On the Carter's Factor Calculation for Slotted Electric Machines

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***Abstract*—The air-gap flux density in a single side slotted unsaturated machine is computed via two dimensions finite element method (2D-FEM) and via some analytical approximations. The Carter's factor values are calculated using different equations and a comparison between the obtained results is presented, allowing for pertinent conclusions concerning the flux density analytical estimation or the Carter's factor calculation.**

***Index Terms*—Air-gap flux density, Carter's factor, analytical models, unsaturated slotted machines.**

I. Introduction

The air-gap magnetic field values provide valuable information in evaluating electric machines performance. In the slotted electric machines the air-gap flux density varies against the circumferential coordinate and has its peak value in the tooth axis. Obviously the electric machines design and performance estimation procedures require knowledge on the air-gap flux density; therefore its calculation was in the researcher's attention for a very long time. The first important results concerning the air-gap flux density in the induction and synchronous machine were published in the beginning of the twentieth century as were the works of Weber [1] and Carter [2]. Heller's book [3], published firstly in the sixties of the last century, contains a synthesis of his entire work on this domain together with the most important contributions published up to that moment. Quite many late published papers were dedicated to the permanent magnet synchronous machine air-gap magnetic field, such as are [4], [5] and [6]. The air-gap variable equivalent permenace was developed as a method to estimate the air-gap flux density in induction machine [3], [7] but it was extended to other machines such as switched reluctance motor (SRM) [8], transverse flux reluctance linear motor (TFRLM) [9], or to double-slotted magnetic structures [10],[11]. A discussion concerning the analytic approximation of the air-gap flux density variation in single side slotted machines is presented in [12].

There are two ways to deal with the air-gap magnetic field in the case of slotted electric machines:

i) To introduce an enlarged equivalent air-gap, via Carter's factor where the flux density is not varying against the circumferential coordinate and both sides are smooth with no teeth and slots.
ii) To fully consider the air-gap flux density variation, eventually calculating the field harmonics due to the slots openings.

In the case of variable reluctance machines, such as SRM for instance, one has to calculate the actual air-gap flux density variation since the torque is produced by this. In the case of induction or nosalient synchronous machine analytic design procedures are based on the enlarged equivalent air-gap, and consequently on the equivalent constant air-gap flux density calculated value.

In this paper only the case of unsaturated single side slotted air-gap topology is considered. In this case Carter's factor is calculated by using:

i) Dedicated equations obtained via conformal mapping procedure and given in the literature
ii) Equations obtained based on the analytical approximations proposed for the air-gap flux density variation versus circumferential coordinate
iii) Numerical calculation done on the data obtained via a two dimension finite element method (2D-FEM) analysis of the magnetic field computed on a machine simple model.

The obtained results are compared between them and some conclusions concerning the way the equivalent air-gap flux density and corresponding harmonics due to the slot openings should be calculated are presented.

II. Magnetic field calculation via 2D-FEM

A simple machine model, Fig. 1, which contains two slots, a coil on the stator and a nonsalient rotor was considered for the magnetic field calculation via 2D-FEM analysis. The model structure was parameterized to allow adequate variation of the most dimensions, only the tooth pitch t (for symmetry reasons the stator length is equal to two tooth pitches), was kept constant.

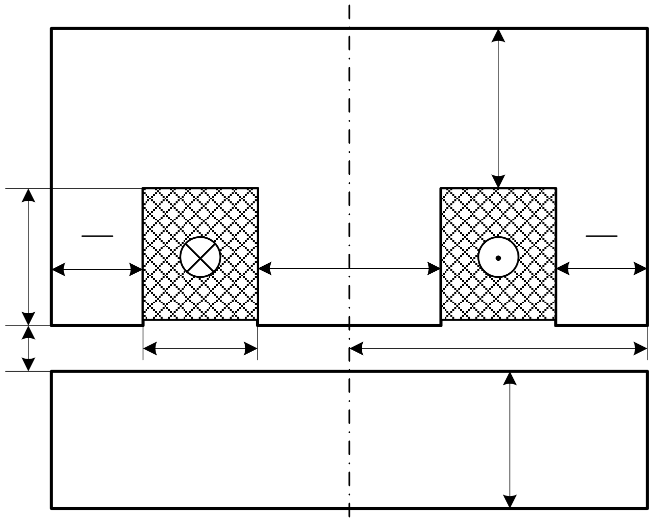

Figure 1. Single side slotted model

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The considered model iron core is unsaturated at quite large values of the air-gap flux density if the ratio between the tooth width and the double of stator and respectively rotor yoke is small, as one can see from the values given in Table 1 for $t/g = 60$, $w_t/2h_{yS} = 0.25$, $w_t/2h_{yR} = 0.167$.

TABLE 1. SATURATION K_{SAT} AND CARTER'S K_C FACTORS

$\mathbf{B_{gmax}}$	T	0.809	1.117	1.414	1.571
$\mathbf{K_{sat}}$	-	1.01	1.022	1.022	1.12
$\mathbf{K_C}$	-	2.492	2.508	2.508	2.518

The air-gap flux density calculated in the middle of the model's air-gap has a smooth and continuous variation, Fig. 2.

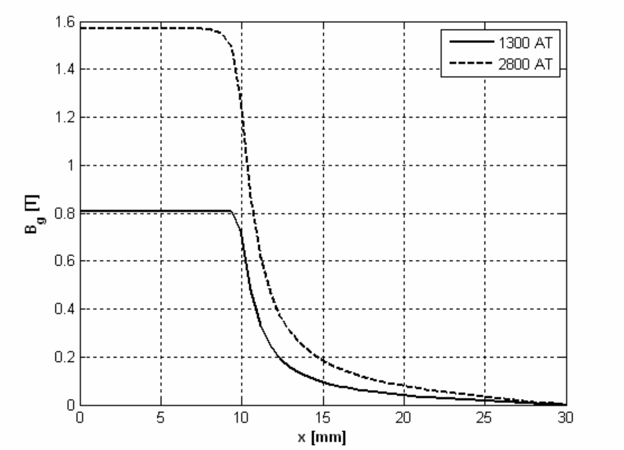

Figure 2. Air-gap flux density variation versus circumferential coordinate, $t/g = 60$, $w_s/2g = 20$

The Carter's factor is calculated as the ratio between the peak, B_{gmax}, and the average value, B_{gav} of the air-gap flux density:

$$K_C = B_{g\max} / B_{gav} \tag{1}$$

The saturation coefficient K_{sat} is:

$$K_{sat} = B_{g\max unsat} / B_{g\max} \tag{2}$$

where the saturated peak air-gap flux density value B_{gmax} is calculated in the tooth axis via 2D-FEM and the unsaturated air-gap flux density value is:

$$B_{g\max unsat} = \mu_0 F / 2g \tag{3}$$

with the following notations μ_0 – air-gap permeability ($4\pi 10^{-7}$H/m), F – coil **mmf** in ampere turns and g – air-gap radial length in meters.

The air-gap flux density average value B_{gav} is calculated as:

$$B_{gav} = \frac{2}{t}\int_0^{t/2} B_g(x)dx \tag{4}$$

where $B_g(x)$ gives the air-gap flux density variation versus circumferential coordinate x.

In the case of 2D-FEM analysis $B_g(x)$ is numerically defined by the air-gap flux density values calculated at equidistant values $x \in [0, t/2]$.

III. ANALYTICAL APPROXIMATIONS OF THE AIR-GAP FLUX DENSITY VARIATION

Besides the analytical approximation based on the air-gap variable equivalent permeance, which will not be discussed here, other quite good approximations of the air-gap flux density variation are:

The one proposed by Weber [1]

$$B_g(x) = B_{g\max}\left(1 - 2\beta \sin^{2a}\left(\frac{x}{t}\right)\pi\right),$$

$$a = \frac{t - w_s}{w_s}, \quad x \in [0, t/2] \tag{5}$$

where the air-gap topology coefficient is [4]:

$$\beta = \frac{B_{g\max} - B_{g\min}}{2B_{g\max}} = \frac{\Delta B_g}{B_{g\max}}$$

$$\beta = 0.5\left(1 - \frac{1}{\sqrt{1 + (w_s / 2g)^2}}\right) \tag{6}$$

The one proposed by Heller [3]

$$y = \frac{\pi}{t/2 - \beta w_t} x - \frac{\pi \beta w_t}{t/2 - \beta w_t} \tag{7}$$

A nonsinusoidal approximation based on a simplified air-gap flux lines topology [5]

$$B_g(x) = B_{g\max}, \quad x \in [0, w_t / 2]$$

$$B_g(x) = B_{g\max}\left[1 + \frac{\pi}{2g}(x - w_t / 2)\right]^{-1},$$

$$x \in (w_t / 2, t/2] \tag{8}$$

An exponential approximation ($x = 0$ in a slot axis) based on a curve fitting procedure [12]

$$B_g(x) = B_{g\max} \frac{1}{1 + 10^{(\beta w_s - x)}}, \quad x \in [0, t/2] \tag{9}$$

A comparison between the 2D-FEM analysis and different proposed approximations of the air-gap flux density referred to its peak value is given in Fig. 3 in the case of $t/g = 37.5$, $w_s/2g = 6.25$ and $F = 3800$ AT.

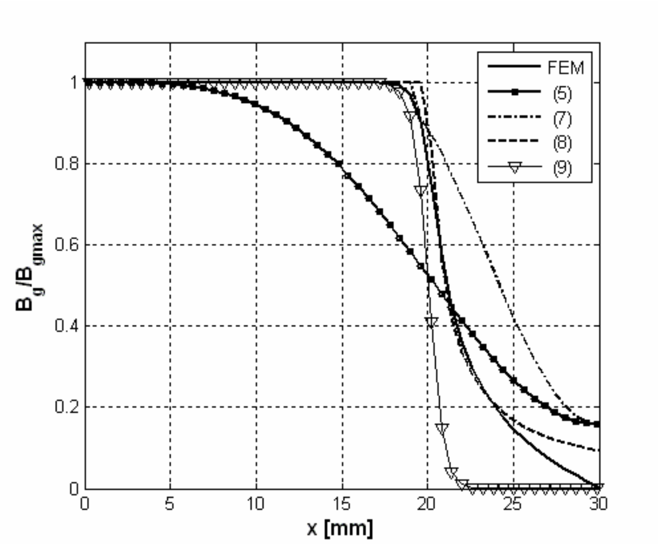

Figure 3. A comparison between 2D-FEM calculation and proposed analytical approximations

IV. Carter's factor calculation

Basically, the Carter's factor is defined by (1), the average air-gap flux density being calculated accordingly to (4). For the considered case of single side slotted machine only the air-gap flux density variation on a half of a tooth pitch is taken due to the symmetry against the tooth axis, which is the origin of the circumferential coordinate *x* as in Fig. 3.

There are quite many equations, mostly obtained via conformal mapping, that allow for Carter's factor calculation, as the following three given here [3,6]:

$$k_{C1} = \left[1 - \frac{1}{t / w_s \left(5g / w_s + 1\right)}\right]^{-1} \quad (10)$$

$$k_{C2} = \left[1 - \frac{2w_s}{t\pi}\left(a\tan(\frac{w_s}{2g}) - \frac{g}{w_s}\ln\left(1 + \frac{1}{4}\left(\frac{w_s}{g}\right)^2\right)\right)\right]^{-1} \quad (11)$$

$$k_{C3} = \left[1 - \frac{w_s}{t} + \frac{4g}{t\pi}\ln\left(1 + \frac{\pi w_s}{4g}\right)\right]^{-1} \quad (12)$$

The Carter's factor values calculated for an unsaturated machine, $K_{sat} < 1.04$, via 2D-FEM in comparison with the ones obtained via equations (10) – (12) are given in Tables 2 and 3 for two values of the tooth pitch air-gap length, *t/g* ratio.

Table 2. Carter's factor values, t/g = 150

K_C \ ws/2g	25	31.25	37.5	43.75	50
FEM	1.446	1.637	1.886	2.225	2.718
(10)	1.435	1.628	1.882	2.231	2.739
(11)	1.424	1.61	1.854	2.187	2.666
(12)	1.433	1.622	1.87	2.209	2.669

Table 3. Carter's factor values, t/g = 37.5

K_C \ $w_s/2g$	6.25	7.813	9.375	10.938	12.5
FEM	1.359	1.515	1.714	1.978	2.341
(10)	1.313	1.461	1.652	1.904	2.25
(11)	1.311	1.455	1.639	1.88	2.207
(12)	1.338	1.49	1.685	1.94	2.293

If the air-gap flux density variation versus *x* coordinate, $B_g(x)$ is analytically defined via the discussed approximations then the Carter's factor can be analytically calculated resulting:

Weber's approximation

$$k_{CW} = \frac{1}{1 - \frac{4\beta}{t}\int_0^{t/2}\left(\sin^{2a}(x\pi / t)\right)dx} \quad (13)$$

If $a = 0.5$ and $a = 1$ the integral can be calculated and the particular equations are:

$$K_{CW(a=0.5)} = \frac{1}{1 - 4\beta / \pi} \quad (14)$$

$$K_{CW(a=1)} = \frac{1}{1 - \beta} \quad (15)$$

Heller's approximation

$$k_{CH} = \frac{1}{\beta w_t / t + (1 - \beta)(1 - \beta w_t / t)} \quad (16)$$

The nonsinusoidal approximation

$$k_{CN} = \frac{1}{\frac{w_t}{t} + \frac{4}{\pi}\frac{g}{t}\ln\left(1 + \frac{4}{\pi}\frac{w_s}{g}\right)} \quad (17)$$

The exponential approximation

$$k_{CE} = \frac{t / 2\ln 10}{\ln\frac{10^c + 10^{t/2}}{10^c - 1}}, \ c = \beta w_S \quad (18)$$

In Tables 4 and 5 the Carter's factor values obtained via Weber and Heller approximation in comparison with the values calculated via 2D-FEM, K_{CFEM}, are given.

Table 4. Carter's factor values, Weber (a = 0.5) and Heller approximations

$w_S/2g$	50	33.3	25	20	16.67	12.5
β	0.49	0.48	0.48	0.47	0.46	0.46
K_{CW} (0.5)	2.66	2.61	2.574	2.53	2.46	2.41
K_{CFEM}	2.72	2.63	2.556	2.49	2.43	2.34
K_{CH}	1.69	1.68	1.676	1.66	1.65	1.64

Table 5. Carter's factor values, Weber (a = 1) and Heller approximations

$w_S/2g$	37.5	25	18.7	15	12.5	9.375
β	0.48	0.48	0.47	0.467	0.46	0.447
K_{CW}(1)	1.95	1.92	1.89	1.876	1.852	1.808
K_{CFEM}	1.88	1.84	1.81	1.783	1.757	1.715
K_{CH}	1.58	1.57	1.56	1.557	1.548	1.532

In Table 4 the constant *a* is 0.5 and in Table 5 is 1 respectively, all the dimensions having adequate values, as for instance if $a = 0.5$ and $w_s/2g = 50$ then $t/g = 150$ and if $a = 1$ and $w_s/2g = 25$ then $t/g = 100$.

Table 6. Carter's factor values, nonsinusoidal and exponential approximations

$w_S/2g$	50	43.75	37.5	31.25	25
β	0.49	0.488	0.486	0.484	0.48
K_{CN}	2.699	2.209	1.87	1.622	1.433
K_{CE}	2.885	2.303	1.923	1.655	1.442
K_{CFEM}	2.720	2.225	1.886	1.637	1.446

In Table 6 the Carter's factor values calculated via nonsinusoidal and exponential approximations are compared with the values obtained via 2D-FEM for t/g = 150 and different $w_s/2g$ values.

In Tables 4, 5 and 6 the air-gap topology coefficient β (6) values are given too.

As previously shown if the ratios of the tooth pitch to double stator or rotor yoke height $t/2h_{yS}$, $t/2h_{yR}$ have small values, the influence upon the iron core saturation is unimportant and consequently the Carter's factors variation is insignificant.

To further prove this statement in Tables 7 and 8 the variation of saturation and Carter's factors function of coil **mmf** are given in two different cases; t/g = 150, $w_s/2g$ = 50 in Table 7 and t/g = 37.5, $w_s/2g$ = 6.25 in Table 8.

In both cases the tooth pitch to yoke height ratios are the same, $t/2h_{yS}$ = 0.75, $t/2h_{yR}$ = 0.5 and the values of coil **mmf** F and of the peak B_{gmax} and average B_{gav} air-gap flux density are given.

As it can be seen the peak air-gap flux density does not vary in the same ratio as the coil **mmf** does due to the slightly saturation of the iron core.

TABLE 7. CARTER'S AND SATURATION FACTORS, T/G = 150; $W_S/2G$ = 50

F	A	600	800	1000	1200
$\mathbf{B_{gmax}}$	T	0.922	1.222	1.503	1.675
$\mathbf{B_{gav}}$	T	0.339	0.449	0.551	0.612
$\mathbf{K_C}$	-	2.718	2.722	2.729	2.739
$\mathbf{K_{sat}}$	-	1.022	1.028	1.045	1.125

TABLE 7. CARTER'S AND SATURATION FACTORS, T/G = 37.5; $W_S/2G$ = 6.25

F	A	2200	3000	3800	4600
$\mathbf{B_{gmax}}$	T	0.855	1.160	1.433	1.526
$\mathbf{B_{gav}}$	T	0.629	0.852	1.047	1.113
$\mathbf{K_C}$	-	1.360	1.362	1.368	1.372
$\mathbf{K_{sat}}$	-	1.011	1.015	1.042	1.183

V. CONCLUSIONS

A simplified model, in a linear layout, with slots only on one side of the air-gap flux density was considered. On this model the air-gap flux density was calculated for different tooth pitch to air-gap length and slot opening to double air-gap length ratios and coil **mmf**.

Some approximations of the air-gap flux density variation are considered and Carter's factors were calculated.

The following remarks should be made concerning the air-gap flux density analytical approximations and the Carter's factors calculated values:

i) The approximations proposed by Weber and Heller are not that accurate and they are more appropriate for the cases when the slot opening ratio to tooth pitch is less than 0.5

ii) Carter's factor values calculated based on Heller's approximations are far from the values computed via 2D-FEM analysis.

iii) All other approximations give Carter's factor values closed to that computed via 2D-FEM analysis and so are the values calculated by using the Carter's factor equations given in the references.

As an overall conclusion it must be remarked that in the case of single side slotted unsaturated electric machines the Carter's factor calculation done in different ways is accurate and the procedure to be used has to be chosen function of the requirements for the entire model to be developed.

REFERENCES

[1] C.A.M. Weber and F.W. Lee, "Harmonics due to slot openings", A.I.E.E. Trans., vol. 43, 1924, pp. 687-693.
[2] F.W. Carter, "The magnetic field of the dynamo-electric machine", The Jour. Of the I.E.E., vol. 64, 1926, pp.1115-1138.
[3] B. Heller, V. Hamata, "Harmonic field effects in induction machines", New York, Elsevier, 1977
[4] Z. Q. Zhu, D. Howe, "Instantaneous magnetic field distribution in brushless permanent magnet dc motors, Part III: Effect of stator slotting", IEEE Trans. on Magnetics, vol. 28, no. 1, January 1993, pp.143-151.
[5] A.B.Proca, A.Keyhani, A.El-Antably, W.Lu, M.Dai, "Analytical model for permanent magnet motors with surface mounted magnets", IEEE Trans. on Energy Conversion, vol.18, no.3, September 2003, pp.386-391.
[6] D. C. Hanselman, "Brushless permanent-magnet motor design", second edition, Writes' Collective Cranston, Rhode Island, SUA, 2003.
[7] I.-A.Viorel, M.M.Radulescu, "On the calculation of the variable equivalent air-gap permeance of induction motors" (in Romanian), EEA-Electrotehnica, vol.32, no.3, 1984, pp.108-111.
[8] I.-A.Viorel, A.Forrai, R.C.Ciorba, H.C.Hedesiu, "Switched reluctance motor performance prediction", Proc. Of IEE-IEMDC, Milwaukee, USA, 1997, TBI-4.1-4.3.
[9] J.-H.Chang, D.-H.Kang, I.-A.Viorel, Larisa Strete, "Transverse flux reluctance linear motor's analytical model based on finite element method analysis results", IEEE transaction on Magnetics, vol.43, no.4, April 2007, pp.1201-1204.
[10] G.Qishan, Z. Jimin, "Saturated permeance of identically double-slotted magnetic structures", IEE Proc.-B, vol.140, no.5, pp.323-328, 1993.
[11] G.Qishan, E.Andersen, G.Chun, "Airgap permeance of vernier-type, doubly-slotted magnetic structures", IEE Proc.-B, vol.135, no.1, pp.17-21, 1988.
[12] I.-A. Viorel, Larisa Strete, V. Iancu, Cosmina Nicula, "Air-gap magnetic field of the unsaturated slotted electric machines" accepted at ISEF07, Pragu