A hybrid Picard-Newton acceleration scheme for non-linear time-harmonic problems

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1 Introduction

Non-linear quasi-static magnetic problems are governed by the equation

$$\nabla \times (\nu \nabla \times A) + \sigma \frac{\partial A}{\partial t} = J \quad , \tag{1}$$

with ν the reluctivity tensor [Am/Vs], A the magnetic vector potential [Vs/m], σ the electric conductivity [A/Vm] and J the applied current density vector [A/m]. Eq. (1) must be complemented with an appropriate gauge and appropriate boundary conditions in order to determine a unique solution [1]. In case A and J are phasors, applying the Galerkin finite element method to Eq. (1) yields a system of non-linear complex equations.

$$\tilde{\mathbf{r}}(\tilde{\mathbf{A}}) = [\mathbf{K}(\tilde{\mathbf{A}}) + j\mathbf{L}]\tilde{\mathbf{A}} - \tilde{\mathbf{T}} = \mathbf{0} \quad , \tag{2}$$

with $\tilde{\mathbf{A}}$ the (n x 1) column vector of the phasor-valued connectors, $\mathbf{K}$ the stiffness matrix, $\mathbf{L}$ the eddy current matrix, $\tilde{\mathbf{T}}$ the source current vector and $\tilde{\mathbf{r}}$ the residual vector [2,3]. This paper presents a combined Picard-Newton scheme for iteratively solving Eq. (2). The solution process is started by performing Picard-iterations. As soon as an estimator indicates that the expected convergence rate of the Newton-strategy is close to quadratic, Newton-iterations take over the solution process. This hybrid approach yields a shorter overall computation time. The method is illustrated for the two-dimensional simulation of the short-circuit operation of a 400 kW induction motor.

2 Picard vs. Newton scheme

The Picard-method (equiv. successive substitution) is obtained by successively evaluating the stiffness matrix and solving the complex system of equations. This system is complex symmetric and of size (n x n). For this purpose, either the Conjugate Orthogonal Conjugate Gradient method [4] (COCG) or a variant of the Quasi-Minimal Residual method (QMR) for complex symmetric matrices [5] can be used.

The basic idea behind the Newton-method is to set the first order Taylor series expansion of the residual to zero. However, when working with complex variables, the Taylor series expansion is only defined if the residual is an analytic function of $\tilde{\mathbf{A}}$. Unfortunately, in magneto-dynamic problems, this is generally not the case [6]. Consequently, in order to use a Newton-scheme, one has to derive the Jacobian from the equivalent real representation of Eq. (2). In this case, a real positive definite, but non-symmetric system of size ($2n$ x $2n$) has to be solved [6]. As a consequence, system solvers such as the Bi-Conjugate Gradient method (BiCG) or the Generalized Minimal Residual method (GMRES) can be used.

Unfortunately, the computational cost for applying BiCG on the real equivalent system is approximately twice the one for applying COCG on the complex symmetric system. This favours the Picard-approach. On the other hand, the Picard-strategy makes no use of any information about the differential reluctivities in the elements. For this reason, it features a lower asymptotic convergence rate of the non-linear residual when compared to the Newton-strategy. However, it is observed in practice that the initial convergence rate of both strategies is more or less the same. Moreover, if both strategies start from the zero solution, the first iterate is identical. Therefore, it is suggested to initiate the solution process by Picard-iterations and to switch to Newton-iterations as soon as a significantly better convergence rate can be expected.

3 Truncation error estimator

One can show that the solution of Eq. (2) can be obtained by minimizing half the square of the residual norm:

$$F(\tilde{\mathbf{A}}) = \tfrac{1}{2}\left\|\tilde{\mathbf{r}}(\tilde{\mathbf{A}})\right\|^2 \quad . \tag{3}$$

For a particular iterate $\tilde{\mathbf{A}}_k$, the second order Taylor series expansion of F forms a quadratic model $F_k^{qm}(\tilde{\mathbf{A}})$ in a multi-dimensional space. The closer an iterate $\tilde{\mathbf{A}}_k$ approaches the solution, the smaller the steps towards the solution are, the better this quadratic model describes the function in the vicinity of the solution. As a consequence, the quality of the quadratic model can be monitored by continuously evaluating

$$\kappa_k = \left| 1 - \frac{F(\tilde{\mathbf{A}}_k) - F_k^{\mathrm{qm}}(\tilde{\mathbf{A}}_{k+1})}{F(\tilde{\mathbf{A}}_k) - F(\tilde{\mathbf{A}}_{k+1})} \right| + \left| 1 - \frac{F_{k+1}^{\mathrm{qm}}(\tilde{\mathbf{A}}_k) - F(\tilde{\mathbf{A}}_{k+1})}{F(\tilde{\mathbf{A}}_k) - F(\tilde{\mathbf{A}}_{k+1})} \right| \quad (4)$$

for two subsequent (Picard)-iterates. This parameter is an estimator for the truncation error of the quadratic model and it is therefore expected to indicate when quadratic convergence of the Newton-scheme can be assumed. One can even improve the estimator by taking into account the relaxation factor of the applied line-search algorithm. This cannot be outlined here.

4 Example

The technique is applied for the two-dimensional simulation of the short-circuit operation of a 400 kW induction motor (Fig. 1). Fig. 2 shows κ and its improved version, while performing Picard-iterations. Initially, it is zero, due to the severe relaxation close to the initial zero solution.

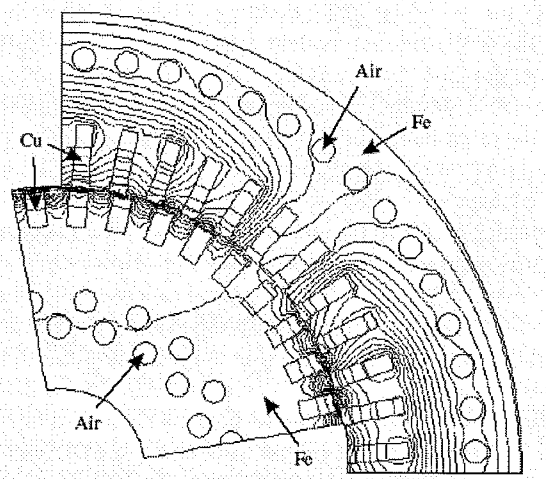

Fig. 1: Magnetic field in a short-circuited induction motor.

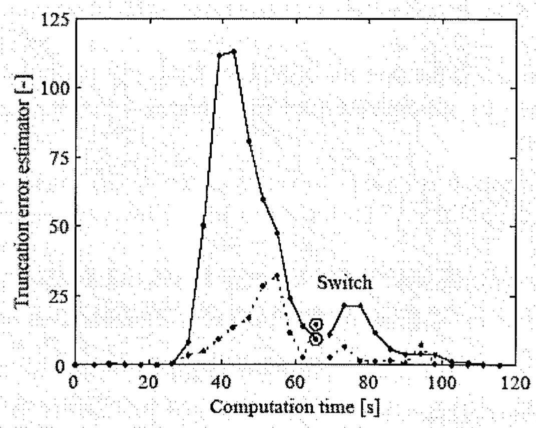

Fig. 2 : The value of the basic (dotted) and improved (solid) truncation error estimator during the convergence process of the Picard-method.

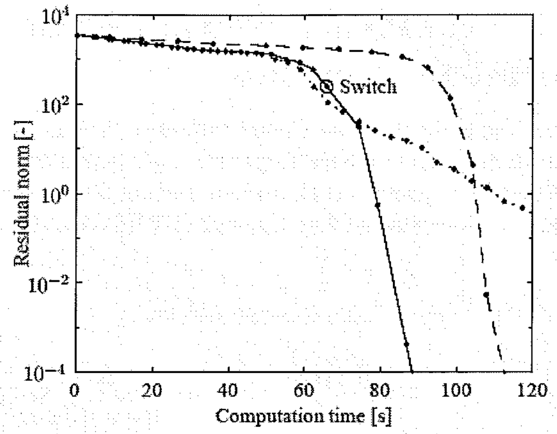

Fig.3: The residual norm as a function of computation time, for the Newton method (dashed), the Picard method (dotted) and the hybrid Picard-Newton method (solid).

Once over the top, the error estimator steadily decreases and it is decided to move onwards using the Newton scheme. Fig. 3 shows the effect on the overall computation time.

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SCOPE – B7

SURFACE WAVE IMPEDANCE MODELLING WITH 2D FINITE ELEMENTS

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Abstract:

A methodology for deriving a surface wave impedance that can be used within an asymptotic UTD ray-tracing model of surface wave diffraction is described in this paper. Fock theory can be used to calculate electromagnetic surface wave propagation on non-conducting, convex surfaces that are represented by an impedance boundary condition. A numerical estimate of the surface wave impedance presented to TE electromagnetic radiation propagating over a conducting, convex surface uniformly coated with a thin, lossy layer can be found from a 2D finite element time domain analysis. Surface wave impedance values derived from the CEM modelling are subsequently plugged into an asymptotic model in order to produce comparative prediction data with which to validate the approach. Simple curve-fitting procedures can be used to characterise the impedance over a range of frequencies and material properties for which the impedance boundary approximation is valid. The material corrections for the TM polarisation state are an order of magnitude smaller than the standard Fock theory predictions for a perfectly conducting body and have therefore been neglected. The problem is relevant to the performance prediction of antennas installed on non-perfectly conducting, coated bodies on which impedance boundary conditions apply. Only surfaces with constant curvature that were uniformly coated with linear, isotropic homogeneous materials were considered for this study.

Keywords: surface waves, impedance boundary conditions, Fock theory, surface diffraction, UTD, asymptotics, installed antenna performance, finite element modelling, computational electromagnetics, material layers