# **State Control of an Electromagnetic Guiding System for Ropeless Elevators**

Benedikt Schmülling, Oliver Effing, and Kay Hameyer Institute of Electrical Machines, RWTH Aachen University Schinkelstraße 4 52056 Aachen, Germany Tel.:  $+49 / (241) - 8097667$ Fax: +49 / (241) – 80 92270 E-Mail: Benedikt.Schmuelling@iem.rwth-aachen.de URL: http://www.iem.rwth-aachen.de

# **Keywords**

«Control methods for electrical systems», «Actuator», «Magnetic bearings», «Active damping», «Linear drive».

# **Abstract**

One major challenge in modern elevator construction is the design of systems, which satisfy the requirements of very high buildings. In this sense conventional elevators with mechanical guides came to their application limitations. An opportunity to optimize the passenger traffic in very high buildings and also in very deep mining applications is an elevator system driven by linear motors instead of traction sheave and ropes. This system can be improved by using non-contact electromagnetic guides instead of a conventional mechanical guiding system. The aim of this work is the implementation of a feedback control for such an active magnetic guiding system for an elevator car in a simulation model.

At first, an overview over the applied elevator system is presented. Functionality of drive and guiding system is explained. Afterwards, the design of the simulation model and the derivation of a multi variable state control for the elevator car guiding are shown. Finally, results are presented and analyzed, which show the feasibility of the entire elevator system.

# **Introduction**

In recent years the demands on elevator systems increased due to the construction of more and more very high buildings. The height of skyscrapers is primarily not longer limited by structural restrictions. Today, the confining factors are the essential vertical escape and transportation routes. In buildings with 100 storeys a third of the total area is assigned to elevators, engine rooms, and other traffic areas. The elevator system attracts notice to the architectural planning of skyscrapers, since the economic efficiency is critically defined by the ratio of traffic area and useful area.

A novel concept in elevator construction determines the location of the elevator drive not at the top of the shaft, but directly attached to the elevator car. This can be realized by using linear motors, with a fixed part at the shaft wall and a moving part at the elevator car [1]. The major benefit of this concept is the possibility to operate the entire system with more than one car in one shaft. Thus, new transportation strategies can be implemented to decrease the average transportation time. More persons can be forwarded with less elevator shafts and the economic efficiency of buildings rises.

An important part of this elevator system is the guiding apparatus. It should ensure a wear free operation and a high ride comfort at the same time. Common elevators deploy roller guides or slideways for their operation. However, for application in a linear driven ropeless elevator for a very high building, conventional guiding systems don't meet the requirements. A solution is the use of non-contact electromagnetic guides. An evident benefit of these guiding systems is the wear free operation, since they do not need any lubricants. A further advantage is the opportunity to control the ride comfort by adjusting the damping rate of the guiding system. For this, a closed loop system is demanded. Here, an electromagnetic guide is an active magnetic actuator with permanent magnets and an excitation winding. The guidance force depends on the current linkage and the distance between actuator and guide rail. The control of multiple actuators requires a multi-variable control system. Aim of the project described is to apply an active magnetic guiding to a ropeless elevator test bench. The following sections show how this guiding system can be modeled and controlled.

# **Ropeless Elevator Test Bench**

Different types of linear motors are suitable for the application in vertical transportation systems in principle. For the ropeless elevator test bench developed at the Institute of Electrical Machines a long-stator type synchronous linear motor was chosen since the high manufacturing tolerances in very high shafts determine large air gaps. Considering these constraints the synchronous motor yields the most constant force density. The two reaction rails of the linear motor feature permanent magnets and are mounted on both sides of the elevator car. In permanent magnet synchronous machines high normal forces arise due to the attracting forces between permanent magnets and armature yoke. For this reason the armature is constructed ironless, since the running smoothness of the application is of high interest. Two armatures are mounted along the elevator shaft on two opposing walls. The design of the linear motor is described in [2]. Fig. 1 presents the test bench of the ropeless elevator.



Fig. 1: Ropeless elevator test bench.

Measurements result in a normal peak force *FNORMAL* of less than *100 N* for one linear motor, whereas the nominal propulsion force  $F_N$  reaches 16.2 kN. Due to this marginal force  $F_{NORMAL}$  the application of electromagnetic guides is feasible.

# **Vertical Electromagnetic Guiding**

Electromagnetic guiding of an elevator car makes high demands to guiding topology and control system. Compared to magnetic levitation systems for horizontal applications they possess no rated force in a defined direction. That means the displacement of the elevator car from its designated position can occur in every direction.

### **Degrees of Freedom**

The non-guided ropeless elevator behaves like a rigid body. It is only fixed in one degree of freedom (DOF) by the linear motor. This is the degree of freedom in vertical direction, here called z direction. The other five degrees of freedom are the translatory movements in x and y direction and the rotary movements  $\alpha$ ,  $\beta$ , and  $\gamma$  about the axes of a Cartesian coordinate system. These five degrees of freedom need to be controlled by the active magnetic guides. Fig. 2 shows the elevator car with all six degrees of freedom.



Fig. 2: The six degrees of freedom of a rigid body.

According to this mobility the elevator car requires at least five positioning forces.

# **Elevator Topology and Actuators**

For the placement of guiding systems in elevator shafts different topologies exist. This means, there are a number of possibilities to arrange the separate guides on the walls. Each topology subjects to different basic conditions, which depends on used material, shaft architecture, and drive. Here, a topology is chosen, where two guide rails are mounted in a center position on two opposing shaft walls.

To form a guiding shoe different electromagnetic actuators can be used. Examples are presented in [3] and [4]. In this work, u-shape actuators are deployed. These actuators are able to produce an attracting force only in one direction. They can be designed electrical or hybrid excited. A hybrid excited actuator consists of permanent magnets, which yield offset excitation and furthermore, it consists of an electrical winding to adjust the cumulative excitation. The winding is coiled around the two shanks of the u-shaped iron yoke. It is connected in series circuit. The permanent magnets are mounted onto the two shanks of the yoke. To control one DOF by means of these u-shaped actuators, it is necessary to couple at least two fronting actuators. These coupled actuators are positioned along a line of action of one positioning force. Twelve actuators in four packages establish the entire guiding system. Two packages are arranged at the top and two further packages are arranged at the bottom of the elevator car. The construction of the actuator packages and their mounting on the elevator car is presented in Fig. 3.



Fig. 3: Actuator packaging on one elevator car.

In the left part of the figure the u-shaped hybrid magnets with coils and permanent magnets can be seen. On the right it shows, that forces with the same index number are located along one action line of positioning forces. These overall six positioning forces face the five remaining DOF. Thus, the control equations system is over-determined.

# **Modeling of the Simulation Area**

The entire elevator model contains the dynamic moving equations of the elevator car and the dynamic electromagnetic equations of all twelve actuators. For simulation these equations are implemented in MATLAB/Simulink.

#### **Electromagnetic Equations**

According to Maxwell's equations the formula of the pulling force  $F_M$  between actuator and guide rail is

$$
F_M(\Phi) = \frac{\Phi^2}{A \cdot \mu_0},\tag{1}
$$

with  $\Phi$  as magnetic flux, *A* as air gap area of one pole, and  $\mu_0$  as permeability of the vacuum. The magnetic flux can be calculated with a magnetic equivalent circuit akin to the calculation of current with an electrical equivalent circuit. Thus, the magnetic flux Φ results in

$$
\Phi(\delta,\Theta) = \frac{\Theta + 2 \cdot H_c \cdot h_{PM}}{2 \cdot \left(H_c \cdot h_{PM} / B_R \cdot A\right) + 2 \cdot \left(\frac{\delta}{\delta \mu_0 \cdot A}\right)}.
$$
\n(2)

Here,  $\Theta$  is the entire current linkage of the electrical windings, and  $\delta$  is the height of the air gap between actuator and guide rail. The properties of the permanent magnets are  $B_R$  as the remanent flux density,  $H_C$  as the coercivity, and  $h_{PM}$  as the height. An equation of the force  $F_M$  subject to current linkage  $\Theta$  and air gab height  $\delta$  occurs by merging the formulae (1) and (2):

$$
F_M(\delta, \Theta) = \frac{1}{\mu_0 \cdot A} \cdot \left[ \frac{\Theta + 2 \cdot H_c \cdot h_{PM}}{2 \cdot \left( \frac{H_c \cdot h_{PM}}{B_R \cdot A} \right) + 2 \cdot \left( \frac{\delta}{\mu_0 \cdot A} \right)} \right]^2 \tag{3}
$$

The current linkage Θ is supplied by the voltage *U* connected to the actuator winding. This voltage is defined as follows:

$$
U(\delta, \Theta) = \frac{R}{N} \cdot \Theta - U_i(\delta, \Theta)
$$
 (4)

$$
U_i(\delta, \Theta) = -N \cdot \frac{d}{dt} \cdot \Phi(\delta, \Theta)
$$
\n(5)

*N* is the number of turns and *R* the ohmic resistance. Equation (4) shows, that the voltage *U* is composed of the line voltage drop and the electromotive force (EMF) *U<sup>i</sup>* . This EMF depends on the temporal variation of the magnetic flux. Equation (3) and (4) describe the entire characteristic of one actuator.

For the realization of a feedback control system a linearized description of the actuator performance is required. Therefore, a Taylor series expansion is accomplished, which requires a determination of a working point. This working point is defined by the desired values of the elevator position since no control action is necessary. Here, the working point is set to

$$
\delta = \delta_0 = 3mm \tag{6}
$$

and

$$
\Theta = \Theta_0 = 0 \tag{7}
$$

With this, the Taylor series expansion is accomplished. The results are

$$
F_M(\Delta \delta, \Delta \Theta) = F_0 + \left[\frac{\partial F_M}{\partial \Theta}\right]_{\Theta_0, \delta_0} \cdot \Delta \Theta + \left[\frac{\partial F_M}{\partial \delta}\right]_{\Theta_0, \delta_0} \cdot \Delta \delta \tag{8}
$$

and

$$
\Phi(\Delta \delta, \Delta \Theta) = \Phi_0 + \left[\frac{\partial \Phi}{\partial \Theta}\right]_{\Theta_0, \delta_0} \cdot \Delta \Theta + \left[\frac{\partial \Phi}{\partial \delta}\right]_{\Theta_0, \delta_0} \cdot \Delta \delta. \tag{9}
$$

 $F_0$  and  $\Phi_0$  are offset values;  $\Delta$ -values are derivations to the working point values. To standardize the notation the partial derivatives are substituted by coefficients. This results in the linearized magnetic force equation and the linearized flux equation:

$$
F_M(\Delta \delta, \Delta \Theta) = F_0 + F_\Theta \cdot \Delta \Theta + F_\delta \cdot \Delta \delta \tag{10}
$$

$$
\Phi(\Delta \delta, \Delta \Theta) = \Phi_0 + \Phi_\Theta \cdot \Delta \Theta + \Phi_\delta \cdot \Delta \delta \tag{11}
$$

By using (11) the linearization of equation (4) yields

$$
U = \frac{R}{N} \cdot \Delta \Theta + N \cdot (\Phi_{\Theta} \cdot \Delta \dot{\Theta} + \Phi_{\delta} \cdot \Delta \dot{\delta})
$$
 (12)

Solving equation (12) for the derivative of the current linkage  $\Delta\Theta$  causes to the differential equation of current rise:

$$
\Delta \dot{\Theta} = \frac{1}{N \cdot \Phi_{\Theta}} \cdot \left( U - \frac{R}{N} \cdot \Delta \Theta + N \cdot \Phi_{\delta} \cdot \Delta \dot{\delta} \right)
$$
(13)

Finally, the performance of the hybrid actuators can be described linearly by using the equations (10) and  $(13)$ .

#### **Movement of the Elevator Car**

The mechanical behavior of the elevator car can be deduced from the Newton-Euler method. In principle, this method is used to determine the movement of multi-body systems (MBS). For every single body of the MBS the principles of linear and angular momentum have to be formulated separately.

A further method is the application of Lagrange's equation. The establishment of this method offers more benefits than the Newton-Euler method, since the system needs not to be separated in subsystems. The procedure is founded on the kinetic energy equations *EKIN* of the entire system. This value is differentiated with respect to the position vector *q* and with respect to its time derivative. The sum of this and the vector of the active forces results in the Lagrange's equation of second order

$$
\frac{d}{dt}\left(\frac{\partial E_{\text{KIN}}}{\partial \dot{q}^T}\right) - \frac{\partial E_{\text{KIN}}}{\partial q^T} = d(q, \dot{q}, t),\tag{14}
$$

with *d* as a vector containing the injected forces and torques.

The derivation of the kinetic energy  $E_{KIN}$  leads to the nonlinear equation of motion of the entire mechanical system:

$$
M(q,t)\ddot{q}(t) + g(q,\dot{q},t) = d(q,\dot{q},t) \tag{15}
$$

Here, *M* is the symmetrical mass matrix, which contains the mass, the moments of inertia, and the moments of first order of the body. *g* is the vector of the generalized centrifugal forces and the generalized Coriolis forces.

#### **Implementation of the Simulation Model**

The two parts of the software package MATLAB/Simulink complement one another. The workspace of MATLAB provides the required data, whereas Simulink arranges the active computation. Basis of Simulink is the functional block diagram, which contains several signal processing elements. Partial circuits can be merged to subsystems.

The electrical and mechanical system equations represent the base of the entire simulation model. As aforementioned, the differential equations of the actuators are implemented in the signal flow syntax of Simulink. Input signals are the control voltage *U*, the air gap height δ, and its time derivative. Output values are current linkage and pulling force. The implementation of one actuator is presented in Fig. 4.

Due to its complexity, the equation of motion of the elevator car is not implemented as a block diagram. More favorable is the application of an Embedded MATLAB Function in Simulink. This function contains a MATLAB script and is executed in every single simulation time step. Output value of this function is the position vector *q* considering all DOF.



Fig. 4: Simulink subsystem of one single actuator.

### **State Control**

In principle, every single actuator can be controlled separately. For this, the control algorithm calculates the local manipulated variable from the local air gap heights. By reason of large manufacturing tolerances in high elevator shafts this method is unfavorable, since an independent control of solid-coupled actuators leads to unstable operation conditions.

A further method is the control of the DOF, the so called DOF-control. At this juncture, the decoupling of the actuators occurs not mechanical but in the control loop system. Local measurement values are transformed into global values. This concept observes the absolute position of the elevator car in the shaft, described by the five predetermined DOF. Thus, this control method is realized in this work.

The structure of a control loop can differ between a simple feedback system and a complex and interleaved control system. However, it is reasonable to realize the control loop as simple as possible to afford an easy design and the opportunity to analyze the system with common methods.

#### **Transformation of Local Values**

Due to the five remaining DOF only five air gap measurement signals are required. This leads to the problem that five measurement values confront with six manipulated variables for the six actuating forces. This means the elevator car is guided statically over-determined. The difficulty can be solved by permission of an axial deformation of the elevator car. Thus, torsion φ is established as a further position variable. The restriction that the entire elevator car has to be a solid body is abolished.

To transform the six local air gap heights  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$ ,  $\delta_4$ ,  $\delta_5$ , and  $\delta_6$  (The measuring points of the air gap heights are presented in Fig. 5) to global positioning values, six constellations of actuating are analyzed. Each considered actuator operation affects one single DOF. As an example the offset of the elevator car in positive x direction is presented. This offset occurs, when all actuator pairs adjusted in x direction produce the same force. That means for the actuating forces obtain

$$
F_1 = F_2 = F_3 = F_4 = const.
$$
\n(16)

and

$$
F_{5} = F_{6} = 0. \tag{17}
$$

The action lines of the several forces are shown in Fig. 5.



Fig. 5: Action lines of the actuating forces (left) and air gap measurement (right).

With this exertion of force all air gaps receive the same acceleration in x direction. According to this the offset is calculated by averaging the derivation values of the air gaps:

$$
\Delta x = \frac{1}{4} \left( \Delta \delta_1 + \Delta \delta_2 + \Delta \delta_3 + \Delta \delta_4 \right) \tag{18}
$$

Here,  $\Delta x$  defines the deviation to the mean position of the elevator car in x direction. By employment of this method all further global position variables are calculated and the position vector  $q$ , which is exemplarily used in (15), is formed as follows:

$$
q = (\Delta x \quad \Delta y \quad \Delta \alpha \quad \Delta \beta \quad \Delta \gamma \quad \Delta \varphi)^T \tag{19}
$$

By adoption of these geometrical coherences between local air gaps and global variables the transformation matrix is derived. Local effects like magnet forces and actuator currents are transformed to global variables using this matrix.

#### **State Control Design**

According to the predetermined equations, controllers can be designed for every global positioning value. With the equation of motion (15), the equation of the linearized magnet force (10), and the differential equation of the current rise (13), three formulae for every positioning value exist. From these, the state space model of the open loop system is derived. For the y direction results

$$
\dot{y} = A \cdot y + b \cdot u \Longleftrightarrow \begin{pmatrix} \Delta \dot{\tilde{S}}_{y} \\ \Delta \ddot{\tilde{S}}_{y} \\ \Delta \dot{\tilde{\Theta}}_{yp} \\ \Delta \dot{\tilde{\Theta}}_{yp} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{4}{m} \cdot F_{\delta} & 0 & \frac{2}{m} \cdot F_{\delta} & \frac{2}{m} \cdot F_{\delta} \\ 0 & -\frac{\Theta_{\delta}}{\phi_{0}} & -\frac{R}{N^{2} \cdot \phi_{0}} & 0 \\ 0 & \frac{\phi_{\delta}}{\phi_{0}} & 0 & -\frac{R}{N^{2} \cdot \phi_{0}} \end{bmatrix} \begin{pmatrix} \Delta \tilde{\delta}_{y} \\ \Delta \dot{\tilde{\delta}}_{y} \\ \Delta \tilde{\Theta}_{yp} \\ \Delta \tilde{\Theta}_{yn} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{N \cdot \phi_{0}} \\ \frac{1}{N \cdot \phi_{0}} \end{pmatrix} \tilde{U}_{y}.
$$
\n(20)

Here, ~-values are global variables, *m* is the mass of the elevator car, subscript *p* means positive, and subscript *n* means negative. Air gap height, its derivative, and the current linkage act as state variables. For visualization of the system performance a pole-zero diagram is used. This diagram contains one pole with a positive real part at least. Here, the instability of the open loop model is shown.

By changing to the closed loop model a state extension is accomplished. The model is extended by the integral of the air gap height δ as a further state variable. Thus, the order of the system is increased by one and the rank of the system matrix is 5.

For the following state feedback the controller's law is employed. By merging the controller's law with the state space equation (20) the system matrix of the closed loop system occurs. The eigenvalues of the matrix are computed using the method of pole placement. This method is qualified in a several publications. In [5] and [6] the use of this procedure is specified for the application in magnetic levitation controllers. [7] describes the pole placement for a 6-DOF vehicle. Other papers ([8] e.g.) show the implementation of the pole placement for other intentions.

Here, the poles with a positive real part are mirrored at the imaginary axis. Furthermore, the pole, which appeared by the state extension has to be placed. It is set between the yet existing stable poles, since there is a sufficient distance to the unstable area and no high control expense is required. After calculating the control factors  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ , and  $k_5$  the system matrix  $A_K$  is obtained:

$$
A_{K} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{4}{m} \cdot F_{\delta} & 0 & \frac{2}{m} \cdot F_{\Theta} & \frac{2}{m} \cdot F_{\Theta} \\ \frac{k_{1}}{N \Phi_{\Theta}} & \frac{k_{2}}{N \Phi_{\Theta}} & \frac{k_{3} - N \Phi_{\delta}}{N \Phi_{\Theta}} & \frac{k_{4} - \beta_{N}'}{N \Phi_{\Theta}} & \frac{k_{5}}{N \Phi_{\Theta}} \\ \frac{k_{1}}{N \Phi_{\Theta}} & \frac{k_{2}}{N \Phi_{\Theta}} & \frac{k_{3} - N \Phi_{\delta}}{N \Phi_{\Theta}} & \frac{k_{4}}{N \Phi_{\Theta}} & \frac{k_{5} - \beta_{N}'}{N \Phi_{\Theta}} \end{bmatrix}
$$
(21)

### **Simulation Results**

Functionality of the closed-loop position control is analyzed using the response of the guiding system to several loading cases. The selection of loading cases is geared to the operation of a common elevator with a nominal load of *630 kg*. As customary the load is applied as a force step or as a force impulse. Indeed, this load characteristic is not realistic according to real load of an elevator car, but it yields meaningful system reactions. Reasonable loading cases are eccentric loads and bounces against the elevator car wall. Eccentric loads are force steps in vertical direction anywhere inside the car. This case occurs when one or more persons enter the elevator car, for example. Bounces against the elevator car crop up when persons inside the elevator collide with its wall. Here, a force impulse in horizontal direction is analyzed instead of a force step in vertical direction. This second load case is exemplified in Fig. 6.



Fig. 6: Step response of the positioning variables in the case of a force impulse in horizontal direction.

Let's assume, a person with a body weight of *100 kg* collides with the wall and accomplishes an elastic collision process in x direction. The exact form of the force impact depends on the elasticity of the elevator wall. Acting on the assumption that conventional elevators are made in lightweight construction the compliance is very high. Thus, the term of the force impact is assumed to *0.2 s*. The force impulse results to *1000 N*, if the person moves with *2 m/s* towards the wall. In Fig. 6 can be seen, that the highest offset results in x direction. Further deviations to the meaning values occur in the variables β and γ. All deviations are compensated within *0.4 s*.

The entire series of simulation shows that the employed electromagnetic guiding system for ropeless elevators is able to substitute common guides. All performed test cases show a stable operation.

# **Conclusion**

State control of an active magnetic guiding system for vertical applications is presented and discussed in this paper.

At first, an application example for the deployment of electromagnetic guides, which substitute the functionality of common roller guides or slideways, is given by the description of the ropeless elevator test bench. A further section of this work discusses basics of vertical magnetic guiding. It contains the description of the six degrees of freedom of an unguided elevator car. Furthermore, the used actuators are presented. These actuators are mounted on the elevator car and perform forces to a guide rail in the shaft. They act as final controlling elements of the position control of the elevator car to counter uneven loads and to stabilize its position. Another subsection presents the base of the entire simulation. Therefore, the design and implementation of the simulation model is qualified. In the following part the design of the DOF-Controller is explained. It is revealed, that the measurements of six air gap heights contain all required information about the elevator car position. Therefore, the six possible degrees of freedom were identified and the calculation specifications of their determination from the measured gap signals defined. The designed position controller consists of six parallel controllers, one for every single degree of freedom. The several controllers are rated on the ground of a state space model employing the method of pole placement. An example of the functionality of the multi variable controller is presented. The results show a fast system response to different disturbances. As a final remark it can be noticed that the presented guiding improves the riding comfort of elevator cars, whereas the friction is decremented and the applica-

# tion of lubricants is not longer required.

# **References**

[1] Ishii, T., "Elevators for skyscrapers," *IEEE Spectrum*, 31(9): 42-46, September 1994.

[2] Platen, M., Henneberger, G., "Examination of Leakage and End Effects in a Linear Synchronous Motor for Vertical Transportation by Means of Finite Element Computation," *IEEE Transactions on Magnetics*, 37(5):3640-3643, September 2001.

[3] Morishita, M., Akashi, M., "Electromagnetic Non-contact Guide System for Elevator Cars," *The Third International Symposium on Linear Drives for Industry Applications*, pages 416-419, LDIA, Nagano, Japan, October 2001.

[4] Inaba, H., Shigeta, M., Ando, T., Nokita, A., Konya, M., "Attitude Control System of a Super-high Speed Elevator Car Based on Magnetic Guides," *The 20th IEEE International Conference on Industrial Electronics, Control, Instrumentation, and Automation*, Vol. 2, pages 1028-1033, IECON '94, Bologna, Italy, September 1994.

[5] Lu, Y.-S., Chen, J.-S., "Design of a perturbation estimator using the theory of variable-structure systems and its application to magnetic levitation systems," *IEEE Transactions on Industrial Electronics*, 42(3):281-289, June 1995.

[6] Van Goethem, J., Henneberger, G.: "Design and Implementation of a Levitation-Controller for a Magnetic Levitation Conveyor Vehicle", *8th Int. Symposium on Magnetic Bearing*, Mito, Japan, 2002.

[7] Ghersin, A. S., Sánchez Peña, R. S., "LPV Control of a 6-DOF Vehicle," *IEEE Transactions on Control Systems Technology*, 10(6):883-887, November 2002.

[8] Oliva, A. R., Ang, S. S., Bortolotto, G. E., "Digital Control of a Voltage-Mode Synchronous Buck Converter," *IEEE Transactions on Power Electronics*, 21(1):157-163, January 2006.