The Energy Viewpoint in Computational Electromagnetics*

Francois Henrotte[1] and Kay Hameyer[1]

Institute of Electrical Machines, RWTH Aachen University
Schinkelstrae 4, D-52056 Aachen, Germany
fh@iem.rwth-aachen.de

1 Introduction

Opening a textbook on electromagnetism, it is likely that the first set of equations presented will be Maxwell's equations

$$\operatorname{curl}\mathbf{h} - \partial_t\mathbf{d} = \mathbf{j} \quad (1)$$
$$\operatorname{curl}\mathbf{e} + \partial_t\mathbf{b} = 0 \quad (2)$$
$$\operatorname{div}\mathbf{b} = 0 \quad (3)$$
$$\operatorname{div}\mathbf{d} = \rho^Q \quad (4)$$

complemented by a set of constitutive relations of the form

$$\mathbf{b} = \mu\,\mathbf{h} \quad , \quad \mathbf{d} = \varepsilon\,\mathbf{e} \quad , \quad \mathbf{j} = \sigma\,\mathbf{e} \quad (5)$$

with the mention that the first set are universal (always valid) and the second one contains any relation one would need to 'close the system' and be able to solve it. Electromagnetism is in this way seen as a set of fields whose evolution in time and distribution in space are ruled by partial differential equations (PDE) and constitutive relations. There is no place in this setting for any energy considerations.
Further in the same book however, some energy related notions are likely to be introduced. The magnetic energy, for instance, is usually defined as a functional of $\mathbf{b}$ or $\mathbf{h}$ (or even both). Different materials will be considered, starting with the simplest medium (vacuum) and proceeding in a bottom-up fashion towards more complex materials : linear, anisotropic, nonlinear, etc. Not for long however, because the definitions become quickly rather technical and fall outside the scope of a general monograph.
Classical presentations of the theory of electromagnetism leave thus the impression that energy aspects are by-products of the field theory, somehow accessory and difficult to exploit. The principles of Thermodynamics however are universal and they must apply to electromagnetic phenomena also. Maxwell's equations say actually something yet about electromagnetic energy conservation, but they do so in a way that makes it impossible to disentangle the different energy flows in presence. Moreover, classical presentations of the theory leave unanswered fundamental questions like

- What are the state variables in an electromagnetic system ?
- How are magnetic and electric energy defined in the general case ?
- What are the possible dissipation mechanisms ?
- How is magnetic energy converted into electric energy ?

* Invited Paper at SCEE-2006

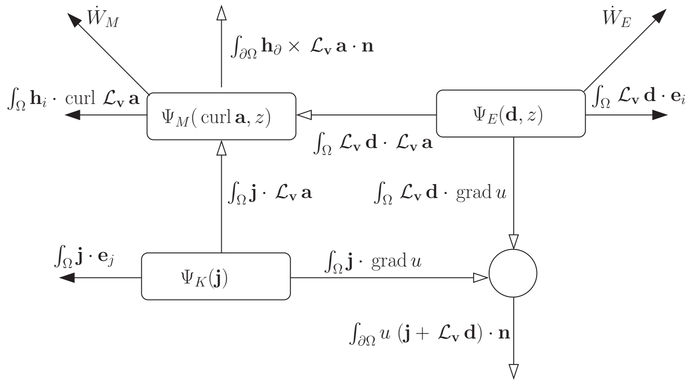

Fig. 1: EM energy flow diagram in the Euclidean space E.

- How is electromagnetic energy converted into other forms of energy ?
- etc.

Those shortcomings are particularly hampering when one deals with problems like the computation of local electromagnetic forces (energy conversion), magnetic hysteresis (energy dissipation) or magnetostriction (both) or multiphysics problems in general. For such problems, it really makes sense to dispose of a theory of electromagnetism where energy aspects are considered from the beginning and throughout.
After pursuing theoretical investigations in those domains, and accumulating along the way pieces of knowledge about how energy behaves in electromagnetic systems, a big picture has eventually, and somewhat unexpectedly, formed that gives rise to an energy-based theory of electromagnetism [1]. This representation of Electromagnetism takes the form of a flow diagram. It provides more information than the classical theory and gives answers to the questions listed above. Being expressed in integral form instead of by a set of PDE's, the governing equations can be established straightforwardly in arbitrary coordinate systems. Finally, the energy-based theory provides operative concepts, which clarify issues like hysteresis modelling and give many clues how to deal in a consistent way with coupling terms in multiphysics problems and parameters in reduced order models.

2 Energy flow diagram

The energy-based theory of electromagnetism is now briefly presented. More details can be found in [1]. The representation in an Euclidean space of the electromagnetic energy flow diagram is depicted in Fig. 1. The diagram consists of four interconnected energy reservoirs, each one associated with a state variable. The state variables are the two electromagnetic potentials, i.e. the magnetic vector potential $\mathbf{a}$ and the electric scalar potential u, and the two fields associated with electric charges, i.e. the electric displacement $\mathbf{d}$ and the current density $\mathbf{j}$.
The $\mathbf{a}-$reservoir contains the magnetic energy

$$\Psi_M(\operatorname{curl}\mathbf{a}, z) \equiv \int_\Omega \rho_M^\Psi(\operatorname{curl}\mathbf{a}, z), \tag{6}$$

which is the integral over the domain Ω under consideration of the magnetic energy density ρ_M^Ψ, a function of the induction $\operatorname{curl}\mathbf{a}$ and possibly also of one or several additional non-electromagnetic quantities (e.g. the strain) represented in a generic way by the unspecified