An energy-based harmonic constitutive law for magnetic cores with hysteresis

Martin Hafner, François Henrotte, Mercedes Herranz Gracia and Kay Hameyer
Institute of Electrical Machines – RWTH Aachen University
Schinkelstraße 4, D-52062 Aachen, Germany
E-mail: Francois.Henrotte@IEM.RWTH-Aachen.de, Mercedes.HerranzGracia@IEM.RWTH-Aachen.de

***Abstract*— In this paper a phenomenogical energy-based harmonic ferromagnetic material model is presented which considers time-dependent hysteresis losses as a part of the material characteristic. The model is based on the effective reluctivity concept and can be applied to simulate hysteresis losses of electromechanic devices. The hysteresis losses of a C-Core are simulated and compared to the results of an iron loss estimation by loss curves.**

I. Introduction

In the (standard) FEA iron loss analysis, hysteresis losses are determined together with eddy current losses by means of measured loss curves. As standard loss measurement gives total losses in function of applied field and frequency, it is not surprising, that a subsequent separation of both nonlinear loss effects is exceedingly difficult. Nevertheless the evaluated iron loss results comply with experiment.

But when the question of hysteresis losses (in the material) arises, the accuracy of total losses is no longer sufficient. An energy-based constitutive material law for the finite element analysis is required that provides a material model (hysteresis) on the one hand and a physical, phenomenological loss tracking (depending strongly on the magnetic behavior) on the other. The derived material representation is implemented in the time harmonic finite element analysis. This provides a joined computation of flux distribution (based on the effective reluctivity concept) and total nonlinear hysteresis losses.

The hysteresis characteristics for this material law is provided by the energy-based vector hysteresis model proposed in [1]. Comparable results would be obtained with any other model that provides a real (ferromagnetic) material curve with a true interpretation in terms of energy.

II. Theoretical setup

A. General frequency domain approaches

The time harmonic material law assumes a linear relation between the magnetic field H [A/m] and the flux density B [T].

$$\underline{\mathbf{H}} = \underline{\nu}\,\underline{\mathbf{B}} \tag{1}$$

Due to magnetic material characteristics (nonlinearity, dissipation), an additional error term can be considered. Considering (1) in time domain, where the complex reluctivity is represented as time operator

$$\nu = \nu_r + \frac{\nu_i}{\omega}\partial_t \tag{2}$$

yields

$$\mathbf{h}(t) = \nu\mathbf{b}(t) + \Delta\mathbf{h}(t) \tag{3}$$

Equation (3) represents a realistic frequency domain material characteristic. A harmonic induction $\mathbf{b}(t)$ leads to a non sinusoidal magnetic field strength $\mathbf{h}(t)$ and a compensation field $\Delta\mathbf{h}(t)$ that forces the sum of both fraction to be sinusoidal.

The first law of thermodynamics states in this case that

$$\mathbf{h}\cdot\partial_t\mathbf{b} = \nu_r\partial_t\left|\mathbf{b}\right|^2 + \frac{\nu_i}{\omega}\left|\partial_t\mathbf{b}\right|^2 + \Delta\mathbf{h}\cdot\partial_t\mathbf{b} \tag{4}$$

where $\nu_r\partial_t\left|\mathbf{b}\right|^2$ is the change of magnetic stored energy. The positive term $\frac{\nu_i}{\omega}\left|\partial_t\mathbf{b}\right|^2$ represents the rate of dissipated energy. Obviously the leak of energy $\Delta\mathbf{h}\cdot\partial_t\mathbf{b}$ depends on the chosen complex material representation ν_r and ν_i.

To provide an interpretation in terms of energy, the following general approaches to minimize the energy difference $\Delta\mathbf{h}\partial_t\cdot\mathbf{b}\rightarrow 0$ are reasonable:

- $\Delta\mathbf{h}(t)\perp\partial_t\mathbf{b}(t)$, where $\Delta\mathbf{h}$ and $\mathbf{b}$ have the same sinusoidal frequency. The assumption of a magnetic field orthogonal to the flux density is unphysical.
- $\Delta\mathbf{h}_n\cdot\mathbf{b} = 0$, where $\Delta\mathbf{h}_n$ are harmonics to the fundamental frequency of $\mathbf{b}$. This assumption, orthogonality in frequency domain, reflects reality.
- $\int_0^T\Delta\mathbf{h}(t)\cdot\partial_t\mathbf{b}(t)dt = 0$, implies that the mean value of the energy error is equal to zero. Mathematically this constraint is fulfilled by an auxiliary reluctivity $\Delta\mathbf{h} = \nu^+\mathbf{b}$, so one obtains

$$\mathbf{h}_{\text{eff}} = \left(\nu_r + \nu^+\right)\mathbf{b}_{\text{eff}} \tag{5}$$

B. Harmonic constitutive law

The reluctivity of (5) is formally known as "effective permeability" $\nu_{\text{eff}} = \nu_r + \nu^+$ and described in literature [2], [3] as a good estimation for time-harmonic finite element computations.

By combining (1) with the results of (5) one obtains

$$|\underline{\nu}|\ \ |\underline{\mathbf{B}}| \equiv \nu_{\text{eff}}\ \mathbf{b}_{\text{eff}} \tag{6}$$

which yields to a constraint for the magnitude of the complex reluctivity.

$$|\underline{\nu}| = \frac{\int_0^T\mathbf{h}^2(t)dt}{\int_0^T\mathbf{h}(t)\cdot\mathbf{b}(t)dt} \tag{7}$$

A periodic interpretation of the energy balance of ferromagnetic material, of (4), yields to the total magnetic energy losses P_μ. The integration of the right side identifies P_μ as the occurring hysteresis losses:

$$P_\mu = \int_0^T\mathbf{h}(t)\cdot\partial_t\mathbf{b}(t)dt \tag{8}$$

The magnetic stored energy is in a state of equilibrium, so the integration leads to no energy contribution.

$$\int_0^T \nu_r \partial_t \left| \mathbf{b}(t) \right|^2 dt \equiv 0 \tag{9}$$

By taking the construction condition of (5) into account, the integration of the dissipation term

$$\begin{aligned} P_\mu &= \int_0^T \frac{\nu_i}{\omega} \left| \partial_t \mathbf{b}(t) \right|^2 dt & (10) \\ &= T \omega \nu_i \left| \underline{B} \right|^2 & (11) \end{aligned}$$

can be determined as hysteresis losses.

III. Implementation

Equation (7) and (11) state an energy based time harmonic constitutive law. The derived complex reluctivity $\underline{\nu}$ constitutes a true correlation of the flux density $\mathbf{B}$ to the magnetic field $\mathbf{H}$ in terms of mean energy and is applicably to the governing equation of the time harmonic (3-D) problems

$$\text{curl}\ (\underline{\nu}\ \text{curl}\, \mathbf{A}) = \mathbf{J}_s \tag{12}$$

where $\mathbf{A}$ is the magnetic vector potential and $\mathbf{J}_s$ the applied current source density.

When applying the finite element method for numerically solving (12), one basically has to update the value of the reluctivity $\underline{\nu}$ as a function of three dimensional flux density solution $\mathbf{B} = \text{curl}\, \mathbf{A}$ after each calculation step. Comparable to the Newton-Raphson method, the algebraic system of complex equations can be solved iteratively to fulfill the steady state condition.

The ν-$\mathbf{B}$ function is implemented as a map that associates to every possible variation of the flux density distribution $\mathbf{B}$ characteristic parameters ν_r and ν_i. The entries of this preliminary calculated look-up table are provided by an energy-based vector hysteresis model.

IV. Validation Concept

There is no measurement setup to determine the hysteresis losses in a non rotating electromagnetic system. Due to this fact, the loss results, obtained from the energy-based time-harmonic calculation, are compared to the hysteresis losses estimated from a post-process routine of the time-domain simulation. In both cases all required characteristics are derived from the same measured hysteresis data to minimize the differences between both numerical approaches. In the time-harmonic case the hysteresis model parameters directly base on the measured hysteresis loops, cmp. [1]. In the other case the points of the hysteresis loss curve $\mathrm{W_h}$ are computed by

$$\mathrm{W_h}\left(\mathrm{B_{max}}\right) = \oint_c \mathrm{H} \cdot d\mathrm{B} \tag{13}$$

where H and B are the measuring points of a hysteresis loop with the maximal measured flux density $\mathrm{B_{max}}$.

V. Results

A C-Core, see fig.1, with a steel volume of 0.002 m^3, is used as test model for the described loss estimation approaches. The transient post-processing tool applies the hysteresis loss curve, compare (13), to the maximum induction B_{max} of each finite element. An additional loss estimation only basing on the fundamental flux density $B_{1,peak}$ can be obtained by a FFT response of the time-domain solution in the same manner.

Figure 2 shows the Hysteresis losses P_μ in function of the effective coil current density J_s obtained by these estimations. For the unsaturated case ($\mathrm{J}_s \leq 10^6 A/m$) the losses are nearby similar. For higher current densities the saturation effect can be noticed. Both transient estimations are congruent, but have a variation of 10% to the time-harmonic loss results.

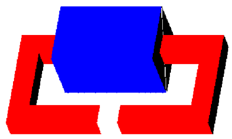

Fig. 1. C-Core as test model for the hysteresis loss calculation

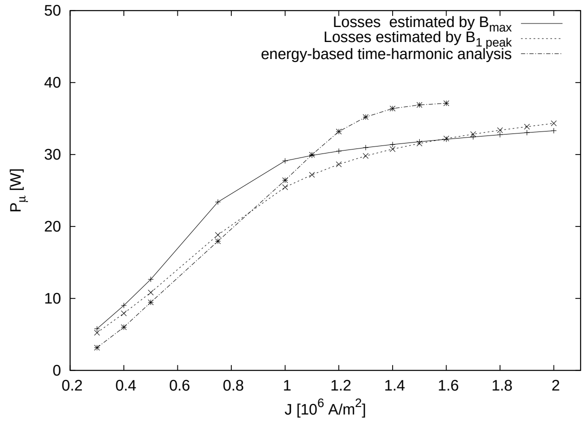

Fig. 2. Hysteresis losses P_μ in function of the effective coil current density J_s for transient and time-harmonic approach.

VI. Conclusion

Time-harmonic analysis are reasonable in early design stages of new devices. This approach generalizes the concept of using effective reluctivity curves for time-harmonic analysis. The generalization is based on the property of an energy-based hysteresis loss calculation during the traditional solving process. In contrast to other loss estimation approaches no additional fitting parameters are required due to the physical and phenomenological interpretation in terms of energy.

References

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