Topology Optimization of Magneto-Thermal Problem considering Heat Inflow Rate

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Abstract — This research provides machine designers with some intuition to consider both, magnetic and heat transfer effects. A topological multi-objective function includes magnetic energy and heat inflow rate to the system, which equals to the total heat dissipation by convection. For the thermal field regarding the heat inflow, introduced as a reaction force, topology design sensitivity is derived by employing discrete equations. The adjoint variable method is used to avoid numerous sensitivity evaluations. As a numerical example, a C-core design excited by winding current demonstrates the strength of the multi-physical approach.

I. INTRODUCTION

Electrical machines are excited by windings connected to the electric power supply and generate mechanical energy. This points out that losses generated in the machine have to be minimized and the output energy should be maximized as much as possible. In the industrial applications, thermal problems attract more and more attention. This is due to temperature rise diminishing properties of materials used for the machines so that the electromagnetic quantities, such as the torque and the efficiency decrease.

Conventional structural optimal techniques, such as sizing or shape/configuration optimization are aimed at the improvement of current designs. On the other hand, topology optimization focuses on obtaining an initial conceptual design. It does not require sophisticated initial design but some geometry, describing the boundary conditions is good enough to start the topology optimization. The topology optimization is now sufficiently mature and can be extended to various physical systems.

In thermal optimization problem to solve, target nodal temperature as a performance index has been studied [1-2]. This aims at minimizing the thermal resistance between input and output points. The optimal pattern is fully depending on where target nodal point is located. This means that the machine designer at least needs to get some idea before the stage of optimization setup. Otherwise the final design obtained from optimization process might result in of a local optimum. Another performance index in thermal systems is the heat flow rate in many heat exchangers such as cooling fins [3]. If the design goal is to obtain a maximum convection heat flow rate, owing to the energy conservation law, an equivalent problem has to be formed to maximize heat inflow rate around the heat source.

The heat dissipation by convection, one of the important performance indexes in cooling systems, is examined and its design sensitivity analysis (DSA) is performed using the adjoint variable method (AVM). For the multi-objective topology optimization, the magneto-thermal problem is solved using a C-core actuator as an example.

II. DESIGN SENSITIVITY EQUATION

A. Electromagnetic System

A finite element equation of any problem governed by specified differential equations and the boundary conditions can be achieved by the variational method. The governing equation of the magneto-static field can be described by using the set of Maxwell's equation and by introducing a vector potential $B = \nabla \times A$;

$$\nabla \times (\frac{1}{\mu} \nabla \times A) = J_s \qquad (1)$$

where J_s and μ are the current density vector and the permeability of material, respectively. B is the magnetic flux density.

The variational equation and the design sensitivity have been introduced [2]. The topological sensitivity expressed by derivative of the magnetic energy with respect to the design variables, is used to give optimal direction in electromagnetic systems.

B. Thermal System

The matrix form for the finite element thermal field can be expressed by;

$$[K_{th}]\{T\} = \{Q\} \ . \qquad (2)$$

K_{th} is the finite element thermal stiffness matrix containing the conduction and the convection terms, T is the temperature vector and Q is the heat source vector.

If we rewrite (2) in a partitioned form as;

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \bar{\bar{T}} \\ T_S \end{Bmatrix} = \begin{Bmatrix} Q \\ \bar{\bar{R}} \end{Bmatrix} \ , \qquad (3)$$

where unknown variables expressed by $^{=}$ are determined by ordinary matrix operations. T_S and R are the known surface temperature and the heat inflow term described as a reaction force. The total heat inflow rate to the system is then defined as;

$$\psi = v^T R \ , \qquad (4)$$

where the vector $v^T = [1,1,\cdots,1]$. Taking differentiation on (4) with respect to the single design variable and rearranging the design sensitivity equation yields;

$$\psi' = v^T \{(K_{21}'T + K_{22}'T_S) + K_{21}T'\} . \qquad (5)$$

And the other equation of differentiation is;

$$K_{11}T' = Q' - (K_{11}'T + K_{12}'T_S) . \qquad (6)$$

Replacing (6) into (5) is expressed by;

$$\psi' = v^T\left(K_{21}'T + K_{22}'T_S\right) + \lambda^T\left\{Q' - \left(K_{11}'T + K_{12}'T_S\right)\right\}. \quad (7)$$

The corresponding adjoint equation to (7) is written as;

$$K_{12}v = K_{11}\lambda\,. \quad (8)$$

The following equation subsequently is obtained to calculate design sensitivity;

$$\psi' = \left[\lambda^T \quad v^T\right]K'T + \lambda^T Q'\,. \quad (9)$$

III. TOPOLOGY OPTIMIZATION

A main purpose of the topology optimization is that an initiative design is conceptually obtained without full understanding of the multi-physics problem. In the electromagnetic problem of the topology algorithm, the performance index is able to be the magnetic energy, the torque, the efficiency, etc. The magnetic energy for the objective value is a good approach in a global optimal point of view.

According to the energy conservation law, to maximize the heat inflow rate results in maximizing the heat out of the machines. In order to consider the heat dissipation by the convection, the heat inflow rate is adopted as one of indexes.

Therefore, a multi-objective function consists of the magnetic energy and the heat inflow rate with a weighting factor (α, β). In order to identify units of two performances, each value is normalized by an initial value. The topology optimization problem takes the form;

$$Max.\ \alpha \times \frac{Energy_{em}}{Initial\ Energy_{em}} + \beta \times \frac{Heat\ Inflow\ Rate}{Initial\ Heat\ Inflow\ Rate} \quad (10)$$

$$subject\ \ to\quad g = \frac{\iiint_{\Omega_1} \rho\, A_r\, t\, d\ \Omega}{0.6\ V_0} - 1 \le 0$$

bounded to $0 \le b \le 1$ for all $b \in \Omega$.

A_r is the area, t is the thickness, and V_0 are the initial volume, respectively.

IV. NUMERICAL EXAMPLE

For the application of a magnetic levitation system, a C-core structure is used to demonstrate the proposed approach to solve the two physical effects. Current applied the coil areas shown in Fig.1, generates magnetic field and joule's heat in the core.

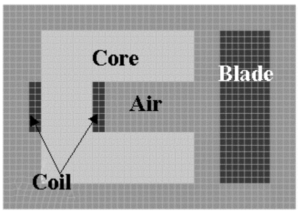
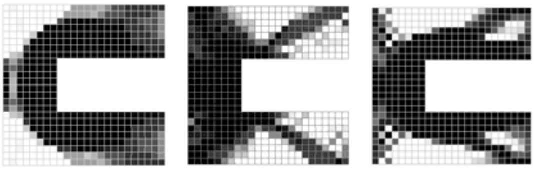


Fig. 1. Numerical Model

In order to review optimal designs of the each field, weighting factors of different sets, such as (1,0), (0,1) and (0.5, 0.5) are chosen. (1,0) is for the magnetic characteristics, (0,1) for the thermal field and (0.5, 0.5) for the both fields. Fig. 2 (b) and (c) are distinct patterns from optimized topologies regarding the target nodal temperature [2]. A final optimum shape is obtained by choosing th threshold larger than 0.5. Table I shows comparison between nonlinear reanalysis of the optimal design (α=0.5, β=0.5) with initial design. Minimum temperature of the optimal core shape is 4[^{0}C] less than that of the initial design shown in Fig. 4.



(a)α=1, β=0 (b) α=0, β=1 (c) α=0.5, β=0.5

Fig. 2. Topological Optimum Result

TABLE I
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Performance (Heat inflow rate) [%]	100	103.91
Heat Transfer Rate per Volume [%]	100	138.45
Magnetic Energy of Design Domain [%]	100	133.51
Volume [%]	100	69.34

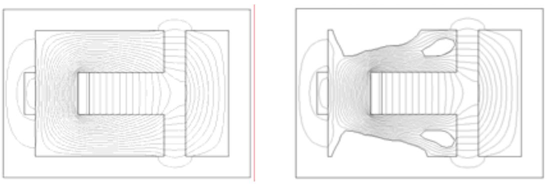


Fig. 3. Magnetic Flux Line of Initial (Left) and Optimal Design (Right)

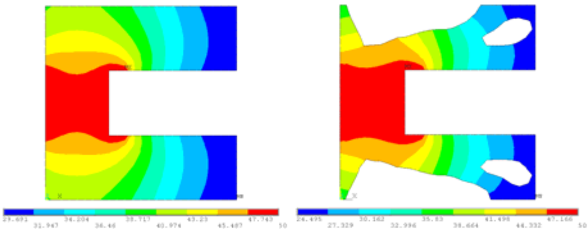


Fig. 4. Temperature [^{0}C] of Initial (Left) and Optimal Design (Right)

V. CONCLUSION

The multi-objective topology optimization is proposed to simulate a magneto-thermal example problem. The DSA regarding the heat inflow rate is derived from the AVM through discrete equations. The proposed approach is validated by a numerical example, core design, and the result is identified by reanalysis comparison. The optimal design is distinct from the optimal shape regarding the target nodal temperature.

VI. REFERENCES

[1] M. Bendsoe and O. Sigmund, *Topology Optimization – Theory, Methods and Application*, Springer, pp. 112, 2003.

[2] H. Shim, H. Moon, S. Wang, "3D Topology Optimization of Magneto-Thermal Systems" COMPEL, Vol. 25, No. 3, pp. 572-580, July, 2006.

[3] G. Ahmadi and A. Razani, "Some optimization problems related to cooling fins," Int. J. Heat Mass Transfer, Vol. 16, pp.2369-2375, 1973.