Influence of the manufacturing process on the magnetic properties of iron cores in induction machines

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Summary

The purpose of this paper is to show how the influence of the manufacturing process on the magnetic characteristics of stator and rotor core laminations can be *a posteriori* assessed from a voltage-current characteristic measured at no load on the finished machine. The implementation of this inverse problem is described and an application example is treated. The obtained tool allows induction machine manufacturers to evaluate manufacturing effects on the resulting magnetic properties of the magnetic steel into account, when they decide on which kind of electrical steel to select for each particular design application.

1 Introduction

We are interested in the calculation of effective *B-H* characteristics of processed (punched, stamped, cut...) laminations in induction machines, on basis of voltage *U* and current *I* measurements performed on that finished machine. The objective is to be able to compare a calculated **effective *B-H* characteristic** with the **reference *B-H* characteristic** of the unprocessed material. The latter is usually measured with an Epstein frame and provided by the manufacturer (See Fig. 1).
To this end, a tool is built to determine the resulting *B-H* characteristic of the steel in the machine that, when used as material characteristic in the analytical model of the machine, delivers a *U-I* characteristic matching the measured one. On the other hand, the tool should help manufacturers to take into account the effects expected from the manufacturing process, when they decide on the best material, the optimum design and the fabrication methods in the given application case.

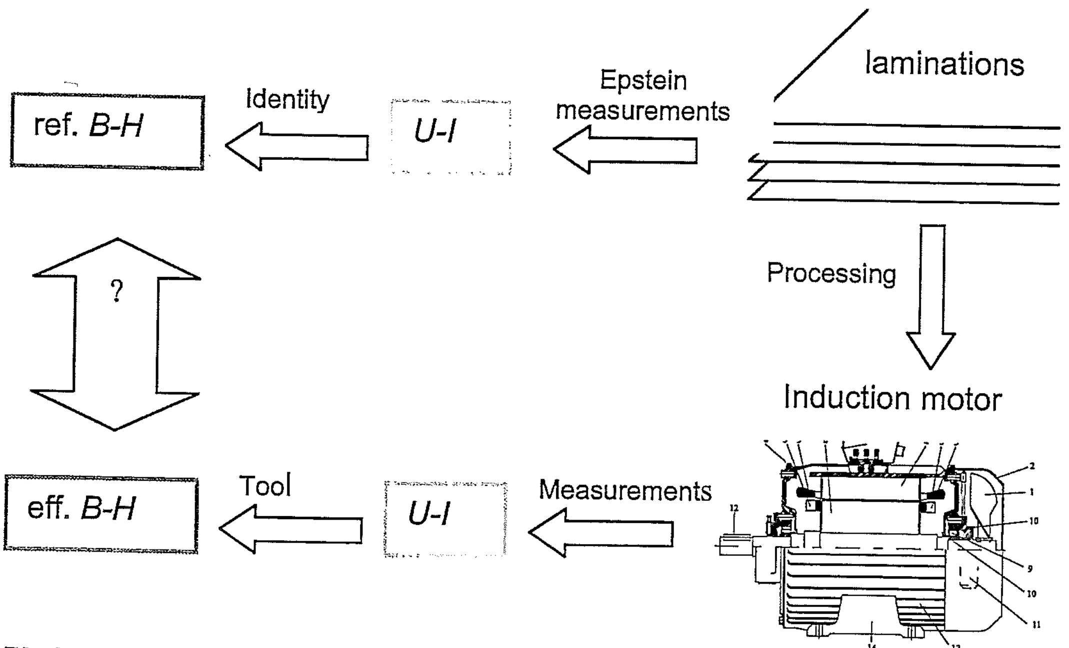

Fig.1. Block diagram describing the project. The "Tool" allows generating an effective *B-H* characteristic that can be compared with the reference *B-H* characteristic measured at the factory.

2 Magnetic *B-H* characteristic

It should be noted that the *reference B-H* characteristic (Epstein frame measurement) is also derived from a *U-I* measurements, exactly as we intend to do for the *effective B-H* characteristic of an electrical machine.
The *B-H* curve deduced from Epstein measurements is a useful representation of the magnetic material for magnetic field calculations based on conventional models or using FEM. It is precisely what (analytical or numerical) models neglecting hysteresis and eddy currents in the magnetic cores need as a material description. The resulting characteristic of the Epstein measurements for the specific magnetic losses *P* of the regarded magnetic steel as function of the maximum value of the induction *B* at a given frequency is mostly used in the calculation of the Fe-losses of the regarded electrical machine. The calculation of Fe-losses is generally done in the post-processing of magnetic field calculation results.
As the Epstein frame is an experimental set-up for the determination of the *B-H* characteristics and the magnetic losses of magnetic materials, it is conceived in such a way that the relation between the measurable quantities (*U* and *I*) and the measured quantities (*B* and *H*) is as close as possible to an identity (See the arrow "Identity in Fig. 1). To this point is the enforcement of the **sinusoidal-*B* condition** of importance, i.e. the current *I*(*t*) injected in the excitation coil is controlled in such a way that the voltage *U*(*t*) induced at the terminals of the measurement coil is sinusoidal. This allows monitoring directly B_{peak} and H_{peak} thanks to the relations

$$B_{peak} = U_{peak} / (2\pi f S)\ ,\ H_{peak}=N\, I_{peak} / L \qquad (2)$$

where *f*, S, N and L are respectively the frequency, the cross section of the measured sample, the number of turns of the excitation coil and the average magnetic path length of the field lines in the magnetic core of the Epstein frame made up from

Epstein strips. The procedure is repeated for different amplitudes of the excitation current, so as to generate the complete B_{peak}-H_{peak} characteristic, which is called simply *B-H* characteristic (anhysteretic or virgin curve) in the following. The material is demagnetised before each measurement.

3 The model for magnetic field calculation

The analytical model for magnetic field calculation that is used to represent asynchronous machines is the one developed by G. Müller and K. Hameyer. We denote it as the Ha-Mü model. It applies to rotating asynchronous machines in the range from 1 to 100 kW. The purpose of this model is not to provide a descriptive model of the asynchronous machine, but rather to evaluate, on basis of basic geometrical and design data of the machine, the effects of substituting one magnetic steel with another on the energetic properties of the regarded machine.

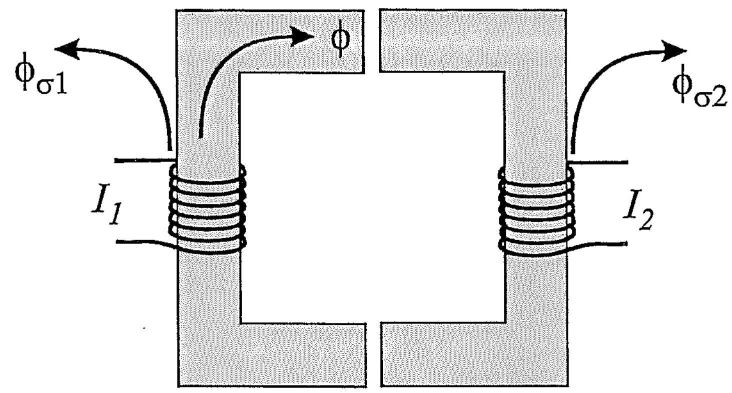

Fig. 2. Magnetic circuit of one pole pair of an asynchronous machine.

One pole of an induction machine can be considered as a magnetic circuit with three independent loops (see Fig. 2). The main loop carries the flux linkage ϕ which links with both stator and the rotor currents and crosses twice the air gap. The two other loops carry respectively the stator and rotor leakage fluxes $\phi_{\sigma 1}$ and $\phi_{\sigma 2}$. The purpose of the Ha-Mü model is to extract the relation between the flux linkage ϕ and the corresponding magneto-motive force $V=I_1+I_2$.

If D is the diameter of the air gap, the pole pitch, i.e. the arc length of the air gap spanned by one pole of the p pole pair machine, is defined by $\tau_p = \pi D/2p$. The flux linkage is then by definition $\phi = B_m \tau_p L$, with L the length of the machine and B_m the average induction over one pole of the machine. The flux linkage flows in magnetic materials only, except when it crosses the air gap. It is the quantity that is the more tightly linked with the *B-H* characteristic of the magnetic cores.

If higher spatial harmonics are neglected, one has the relation $B_m = 2\ B_{fsw}/\pi$ between B_m and the amplitude B_{fsw} of the fundamental spatial induction wave in the air gap. This simplification disregards the contribution of the higher odd order spatial harmonics (5, 7... in a three-phase machine). The error granted this way on the flux linkage is smaller than 10% (1/25+1/49+1/121+1/169+...).

One now needs a link between B_{fsw} and the amplitude B_{max} of the induction field in the air gap because peak values are the relevant quantities as look-up variables in *B-H* characteristics. One states that $B_{max} = \alpha B_m$ where α is an empirical factor. The above described procedure may be also generalized for the case of different values of B_m in the yoke and the teeth.

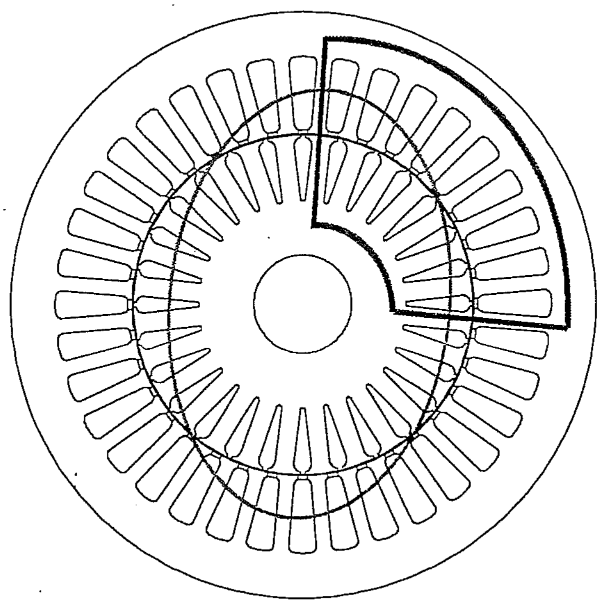

Fig. 3. Cross section of a 2 pole pair induction machine. The distance between the ellipse and the air gap represents the magnitude of the air gap field. The thick line represents the path over which the magnetic field is integrated to obtain the magneto-motive force.

In order to find the relation between the flux linkage ϕ and the magneto-motive force *V*, the magnetic field *H* is integrated along the field line corresponding with that maximum amplitude of the air gap field. This path (Fig. 3) consists of two stator and rotor teeth located, at the particular instant of time considered, closest to the apex of the air gap field. At the feet of these teeth, the maximal value B_{max} is thus assumed. These radial paths are connected with each other by tangential paths through the stator and rotor yokes. In the yokes, an average value of B_m calculated. In total, the considered path circles around one pole of the machine.

4 Link between *U-I* measurements and the parameters of the model

The ϕ-*V* relation calculated by the Ha-Mü model can be also obtained for a real machine by means of a no-load experiment. At the no-load experiment, the machine is supposed to be equilibrated. The equivalent circuit of one phase of an induction machine in no-load operation is given in Fig.4.

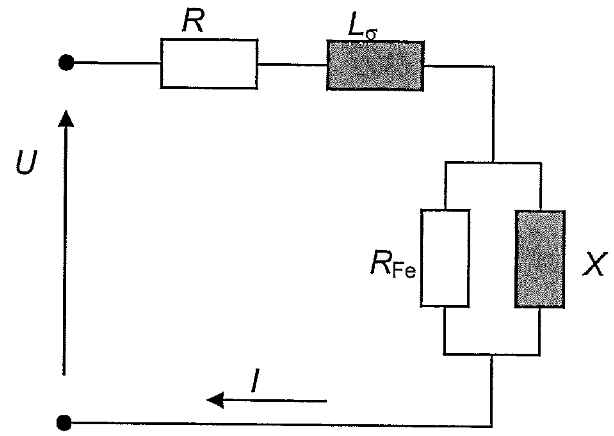

Fig. 4. Equivalent circuit diagram of one phase of an induction machine.

The measurable quantities are the phase voltage U, the line current I, the supplied power P and the phase resistance R. Joule losses are then defined by $P_{Cu} = 3\,R\,I^2$. The curve P–P_{Cu} in function of U^2 is nearly a straight line. Friction losses P_{Rb} are estimated by intersecting this curve with the $U^2=0$ axis. Iron losses are defined by $P_{Fe} = P - P_{Rb} - P_{Cu}$. As the stator phase leakage inductance $L_{\sigma 1}$ is assumed negligible, the voltage at the terminal of the magnetisation branch of the equivalent circuit is $U_\mu = U_1 - R_1 I_1$.

The resistance representing the iron losses in the equivalent circuit is defined by $R_{Fe} = 3\,(U_\mu)^2 / P_{Fe}$. In order to determine the main reactance X of the machine, one proceeds as follows. The total impedance of one phase of the machine at no-load seen from the supply terminals is

$$Z = R + \frac{X^2}{R_{\mathrm{Fe}}^2 + X^2} R_{\mathrm{Fe}} + j \frac{R_{\mathrm{Fe}}^2}{R_{\mathrm{Fe}}^2 + X^2} X$$

The active power delivered to the machine is then

$$P = P_{\mathrm{Rb}} + 3\Re(Z) I_1^2 = P_{\mathrm{Rb}} + 3\left(R + \frac{X^2}{R_{\mathrm{Fe}}^2 + X^2} \right) I_1^2$$

from where follows after elementary calculations

$$X = R_{\mathrm{Fe}} \left(\frac{3 R_{\mathrm{Fe}} I_1^2}{P_{\mathrm{Fe}}} - 1 \right)^{-\frac{1}{2}}$$

The magneto-motive force V and the flux ϕ in the main magnetic loop of the induction machine can now respectively be defined as

$$V = \sqrt{2}\, n I_\mu = \sqrt{2} \frac{n U_\mu}{X}$$

$$\phi = \sqrt{2} \frac{U_\mu}{p n \omega} 10^6$$

where n is the number of windings per pole pair and per phase. The $\sqrt{2}$ factors arise from the fact that V and ϕ are peak values, whereas U_μ and I_μ are effective values.

5 Calculation of effective B-H characteristic

The problem of determining effective *B-H* characteristics from *U-I* measurements seems at first sight to be what Mathematics calls an inverse problem and what engineers call non destructive testing, i.e. the class of problems that aim at retrieving from surface measurements relevant information about the internal constitution of a given system. There are however considerable differences between the problem as posed here and the standard classes of inverse problems:

- In impedance tomography and most medical imaging inverse problems, one tries to identify regions with considerably modified physical characteristics (magnetic permeability, conductivity, dielectric permittivity...). In this case, we have to detect a small deviation (a few percent) with respect to the *B-H* characteristic of the unprocessed steel.
- Standard inverse problems deal with linear systems, whereas electrical steel has a strongly non-linear *B-H* characteristic and is usually operated precisely in the range where this non-linearity is most visible.
- The engineering problems of tomography and non-destructive testing consist mainly in determining how many probes are required and where they should be positioned for an optimal reconstruction. In this case, the measurement set is entirely fixed from the beginning and consists in the *U-I* no-load characteristic measured on the finished machine. This amounts to probing the main flux path in the machine, which consists of well-defined teeth and yoke pieces. The measurement set is then quite poor and incomplete. In particular it does not allow identifying individually different *B-H* characteristics, found at different places along this path.

However, the effect of fabrication steps for a magnetic component made up of laminations electrical steel do clearly appear in *U-I* measurements, so that it must be possible to extract from them some valuable information about the material's alteration.

Inverse problems are a class of **optimisation** problems. One must here find an effective *B-H* characteristic that minimises the error between measured and calculated *U-I* characteristics. For this optimisation problem, as well as for the Ha-Mü model, we have worked with Mathcad.

Representation: The *B-H* characteristic is represented as a function $H(B)$. This is convenient for the Ha-Mü model, which assumes the air gap induction as the fundamental input variable. It has also the advantage that saturation gives this way no constraint on the correction. The magnetic characteristic is represented by the linear interpolation of couples of points. The correction $dH(B)$ has been chosen multiplicative, i.e. $H(B) = H_0(B)\, dH(B)$ because the H-scale of magnetic characteristics is usually logarithmic. In other words, the correction is additive in the logarithmic scale. In this equation, $H_0(B)$ is an **initial guess** for the *B-H* characteristic. It makes sense to choose the *B-H* characteristic of the unprocessed material (the so-called reference *B-H* characteristic) as initial guess, but it is not mandatory. In that case, the correction *dH*(*B*) is directly equal to the effect of the processing of the laminations. Both characteristics, i.e. *H*(*B*) and *dH*(*B*), are represented by linear interpolation of a set of (*B,H*) pairs on the range [0, 2] Tesla, and extrapolated linearly for larger values of the induction.

Optimisation method: The optimisation method that has been implemented is the Gauss-Seidel iteration. When using the Ha-Mü model with 1 material (same *B-H* characteristic in teeth and yokes), the convergence of the Gauss-Seidel iteration is however observed to be difficult and unreliable, because the relation between the *B-H* characteristic and the *U-I* characteristic is made more complicated (less regular) by the different magnetisation levels occurring in teeth and yokes. On the other hand, the Ha-Mü model with 2 material gives regular relations between the *B-H* characteristic and the *U-I* characteristic and yields no convergence problem at all. Considering that

- it is technically relevant to consider *a priori* that the material has been affected differently by processing in teeth and yokes,
- Mathcad offers very few possibilities to implement more involved iteration methods,

the standard Ha-Mü model with 2 materials is always used.

Interactivity: Due to Mathcad limitations, the optimisation process cannot however be fully automated. The user's intervention is required at each iteration step. On the other hand, as Mathcad provides minimisation tools for functions of one variable only. These limitations have however advantages as well. The user can progressively learn about the characteristics of the objective function and adapt the minimisation strategy at each iteration in function of the last results. Trial and error is possible. This helps figuring out what efficient optimisation strategies should be. The user can also check the physical significance of the obtained *B-H* characteristic and re-orient the optimisation process accordingly if they are not realistic; i.e, it is the role of the user to decide at each optimisation step which *B-H* characteristic to modify, according to the physical significance of the alteration proposed by the program. The tool has been designed so as to assist as much as possible the qualified user in taking good decisions and to deliver at the end a relevant global measure of the *B-H* characteristic deviation that is imputable to the processing of the laminations.

A characteristic of the Gauss-Seidel method is that the search directions are fixed. It is therefore desirable to have a tool to seek for efficient search directions. The search directions are determined by the number and the location of the interpolation points (left column of the correction parameter matrices dH). The interpolation with n points of the multiplicative correction can be seen as the sum of n triangular (hat) functions. The location of the interpolation points must be chosen in such a way that the images of the corresponding hat functions are aligned with (i.e. not orthogonal to) the distribution of the error to minimise. This can be done by considering the sensitivity indicator, i.e. the figure that shows the residual error (red curve) and the image by the Ha-Mü model of the incremental hat function associated with the current optimisation step (blue curve).

6 Application of the model for calculation of effective *B-H* of the magnetic steel in an electrical machine

The optimisation parameters, say of the magnetic material, are contained in a matrix *dH* called correction parameter matrix. The first column contains the interpolations points (in Tesla) and the second column the interpolation values (no unit). Since the correction is a multiplicative one, the neutral correction (i.e., no correction) has only 1's in the second column. The number of rows of the matrix *dH* is free. A minimum of 3 rows is however necessary. Experience has shown that having 5 or 6 rows is a good choice. The correction *dH*(*B*) is a piecewise linear interpolated function defined on basis of those optimisation parameters.

A characteristic of the Gauss-Seidel method is that the search directions are fixed. It is therefore desirable to devise additional tools to seek for efficient search directions. the search directions are determined by the number and the location of the interpolation points (left column of the correction parameter matrices *dH*). The interpolation with n points of the multiplicative correction can be seen as the sum of n triangular (hat) functions.
On the other hand, one knows by taking a look to measured *B-H* characteristics of processed materials that a limited number of hat functions allow giving a sufficiently detailed representation of the modification brought to the *B-H* characteristic by the industrial processing of assembling stamped laminations (resulting effective B-H characteristic). The location of the interpolation points must be chosen in such a way that the images of the corresponding hat functions are aligned with (i.e. not orthogonal to) the distribution of the error to minimise. This can be done by considering the sensitivity indicator, i.e. the figure that shows the residual error (solid curve) and the image by the Ha-Mü model of the incremental hat function associated with the current optimisation step (dotted curve), Fig. 6.1.

The first step of the optimisation consists in locating the interpolation points. Starting from the situation presented in Fig. 5, one obtains after a few trial and errors the situation of Fig. 6. When the matching of the two ranges is visually satisfactory, the Gauss-Seidel optimisation step is carried over. Proceeding the same way, one obtains within a few minutes, after iterating over the complete set of optimisation parameters, the effective *B-H* characteristic depicted in Fig. 7. The final error is $4.254\ 10^{-2}$, which means a total decrease of 96.3% with respect to the initial error 1.149.

$$dH := \begin{pmatrix} 0 & 1 \\ 0.5 & 1 \\ 1 & 1 \\ 1.5 & 1 \\ 2 & 1 \\ 5 & 1 \end{pmatrix}$$

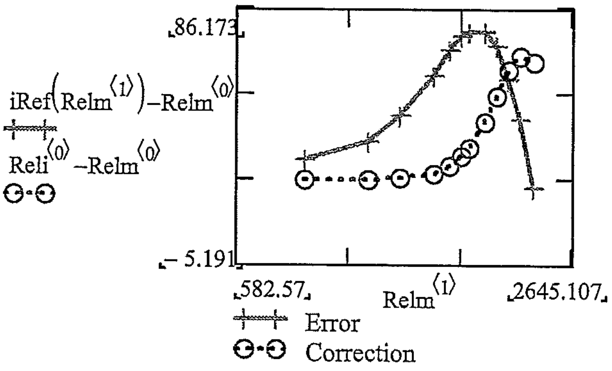

Fig. 5. Sensitivity indicator at the initial location of the interpolation points.

$$dH := \begin{pmatrix} 0 & 1 \\ 0.5 & 1 \\ 1.4 & 1 \\ 1.6 & 1 \\ 2 & 1 \\ 5 & 1 \end{pmatrix}$$

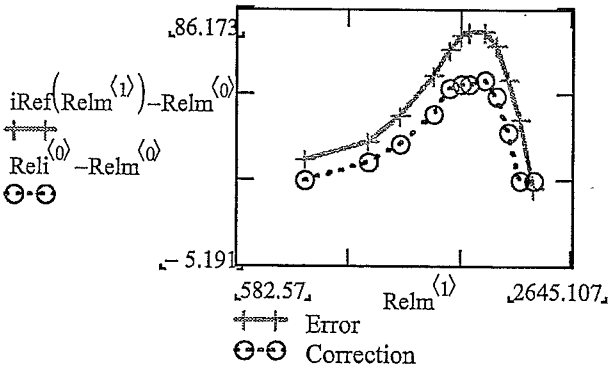

Fig. 6. Sensitivity indicator with adapted interpolation points.

$$dH := \begin{pmatrix} 0 & 2.022 \\ 0.8 & 4.399 \\ 1.4 & 8.283 \\ 1.6 & 1.52 \\ 1.8 & 1.013 \\ 5 & 1 \end{pmatrix}$$

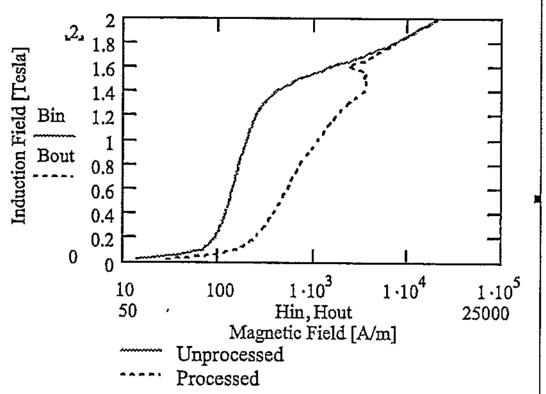

Fig. 7. Final value of the optimisation parameter matrix *dH*, and comparison of the effective (processed) and reference (unprocessed) *B-H* characteristics.

7 Conclusion

A tool to evaluate the effective *B-H* characteristic of magnetic cores in an inductio motor from *U-I* measurement performed on the finished machine has been presente This tool helps machine designers to take alterations due to the manufacturin process into consideration when they have to decide on the best-suited electric steel, the design and the best fabrication methods.