An Efficient Circuit Coupling Based on Parameter Identification and a Temporary Linearization of the Energy Balance of the Electrical Machine

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***Abstract*— In this paper an efficient method for modeling coupled magnetic field and circuit problems is presented. The field problem is solved by applying the Finite Element Method. The non–linear electric circuit system is solved by applying the Newton–Raphson method. The FE and the circuit systems are solved seperately which in turn gives the possibility to consider different time constants for each domain. In opposite to widely used strong coupled methods, this paper introduces a different approach by applying an energy–based parameter identification.**

I. Introduction

The development of electrical machines is often done solely by applying the Finite Element Method to the magnetodynamic domain. The natural excitation source of the magnetic field, the current, must be estimated in advance. Since electrical machines are often fed by voltage sources, a simple FE–model is not capable to regard the interaction of the coupled circuit–field system. Many approaches based on strong and weak coupling as well as on parameter identification have been developed [1],[2].
In this paper a 2D model of a stranded conductor is considered. Though the coupling mechanism is not limited to 2D problems, the model will be used as a prove of concept.

II. Field and Circuit Equations

The field and the circuit equations are well known. Thus only a brief overview will be given here.

A. The Field Domain

The equation covering a 2D problem based on the vector potential $\mathbf{a}$ for a non–conducting region derived from Maxwell's equations is known as [3]:

$$-\nabla \cdot (\nu \nabla a_z \vec{e}_z) = \vec{0}. \tag{1}$$

For the cross section of a stranded conductor, e.g. a coil, the equation is

$$-\nabla \cdot (\nu \nabla a_z \vec{e}_z) = j_z \vec{e}_z \tag{2}$$

with the current density $j_z = I_r \frac{w_r}{S_r}$. Herein w_r is the number of turns, I_r the current and S_r the area of the stranded conductor r. The standard Gelerkin scheme is applied to obtain the FE–formulation.

B. The Circuit Domain

The circuit simulator is based on the modified nodal analysis and non–linearities are treated by the Newton–Raphson method which leads to the following equation system:

$$\mathbf{J} \cdot \vec{s} = \vec{r}. \tag{3}$$

The solution vector is given by $\vec{s} = (\nu_1 \dots \nu_m, x_1 \dots x_n)^T$ with the nodal potentials ν_m and the internal quantities x_n. The internal quantities may be used if it is impossible to explicetly solve the equation. The right–hand side $\vec{r} = (-i_1 \dots - i_m, -F_1 \dots - F_n)^T$ includes the flow variables and the load vector of the internal quantities. The Jacobian matrix **J** includes the derivatives of the flow variables with respect to the nodal values and internal quantities.

III. Energy Balance in Magnetodynamics

The Thermodynamic principles are valid for electromagnetism. Upon this fact an energy based theory has been developed [4]. The theory is based on four energy reservoirs, each of them associated with one state variable: $\mathbf{a}, \mathbf{j}, \mathbf{d}, u$.

A. The Energy Balance

The model contains the magnetic energy reservoir $\Psi_M(\mathbf{a})$, the reservoir of the kinetic energy of charge carriers $\Psi_K(\mathbf{j})$, the electric energy reservoir $\Psi_E(\mathbf{d})$ and the u–reservoir, with the latter one always being empty. In case of magnetodynamics Ψ_E is empty, because of $\mathbf{d} \equiv 0$. Except for superconductors, Ψ_K vanishes as well.
The balance of magnetic energy denotes from the Thermodynamic point of view

$$\partial_t \Psi_M = \dot{W} + \dot{Q} + \dot{W}_m \tag{4}$$

where the dissipation functional due to magnetic hysteresis $\dot{Q}$ and the work delivered by magnetic forces $\dot{W}_m$ are not considered for the sake of clarity. Thus (4) simplifies into

$$\partial_t \Psi_M = \dot{W} = \int_\Omega \mathbf{j} \partial_t \mathbf{a}. \tag{5}$$

B. Parameter Identification and Mapping of State Variables

In case of electromagnetic systems containing coils, a FE–model with the state variables $\mathbf{a}$ and $\mathbf{j}$ is used. While incorporating the vector potential $\mathbf{a}$ directly into the circuit equation, the representation through inductances with the scalar state variable φ_r, the flux, seems to be more straightforward.

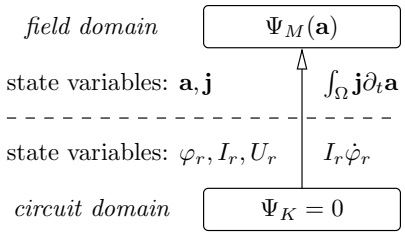

Fig. 1. The energy flow between the field and the circuit domain.

The flow of energy from the circuit domain to the field domain is shown in Fig. 1. A relationship between the two representations demands a definition of φ_r in function of $\mathbf{a}$. Having a look at Fig. 1 the relationship of the field and the scalar state variables between the energy reservoirs Ψ_M and Ψ_K can be obtained:

$$\int_\Omega \mathbf{j} \cdot \frac{\partial \mathbf{a}}{\partial t} \equiv \sum_r I_r \dot{\varphi}_r. \tag{6}$$

With the current only flowing through the stranded conductors, the current density may be expressed as:

$$\mathbf{j} = \sum_r I_r \frac{w_r}{S_r}. \tag{7}$$

Equation (7) in (6) yields:

$$\dot{\varphi}_r = \int_\Omega \frac{w_r}{S_r} \cdot \frac{\partial \mathbf{a}}{\partial t}. \tag{8}$$

If one makes the assumption, that the current distribution functions $\frac{w_r}{S_r}$ do not depend on time, which is true for stranded conductors, the mapping

$$\varphi_r = \int_\Omega \frac{w_r}{S_r} \cdot \mathbf{a} \tag{9}$$

between the scalar and the field state variables has been found.

IV. Coupling Of Circuit and Field Domain

To identify the magnetic energy, two different approaches are possible. Firstly, a look–up table of the magnetic energy is computed before the coupled simulation over the complete state space of the system. The major drawback is the exponentially growth of the table if the number of state parameters increases.

Secondly, the magnetic system is linearised around a given working point. The linearised model is described by

$$R_r^{-1}\left(\partial_t \delta\varphi_r - \delta U_r\right) = \delta I_r \tag{10}$$

$$L_{rs}^{\partial} \delta I_s = \delta\varphi_r \tag{11}$$

where $\delta\varphi_r$, δU_r and δI_r are the linear, scalar state variables. It can be shown that the tangent inductance matrix is given by

$$L_{rs}^{\partial} = W_{ri} J_{ij}^{-1} W_{sj} \quad \text{with} \quad W_{ri} = \int_\Omega \frac{w_r}{S_r} \cdot \alpha_i, \tag{12}$$

where α_i is the i^{th} shape function of the FE–formulation and J_{ij} is the corresponding entry of the Jacobian matrix of the non–linear FE formulation.

The equations (10) and (11) are incorporated within the circuit simulator by using the Jacobian matrix given by (3). Therefore δI_r must be included as an internal quantity of the solution vector $\vec{s}$. With the derivatives of the linearised model

$$\frac{\partial \delta I_r}{\partial \delta U_r} = -R_r^{-1}, \quad \frac{\partial \delta I_r}{\partial \delta I_s} = \frac{L_{rs}^{\partial}}{R_r} \cdot \partial_t, \quad \frac{\partial \delta\varphi_r}{\partial \delta I_s} = L_{rs}^{\partial}, \tag{13}$$

the equation system of the circuit simulator becomes:

$$\begin{pmatrix} -R_r^{-1} & L_{rs}^{\partial} R_r^{-1} \partial_t \\ 0 & L_{rs}^{\partial} \end{pmatrix} \cdot \begin{pmatrix} \delta U_r \\ \delta I_r \end{pmatrix} = \begin{pmatrix} \delta I_r \\ \delta\varphi_r \end{pmatrix}. \tag{14}$$

By applying this approach the circuit simulator is capable of treating high dynamic and complex circuit systems coupled with electromagnetic transducers, which are linearised around a constantly adjusted working point.

The flowchart in Fig. 2 outlines the simulation chain. Due to the small time constant of the circuit system compared to the one of the electrical machine, the simulation is repeated until the difference in one of the currents since the previous FE–calculation exceeds the given limit $\hat{i}$. Then, a new FE–calculation is triggered and L_{rs}^{∂} updated. If the change in one of the inductances exceeds the given limit $\hat{l}$, the circuit simulator steps back in time and a new FE–calculation is carried out with the adjusted δI_r.

This reduces the number of simulation steps performed on the FE–domain compared to a strong coupled approach.

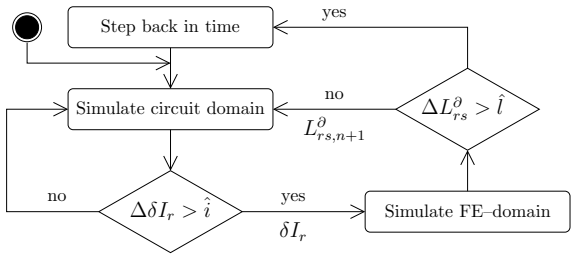

Fig. 2. Flow chart of the coupled simulation.

V. Conclusion

The presented method allows an efficient simulation of coupled problems. The order reduced model and the state mapping between the circuit and the field domain are obtained through an energy based approach. In opposite to strong coupled methods, each domain can be simulated with its own time constant which reduces the computation time compared to strong coupled methods, wherein the smallest time constant of the system is determined. Further details and a comparison with alternative coupling methods will be presented in the full paper.

References

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