# Topology Optimization of Magneto-thermal Systems Considering Eddy Current as Joule Heat

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**Abstract— This research presents a topology optimization to manipulate the main heat flow in a magneto-thermal system. The heat generated by eddy current is considered in the design domain assuming an adiabatic boundary. A topology design sensitivity is derived by employing the discrete system equations combined with the adjoint variable method. As a numerical example, an iron domain heated by eddy currents is dealt with the proposed method.**

## I. Introduction

Most research works associated with the optimization of magneto-thermal systems have been focused on Joule heat, generated in coils. The optimization regarding only solid conduction does not result in good design, because eddy current effects are not neglected. The eddy current problem in shape optimization has been studied [1], but never been applied to topology optimization.

In this paper, topology optimization of a magneto-thermal system is discussed including eddy currents being the main source of the generated Joule heat. The optimization progresses for maximizing target nodal temperature with volume constraint.

## II. Topology Sensitivity Equation

The single governing equation of the transient magnetic field can be described by using the set of Maxwell's equation and by introducing a vector potential $B = \nabla \times A$

$$\nabla \times (\frac{1}{\mu} \nabla \times A) = J_s - \sigma \frac{\partial A}{\partial t} \tag{1}$$

where $J_s$, $\mu$, $\sigma$ and $t$ are the current density vector, the permeability of material, the electric conductivity and the time, respectively.

For the thermostatic field, the governing equation can be expressed by using the law of energy conservation and the Fourier's law.

$$\nabla \cdot (k \cdot \nabla T) = -q \tag{2}$$

where $k$, $T$ and $q$ are the thermal conductivity, the temperature and the heat source term, respectively. The main heat can be classified into the Joule heat generated by the coil, the eddy current and the heat flowing into or out of the domain.

The material interpolation method defines artificial materials such as permeability and electric conductivity for the magnetic domain, and thermal conductivity for the thermal field. In addition, convection coefficient, $h_c$, at the boundary should be taken consideration while the topology commits any hole in the design domain. All components have to be composed by a function of polynomial degree p of defining the material density, $b$. This yields the modified equivalent material coefficients;

$$\mu = \mu_0 + (\mu_0 \mu_r - \mu_0) b^P \tag{3}$$

$$\sigma = b^P \sigma_{initial} \tag{4}$$

$$k = b^P k_{initial} \tag{5}$$

$$h_c = h_{c.initial} (1 - b^{1/p^3}) \tag{6}$$

where subscript 'initial' indicates the initial value prior to the optimization.

In order to obtain the sensitivities of all design variables from a single evaluation, the adjoint variable method is applied and then the sensitivity equation of performance index, $\psi(T,b)$ can be expressed by;

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \lambda_T{}^T \left[ \frac{\partial q}{\partial b} + \frac{\partial q_{eddy}}{\partial \sigma} \frac{\partial \sigma}{\partial b} + \frac{\partial q_{conv}}{\partial h_c} \frac{\partial h_c}{\partial b} - \frac{\partial}{\partial b} (\widetilde{K}_{th} T) \right] + \lambda_A{}^T \left[ \frac{\partial J_S}{\partial b} - \frac{\partial}{\partial b} (\widetilde{K}_{mag} A) - j\omega \frac{\partial}{\partial b} (\widetilde{M}_{mag} A) \right] \tag{7}$$

$q_{eddy}$, $q_{conv}$ are heat by eddy current and convection, and $K_{th}$, $K_{mag}$, $M_{mag}$ are finite element thermal stiffness and magnetic stiffness matrices, and magnetic mass matrix, respectively[2]. $\lambda_T$, $\lambda_A$ are adjoint variable of the thermal system and the magnetic system, which are calculated by the derivatives of the adjoint equations, respectively.

## III. Numerical Example

The numerical model, a quarter section, consists of three materials that are coil, air, iron surface. In this paper, the objective is to maximize nodal temperature on the metal domain, where the eddy current generates the main Joule heat. This means that the optimal design controls efficiently the most heat to be radiated outside. And the constraint is volume used for the optimal design, the 50% out of the original one.

TABLE I Comparison between Original and Optimal Design

| | Nodal Temp [%] | Heat transfer rate[%] | Magnetic energy of iron domain[%] | Volume [%] |
|---|---|---|---|---|
| Original | 100 | 100 | 100 | 100 |
| Optimal | 175.17 | 258.94 | 504.08 | 41.67 |

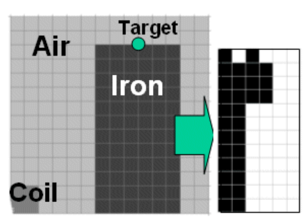

Fig 1. Design Model and Optimal Design

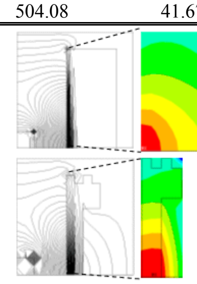
Fig 2. Magnetic Flux and Temperature Distribution (Original – upper, Optimal - lower)

## IV. References

[1] Park, Kwak, Lee, Hahn, Lee, "Design Sensitivity Analysis for Transient Eddy current Problems using Finite Element Discretization and Adjoint Variable Method" *IEEE Transactions on Magnetics*, Vol. 32, No. 3, pp. 1242-1245, May 1996.

[2] *ANSYS Theory Reference*, ANSYS Inc., ANSYS Version 8.0

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