Kriging Models and Torque Improvements of a Special Switched Reluctance Motor

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Abstract – A Special Switched Reluctance Motor for a fractional horsepower and high speed hand tool is studied in order to improve its torque characteristic. This task is accomplished by optimizing some rotor geometrical parameters with the aid of a numerical approach based on the application of Simulated Annealing and Kriging Method. Finite Element simulations are used to evaluate the approximation points for the Kriging model. The numerical results are compared to tests on a prototype.

Index Terms – Reluctance Motors, Finite Element Method, Optimization, Kriging

I. Introduction

Switched Reluctance Motors (SRM) are nowadays a well established choice for variable speed applications. The 4:2 pole – 2 phase Switched Reluctance Motor (SRM) [1-5] has been proposed as a drive for hand tools, instead of the traditional fractional horsepower motors, like shaded-pole and universal types. This configuration is particularly well suited for unidirectional low cost drives due to its simple motor construction and simple drive topology with only four power switches. In order to have a better torque characteristic, the SRM has a specially designed rotor poles with an original geometry that ensures starting torque in a defined direction at any rotor position.

This work deals with the improvement of the magnetic torque characteristics of this SRM, using an optimization approach based on the creation of a Kriging model [8] and the application of Simulated Annealing (SA) in order to minimize the torque ripple and maximize the starting torque at the initial position. The Kriging model is generated using finite element simulations of the SRM. Tests on a prototype are used to validate the optimized results.

II. Switched Reluctance Motor Characteristics

In most cases the 4:2 pole SRM presents a low starting torque [6]. The proposed SRM displays a specially designed rotor poles to ensure higher starting torque in a defined direction at any rotor position.

The basic characteristic of the rotor poles is its geometric asymmetry, which consists of one region with a small uniform air-gap and another with a variable air-gap, which increases toward the quadrature axis.

The complete motor structure is depicted in Fig. 1, while the pole details are shown in Fig. 2. The general geometric characteristics of this SRM are summarized in Table I. As an outcome of its salient poles, the SRM presents an intrinsic non-negligible torque ripple that produces, consequently, vibration and audible noise.

The analytical treatment of this characteristic is very cumbersome and tedious because it is based on a trial and error procedure involving many geometric parameters.

This work presents a numerical approach to the problem by using the Finite Element Method coupled to an optimization algorithm which includes the Kriging model and the Simulated Annealing method.

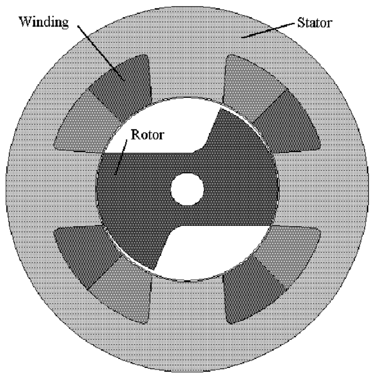

Fig. 1. Proposed geometry for 4:2 pole – 2 phase SRM.

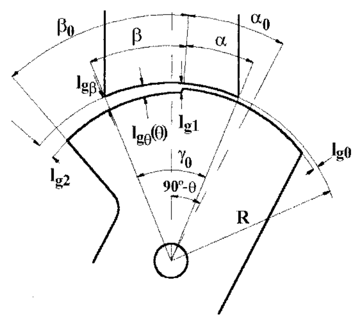

Fig. 2. Details of the rotor geometry.

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TABLE I. GEOMETRIC CHARACTERISTICS OF THE SRM

Characteristic	Value
Stator outer diameter	90 mm
Stator yoke thickness	10 mm
Rotor diameter	2R = 45 mm
Core length	L_c = 35 mm
Stator pole arc	$\gamma_0 = 45°$
Rotor pole arc	$2\alpha_0 = 45°$
Main airgap thickness	l_{g0} = 0.3 mm

III. FINITE ELEMENT SIMULATIONS

The magnetostatic finite element simulations were carried out considering nonlinear ferromagnetic material. The Torque vs. Angular Position curve was obtained by feeding one phase of the stator winding with the rated magnetomotive force in the angular range of 0° to 90°.

The Torque vs. Angular Position obtained for the SRM before the optimization process is depicted in Fig. 4.

IV. THE OPTIMIZATION PROBLEM

The optimization problem could be defined with an objective function, which allows to minimize the torque ripple and to maximize the starting torque at θ=0°. The problem is constrained: the mean torque must have a minimum prescribed value.

In order to accomplish those tasks the geometric parameters β_0, l_{g1} and l_{g2} were chosen as the most significant for the optimization process.

The objective function was adopted represented by (1) where the objective function represents the difference between the mean torque $\overline{C}$ and the torque calculated at n different rotor positions θ_i. In order to favor the starting torque, $C(\theta_0)$, it has a different contribution to the value of the objective function, as shown in (1).

$$\text{Min} f(\beta_0, l_{g1}, l_{g2}) = \overline{C} - C(\theta_0) + \frac{1}{n}\sum_{i=1}^{n}\left|C(\theta_i) - \overline{C}\right| \quad (1)$$

The variation domain of each geometric parameter to be optimized was defined following the values presented in Table II.

TABLE II. PARAMETERS VARIATION DOMAIN

Parameter	Reference Value	Minimum Value	Maximum Value
β_0	45°	30°	60°
l_{g1}	0.6 mm	0.4 mm	0.6 mm
l_{g2}	1.2 mm	0.6 mm	1.8 mm

V. OPTIMIZATION METHODOLOGY

A rigorous framework for the optimization of electromagnetic devices analyzed by Finite Element Method could be achieved when approximated functions are used to replace the objective function and the constraints of the optimization problem. The approximated functions are created using a reduced number of objective function evaluations. This approach allows an important reduction in the number of simulations by finite elements. There is another important advantage: stochastic optimization methods, such as the Simulated Annealing (SA) or Genetic Algorithm (GA), could be used without a high computational cost [7].

A. Kriging

Kriging [8], [9] is a general term used for a family of methods for minimum error variance estimation. In this work, Kriging will replace the objective function during the optimization process. Kriging can exploit the spatial correlation of data in order to build interpolations, thus the correlation function choice highly affects the quality of the approximation. There are several approaches to present Kriging models. In this work, we will adopt the Maximum Likelihood Estimates (MLE), which is more suitable to Design and Analysis of Computer Experiments (DACE).

The Kriging mathematical framework is based on well-established statistical concepts, but these concepts could be understood with a short and intuitive approach. Kriging could be understood as follows: the goal is to interpolate a set of N samples. The interpolation function wil be defined with two sets of functions: the first set will describe the general trend, e.g., it will follow the general tendency of the function to be modelled. It is generally a constant, but there are some works that this function is a polynomial of order 1 or 2. The other set follows the fluctuations around the general trend and it is usually made up with a set of Gaussians.

The mathematical problem is then how to deal with the general trend and the fluctuations. A statistical approach can solve this problem and the Kriging model is defined as follows:

$$y(x) = f(x) + Z(x) \quad (2)$$

where y(x) is the interpolation function, f(x) is a known function (in this work constant and equals to β) and Z(x) is the realization of a random process with mean zero, variance equals to σ^2 and covariance non-zero. So, on (1), f(x) is the general trend and the localized fluctuations are created by Z(x). Our mathematical problem is then to calculate β and σ^2. The covariance matrix of Z(x) could be written as:

$$Cov[Z(x_i), Z(x_j)] = \sigma^2 \mathbf{R}(R(x_i, x_j)) \quad (3)$$

where **R** is the correlation matrix and $R(x_i,x_j)$ is the correlation function. A Gaussian correlation function is usually adopted:

$$R(x_i,x_j) = e^{\sum_{k=1}^{N_{par}} -\theta_k (|x_i - x_j|_k)^2} \quad (4)$$

1- x_i is the an optimization variable and $|x_i\text{-}x_j|_k$ is the distance between x_i and x_j on k-direction.
2- the optimization problem is a multidimensional problem (N_{par} parameters) and the correlation function is defined as a product of correlation functions (different in each k-direction).
3- The parameter θ_k is constant for each k-direction and measures how the data are correlated in this direction

With the MLE approach is possible to evaluate some parameters of the correlation function. If the global model β and $\theta = [\theta_1\ \theta_2 \ldots\ldots \theta_N]$ are fixed, then the best linear unbiased predictor (BLUP) of y(x) is:

$$y^*(x) = \beta(\theta) + \mathbf{r}^t(x,\theta) \times \mathbf{R}(\theta)^{-1} \times (\mathbf{y} - \mathbf{f}\beta(\theta)) \quad (5)$$

where $y^*(x)$ is the estimated value at x, $\mathbf{y}$ is a Nx1 vector filled with the sampled values, $\mathbf{f}$ is a Nx1 vector filled by ones. The vector $\mathbf{r}^t(x)$ is equal to the correlation between x and the N sample points:

$$\mathbf{r}^t(x) = [R(x, x_1) \quad R(x, x_2) \quad \ldots\ldots \quad R(x, x_N)]^t \quad (6)$$

So, (4) is a family of interpolating curves, which depends on the parameters β, θ and σ^2. If θ are fixed, the MLEs of β and σ^2 have an explicit expression. The *estimated* global model β is:

$$\beta(\theta) = (\mathbf{f^t}\, \mathbf{R}(\theta)^{-1}\mathbf{f})^{-1}(\mathbf{f^t}\, \mathbf{R}(\theta)^{-1}\mathbf{y}) \quad (7)$$

and the *estimated* variance σ^2 between the global model β and y is:

$$\sigma^2(\theta) = [\,\mathbf{y} - \mathbf{f}\,\beta(\theta))^t\, \mathbf{R}(\theta)^{-1}(\mathbf{y} - \mathbf{f}\,\beta(\theta))]/N \quad (8)$$

The MLE of θ is then obtained by maximizing:

$$-(N\ln(\sigma^2(\theta)) + \ln(|\mathbf{R}(\theta)|))/2 \quad (9)$$

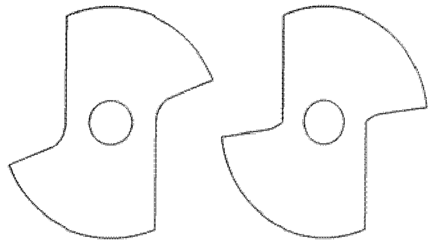
The solution of this unconstrained non-linear problem gives us the value of θ and (4) allows us to evaluate the function for any x.

V. Results

A. Numerical Simulation

The surrogate function was created using a regular grid of 5 points by direction, resulting in 125 evaluations of the ripple using finite element computations. After application of SA in the Kriging model, the values obtained for the optimization parameters were: $\beta_0 = 60°$, $l_{g1} = 0.5$ mm and $l_{g2} = 1.0$ mm.

Fig. 3 shows the rotor geometry before and after the optimization. Fig. 4 presents the Torque vs. Angular Position before and after optimization.



(a) Before optimization (b) After optimization
Fig. 3. Rotor Geometry

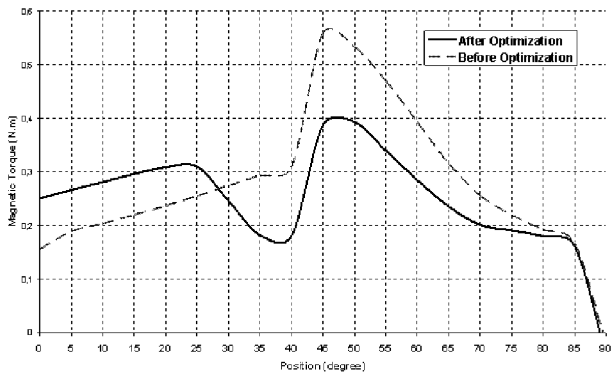


Fig. 4. Magnetic torque developed before and after optimization.

One can observe the torque increase at θ=0° and the reduction of the torque peak which implies in the diminution of the torque ripple.

B. Tests

In order to validate the numerical optimization, prototypes were constructed and tested.

Fig. 5(a) and 5(b) show some pictures of the prototypes.

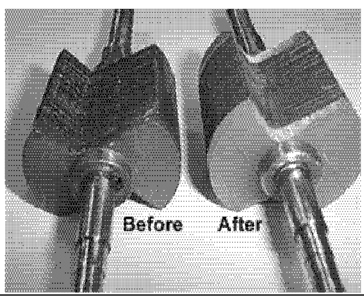


Fig. 5 (a) SRM rotor prototypes

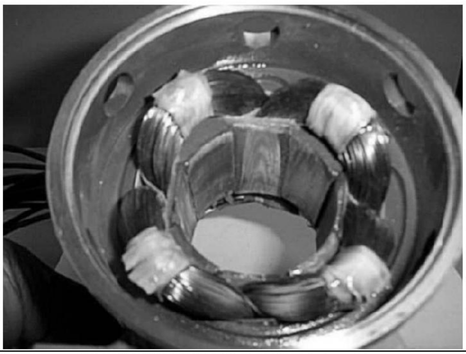


Fig. 5. (b) SRM Stator Prototype

The comparison between the experimental and the simulation curves for the former and the optimized prototypes are presented in Fig. 6.

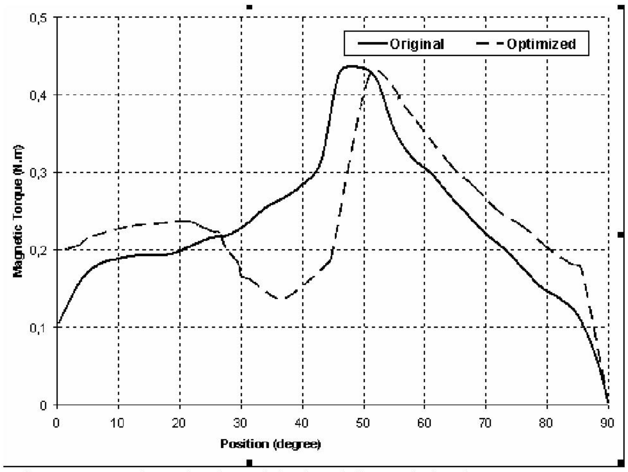


Fig. 6. Comparison for the original and the optimized prototypes – test results

From Fig. 6 one can note that the starting torque at θ=0° is twice bigger in the optimized prototype. The mean torque, calculated in the range of 0° to 90°, the starting torque and the torque ripple are summarized in Table III.

Even though the optimized prototype presents a sag in the torque curve, it is still capable of driving heavier loads because its minimum torque in the 0° to 90° region is higher than the original design. The torque ripple, considered as the difference between the maximum and the minimum values in the 0° to 90° region, has reduced while its frequency has doubled. This last characteristic provides a better mechanical stability for the driven load because its inertia softens the ripple effects at the double the frequency.

TABLE IIII. TORQUE CHARACTERISTICS

	Original Prototype	**Optimized Prototype**
Mean Torque (N.m)	0.23	0.24
Starting Torque (N.m)	0.10	0.20
Torque Ripple (N.m)	0.34	0.23

Moreover, the optimized prototype presents higher torques in the region θ<0°. It allows the electronic drive to switch in a broader range rendering the motor operation more flexible

C. The SRM drive and the vibration tests results

In order to experimentally prove the adopted modeling and optimization procedure, a torque and vibration tests was realized. The objective is to compare and evaluate the torque and vibration characteristics of the SRM when operating with each one of the rotor designs (original and optimized). To accomplish this objective a conventional asymmetrical converter topology [10] [11], depicted at Fig. 7 was used to drive the two phase 4:2 SRM.

The SRM rotor position detection is based on a single optical sensor located in stator motor part; this contributes to improve ruggedness of the motor mechanical structure without excessive complexity of control circuit.

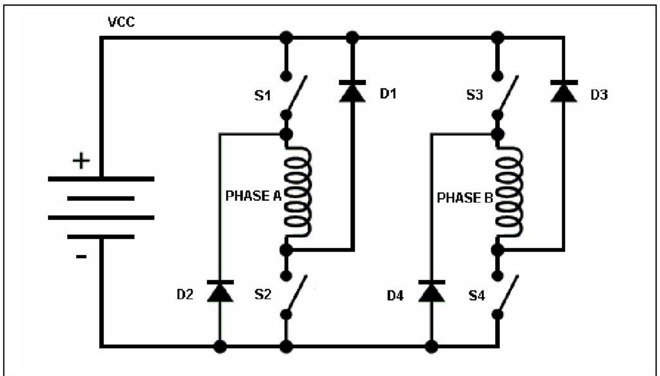


Fig. 7. Conventional Asymmetric Converter

The currents on phases A and B are regulated by a hysteresis (bang-bang) control circuit. Since the phase current limit is not reached, the motor drive works in a single pulse operation mode. The commutation angle was kept fixed at 45° (advanced before the stator/rotor alignment), for the rotors original and optimized.

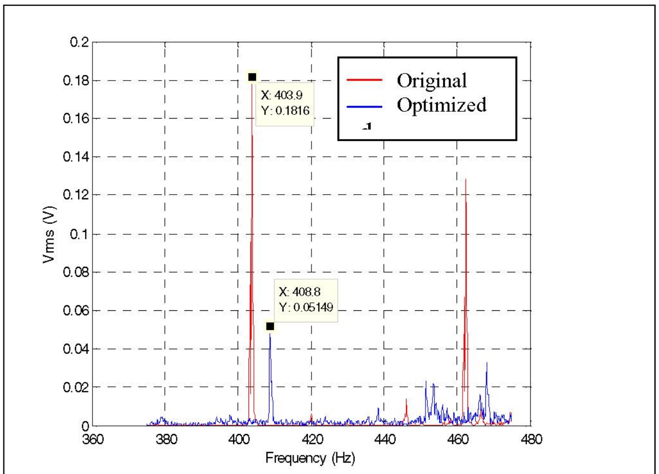


Fig. 8. Vibration analysis

The SRM was tested at the nominal speed and torque, about 6000 rpm (or 100 Hz) and 0.28 Nm, respectively. At the nominal rotor speed, the main component of the torque ripple frequency is four times higher or 400 Hz. The torque ripple was obtained indirectly through the vibration-signal acquisition followed by a Fast Fourier Transform (FFT) analysis in the range of 400 Hz.

The comparison between the vibration results curves for the original and the optimized prototypes running at about 6000 rpm are presented in Fig. 8. From Fig. 8 one can note that the optimized prototype presents a torque ripple reduction of 72.65% than the original design.

VI. Conclusions

An optimization procedure was applied in a two phase 4:2 SRM in order to reduce its torque ripple and to increase its starting torque. This procedure consisted in magnetostatic nonlinear FEM simulations in order to obtain the torque characteristics, the Kriging model to interpolate the data and to construct the surrogate function and the Simulated Annealing to minimize this function.

Three geometric parameters, namely β_0, l_{g1} and l_{g2}, were chosen as the most significant for the optimization purposes. After the numerical optimization, a prototype was made and tested. Although some discrepancies, the results comparison showed that the optimization accomplish the proposed tasks. Considering the original design as the reference, the optimized prototype presents a starting torque 25% bigger and a torque ripple 57.5% smaller.

From the SRM tests, and, at the same nominal speed and torque conditions, was observed a significant vibration reduction of 72.65% when using the optimized rotor, which confirms the numerical results and the torque characteristics improvements proposed.

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