An energy-based model of magnetostriction with hysteresis

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***Abstract*— A parametric energy-based model for the magnetostriction of ferromagnetic materials with hysteresis is discussed and compared with measurements.**

I. INTRODUCTION

Designers of electromagnetic devices are often confronted with materials where conversion and dissipation of energy occur at the microscopic level (e.g. magnetostrictive and piezoelectric materials…). As FE models are macroscopic, the constitutive laws of such materials, which often have a complex microstructure, are conveniently represented by means of an energy-based approach. The idea of this paper is to build a complete parametric multi-physics model of magnetostrictive materials, in terms of which measurements can be interpreted.

II. MODEL OF MAGNETOSTRICTION

This papers deals more particularly with ferromagnetic materials exhibiting magnetostriction. Numerous aspects (saturation, anisotropy, hysteresis, losses…) need to be consistently combined with each other. A quantitative model describing large magnetostriction effect observed in several ferromagnetic shape memory alloys has been proposed by Likhachev and Ullakko [1]. The material is considered as a composite of three martensitic phases aligned with the crystallography axes [100], [010] and [001]. Let $x, y, z \in [0, 1]$, $x + y + z = 1$, be the respective proportions of the three phases. For the phase aligned with [100], a spontaneous elongation ε_0 along the x-axis is observed and a contraction $-\varepsilon_0/2$ along the other two axes. On the other hand, for an applied magnetic field h_x along the x-direction, a magnetisation $M_a(h_x)$ along the x-direction is observed and a slower magnetisation $M_t(h_x)$ in the other two directions (Fig. 1). As it goes similarly for the other two phases, the magnetisation $\mathbf{M}$ and the strain ε can be written as :

$$\mathbf{M} \equiv A \cdot \mathbf{x} = \begin{pmatrix} M_a(h_x) & M_t(h_x) & M_t(h_x) \\ M_t(h_y) & M_a(h_y) & M_t(h_y) \\ M_t(h_z) & M_t(h_z) & M_a(h_z) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{pmatrix} = \varepsilon_0 \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \qquad (1)$$

Solving for $\mathbf{x}$, the magnetisation $\mathbf{M}$ can be expressed as a function of $\mathbf{h}$ and ε, and using the thermodynamic Maxwell relation $\partial_\varepsilon \mathbf{M} = -\partial_\mathbf{h} \sigma$, the stress tensor σ can be determined by integration.

In this paper, the above description is assumed to be representative in principle for magnetostrictive materials in general. The uniaxial thermodynamic discussion of [1] is further developed in order to incorporate a valid model of anisotropic saturation, a more realistic representation of the magnetisation curve and finally to take hysteresis into account as well according to the approach presented in [2]. At the end, one disposes of a complete 3D energy-based model for the material behaviour. The model has a limited number of parameters that can be identified by fitting with measurements.

Its is important to note how beneficial it is to dispose of a complete material model. Measurements are indeed often insufficient (e.g. uniaxial, limited range). Moreover, form effect [3] and hysteresis are always present. A direct identification of the parameters of an empirical magnetostriction model is therefore hazardous. On the other hand, the identification, of the parameters of a complete material model, like the one presented here, can be done meaningfully on basis of incomplete data (e.g. uniaxal). The internal consistency of the model makes up indeed for the lack of available measurements and ensures a reasonable description over the whole application domain of the model.

III. APPLICATION

A magnetic standard Si-steel ring core has been considered for the experimental validation that has been carried by the EELAB group of the University of Ghent. A coil wound around the core creates a magnetic field at a frequency of 0.1 Hz. The induction field and the deformation, are measured in radial and tangential directions. In such a closed-loop magnetic circuit, magnetostrictive effects cease to be overridden by reluctivity forces and form effect can be decoupled from the (strictly speaking) magnetostriction [3]. Fig. 1 shows a comparison the of the computed and measured magnetostriction typical butterfly loops.

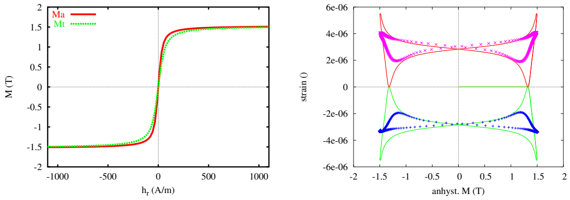

Fig. 1. (Left) Magnetisation curve in the axial and tangential direction, for one given phase. (Right) Magnetostriction curves obtained with the model (radial and tangential).

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